TEST FOR DIVERGENCE: If $\lim_{n\to\infty} a_n$ does not exist or if $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

GEOMETRIC SERIES: Each term of a geometric series is obtained from the preceding one by multiplying it by a common ratio r. Namely,

$$a + ar + ar^2 + ar^3 + ar^4 + \cdots + ar^n + \cdots = \sum_{n=1}^{\infty} ar^{n-1} \qquad a \neq 0.$$

The geometric series converges when $|r| < 1$ and its sum is given by

$$\sum_{n=1}^{\infty} ar^{n-1} = \frac{a}{1-r} \qquad |r| < 1.$$

When $|r| \geq 1$, the geometric series is divergent.

p**-SERIES**: The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if $p > 1$ and is divergent if $p \leq 1$.

THE INTEGRAL TEST: Suppose f is a continuous, positive, decreasing function on $[1, \infty)$ and let $a_n = f(n)$. Then the series $\sum_{n=1}^{\infty} a_n$ is convergent if and only if the improper integral $\int_1^{\infty} f(x)\,dx$ is convergent.

- If $\int_1^{\infty} f(x)\,dx$ is convergent, then $\sum_{n=1}^{\infty} a_n$ is convergent.
- If $\int_1^{\infty} f(x)\,dx$ is divergent, then $\sum_{n=1}^{\infty} a_n$ is divergent.

COMPARISON TEST: Suppose that (a_n) and (b_n) are sequences of non-negative terms for which (eventually) $a_n \leq b_n$. The following statements are true:

- If $\sum a_n$ diverges, then $\sum b_n$ diverges.
- If $\sum b_n$ converges, then $\sum a_n$ converges.

LIMIT COMPARISON TEST: Suppose that $\sum a_n$ and $\sum b_n$ are series with positive terms. If

$$\lim_{n\to\infty} \frac{a_n}{b_n} = c$$

where c is a finite number and $c > 0$, then either both series converge or both diverge.

ALTERNATING SERIES TEST: Let (a_n) be decreasing sequence of positive numbers such that $\lim a_n = 0$. Then the alternating series $\sum_{n=1}^{\infty} (-1)^n a_n$ converges.

THE RATIO TEST

- If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.
- If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = L > 1$ or if $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
- If $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$, then the ratio test is inconclusive and therefore no conclusion can be made about the convergence or divergence of the series. In this case, use another test to determine convergence or divergence.

THE ROOT TEST

- If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L < 1$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent.
- If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = L > 1$ or if $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty$, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.
- If $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$, then the root test is inconclusive and therefore no conclusion can be made about the convergence or divergence of the series. In this case, use another test to determine convergence or divergence.