Definition of the Area of a Surface of Revolution: Let $y = f(x)$ have a continuous derivative on the interval $[a,\ b]$. The area S of the surface of revolution formed by revolving the graph of f about a horizontal or vertical axis is

$$S = 2\pi \int_a^b r(x)\ \sqrt{1 + [f'(x)]^2}\ dx \qquad (y \text{ is a function of } x)$$

where $r(x)$ is the distance between the graph of f and the axis of revolution. If $x = g(y)$ has a continuous derivative on the interval $[c,\ d]$, then the surface area is

$$S = 2\pi \int_c^d r(y)\ \sqrt{1 + [g'(y)]^2}\ dy \qquad (x \text{ is a function of } y)$$

where $r(y)$ is the distance between the graph of g and the axis of revolution.

EXAMPLES:

1. Find the area of the surface obtained by rotating $9x = y^2 + 18, \quad 2 \leq x \leq 6$ about the $x-$axis.

2. Find the area of the surface obtained by rotating $x = 1 + 2y^2, \quad 1 \leq y \leq 2$ about the $x-$axis.

3. Find the area of the surface obtained by rotating $y = 1 - x^2, \ \ 0 \leq x \leq 1$ about the $y-$axis.