Moments and Center of Mass: One-Dimensional System: Let the point masses $m_1, m_2, \ldots, m_n$ be located at $x_1, x_2, \ldots, x_n$.

- The **moment about the origin** is $M_0 = m_1x_1 + m_2x_2 + \cdots + m_nx_n$.
- The **center of mass** is $\bar{x} = \frac{M_0}{m}$, where $m = m_1 + m_2 + \cdots + m_n$ is the total mass of the system.

**Example:** Find the center of mass of the linear system shown below:

![Linear System Diagram]

Moments and Center of Mass: Two-Dimensional System: Let the point masses $m_1, m_2, \ldots, m_n$ be located at $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$.

- The **moment about the $y$-axis** is $M_y = m_1x_1 + m_2x_2 + \cdots + m_nx_n$.
- The **moment about the $x$-axis** is $M_x = m_1y_1 + m_2y_2 + \cdots + m_ny_n$.
- The **center of mass** $(\bar{x}, \bar{y})$ (or center of gravity) is

$$
\bar{x} = \frac{M_y}{m} \quad \text{and} \quad \bar{y} = \frac{M_x}{m}
$$

where $m = m_1 + m_2 + \cdots + m_n$ is the total mass of the system.

**Example:** Find the center of mass of the system of point masses $m_1 = 6, m_2 = 3, m_3 = 2$, and $m_4 = 9$, located at $(3, -2), (0, 0), (-5, 3)$, and $(4, 2)$.
A **planar lamina** is a thin, flat plate of material of uniform density. (For a planar lamina density is a measure of mass per unit of area.)

**Moments and Center of Mass of a Planar Lamina:** Let $f$ and $g$ be continuous functions such that $f(x) \geq g(x)$ on $[a, b]$ and consider the planar lamina of uniform density $\rho$ bounded by the graphs of $y = f(x)$ and $y = g(x)$, and $a \leq x \leq b$.

- The **moments about the $x$–axis and $y$–axis** are
  
  \[
  M_x = \int_a^b \rho \left( \frac{f^2(x) - g^2(x)}{2} \right) \, dx
  \]
  \[
  M_y = \int_a^b \rho x (f(x) - g(x)) \, dx
  \]

- The **center of mass** $(\bar{x}, \bar{y})$ is given by
  
  \[
  \bar{x} = \frac{M_y}{m} \quad \text{and} \quad \bar{y} = \frac{M_x}{m},
  \]
  
  where $m = \int_a^b \rho (f(x) - g(x)) \, dx$ is the mass of the region.

**Example:** Find the center of mass of the lamina of uniform density $\rho$ bounded by the graph of $f(x) = 4 - x^2$ and the $x$–axis.
Since the density $\rho$ is a common factor of both the moments and the mass, it cancels out of the quotients representing the coordinates of the center of mass. Thus, the center of mass of a lamina of uniform density depends only on the shape of the lamina and not on its density. For this reason, the point $(\bar{x}, \bar{y})$ is sometimes called the center of mass of a region in the plane, or the **centroid** of the region. In other words, to find the centroid of a region in the plane, you simply assume that the region has a constant density of $\rho = 1$ and compute the corresponding center of mass.

**EXAMPLE:** Find the centroid of the region bounded by the graphs of $f(x) = 4 - x^2$ and $g(x) = x + 2$.

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**The Theorem of Pappus:** Let $R$ be a region in a plane and let $L$ be the line in the same plane such that $L$ does not intersect the interior of $R$. If $r$ is the distance between the centroid of $R$ and the line, then the volume $V$ of the solid of revolution formed by revolving $R$ about the line is

$$V = 2\pi r A$$

where $A$ is the area of $R$. (Note that $2\pi r$ is the distance traveled by the centroid as the region is revolved about the line.)

**EXAMPLE:** Find the volume of the torus formed by revolving the circular region bounded by $(x - 2)^2 + y^2 = 1$ about the $y$–axis.