

Arc Length Formula for $y = f(x)$: If f' is continuous on $[a, b]$, then the length of the curve $y = f(x)$, $a \leq x \leq b$, is

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

Using our other notation for derivatives, we can rewrite the arc length formula as

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx$$

Arc Length Formula for $x = g(y)$: If g' is continuous on $[c, d]$, then the length of the curve $x = g(y)$, $c \leq y \leq d$, is

$$L = \int_c^d \sqrt{1 + [g'(y)]^2} dy$$

Using our other notation for derivatives, we can rewrite the arc length formula as

$$L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

EXAMPLES: Find the length of the curve.

1. $y = \ln(\cos x)$, $0 \leq x \leq \frac{\pi}{3}$

2. $y^2 = 4x$, $0 \leq y \leq 2$