Comparison Test: Suppose that \((a_n)\) and \((b_n)\) are sequences of non-negative terms for which (eventually) \(a_n \leq b_n\). The following statements are true:

- If \(\sum a_n\) diverges, then \(\sum b_n\) diverges.
- If \(\sum b_n\) converges, then \(\sum a_n\) converges.

Examples: Determine whether the series converges or diverges.

1. \(\sum_{n=2}^{\infty} \frac{1}{n - \sqrt{n}}\)

2. \(\sum_{n=1}^{\infty} \frac{7}{3^n + 4}\)

3. \(\sum_{n=1}^{\infty} \frac{n^2 - 1}{3n^4 + 1}\)

4. \(\sum_{n=1}^{\infty} \frac{\ln n}{n}\)

5. \(\sum_{n=1}^{\infty} \frac{n - 4}{n^3 + n + 1}\)

6. \(\sum_{n=1}^{\infty} \frac{n!}{n^n}\)
**Limit Comparison Test:** Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If
\[
\lim_{n \to \infty} \frac{a_n}{b_n} = c
\]
where $c$ is a finite number and $c > 0$, then either both series converge or both diverge.

**HINT:** when testing many series, we can find a suitable comparison series by taking the highest powers in the denominator and numerator.

**EXAMPLES:** Determine whether the series converges or diverges.

1. $\sum_{n=1}^{\infty} \frac{5}{2n + 3}$

2. $\sum_{n=2}^{\infty} \frac{2n^2 + 3n}{\sqrt{5 + n^5}}$

3. $\sum_{n=2}^{\infty} \frac{n}{n^3 - n^2 - 1}$

4. $\sum_{n=1}^{\infty} \frac{2n^2 + 7n}{3^n (n^2 + 5n - 1)}$

5. $\sum_{n=1}^{\infty} \frac{n + 5}{\sqrt{n^7 + n^2}}$