Definition: A power series is a series of the form
\[ \sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + \cdots \]
where \( x \) is a variable and the \( c_n \)'s are the coefficients of the series. For each fixed \( x \), the above series is a series of constants for which we can determine convergence or divergence. Note that a power series may converge for some values of \( x \) and diverge for other values of \( x \).

Definition: A power series about \( a \) is a series of the form
\[ \sum_{n=0}^{\infty} c_n (x - a)^n = c_0 + c_1 (x - a) + c_2 (x - a)^2 + \cdots \]
where we adopt the convention that \((x - a)^0 = 1\) even when \( x = a \). Notice that when \( x = a \) all of the terms except the first one are zero. Hence, the power series about \( a \) always converges when \( x = a \).

A power series in \( x \) can be viewed as a function of \( x \), \( f(x) = \sum_{n=0}^{\infty} c_n x^n \), where the domain of \( f \) is the set of all \( x \) for which the power series converges.

Theorem 1. For a given power series \( \sum_{n=0}^{\infty} c_n (x - a)^n \) there are only three possibilities:
- The series converges only when \( x = a \).
- The series converges for all \( x \).
- There is a positive number \( R \) such that the series converges if \( |x - a| < R \) and diverges if \( |x - a| > R \).

The number \( R \) in the above theorem is called the radius of convergence of the power series. When the series converges for only \( x = a \) then we say that \( R = 0 \), and when it converges for all \( x \) we say that \( R = \infty \).

The interval of convergence of a power series is the interval that consists of all values of \( x \) for which the series converges. When the series converges at a single point, then the interval is a single point \( a \). When the series converges for all \( x \), the interval is \( (-\infty, \infty) \). When the series converges for certain values of \( x \) the interval is \( a - R < x < a + R \). Note that series might converge at one or both endpoints or it might diverge at both endpoints.
EXAMPLES: Find the radius of convergence and the interval of convergence of the series.

1. \( \sum_{n=1}^{\infty} \frac{x^n}{n} \)

2. \( \sum_{n=0}^{\infty} \frac{(-1)^n(x + 1)^n}{2^n} \)

3. \( \sum_{n=1}^{\infty} \frac{(3x - 2)^n}{n3^n} \)

4. \( \sum_{n=1}^{\infty} n^n x^n \)

5. \( \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!} \)