MATH 12003 **COMPARISON TESTS** **SECTION 12.4**

Comparison Test: Suppose that (a_n) and (b_n) are sequences of non-negative terms for which (eventually) $a_n \leq b_n$. The following statements are true:

- If $\sum a_n$ diverges, then $\sum b_n$ diverges.
- If $\sum b_n$ converges, then $\sum a_n$ converges.

EXAMPLES: Determine whether the series converges or diverges.

1. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n - \sqrt{n}}$

2. $\displaystyle\sum_{n=1}^{\infty} \frac{7}{3^n + 4}$

3. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 - 1}{3n^4 + 1}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n - 4}{n^3 + n + 1}$

6. $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

Limit Comparison Test: Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both diverge.

HINT: when testing many series, we can find a suitable comparison series by taking the highest powers in the denominator and numerator.

EXAMPLES: Determine whether the series converges or diverges.

1. $\displaystyle\sum_{n=1}^{\infty} \frac{5}{2n+3}$

2. $\displaystyle\sum_{n=2}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}}$

3. $\displaystyle\sum_{n=2}^{\infty} \frac{n}{n^3-n^2-1}$

4. $\displaystyle\sum_{n=1}^{\infty} \frac{2n^2+7n}{3^n\left(n^2+5n-1\right)}$

5. $\displaystyle\sum_{n=1}^{\infty} \frac{n+5}{\sqrt[3]{n^7+n^2}}$