Topic 2: Clock Arithmetic

- **Addition**: The sum of two numbers is found by adding the two numbers as whole number, except that when this sum is greater than the clock number, we subtract the clock number.

  For example,

$$5 \oplus 6 \ \ (7 \text{ clock}) = 5 + 6 - 7 = 4$$
$$3 \oplus 9 \ \ (10 \text{ clock}) = 3 + 9 - 10 = 2$$

  Example 1: Calculate the following in the indicated clock:

  (a) $7 \oplus 5 \ \ (9 \text{ clock})$ (b) $3 \oplus 2 \ \ (4 \text{ clock})$

- **Subtraction**:

  1. **Counting backwards**:

  2. **Missing Addend Approach**:

$$3 \ominus 4 = ? \ \ (6 \text{ clock}) \quad \text{if and only if} \quad 4 \ \oplus \ ? = 3 \ \ (6 \text{ clock})$$
$$2 \ominus 5 = ? \ \ (6 \text{ clock}) \quad \text{if and only if} \quad 5 \ \oplus \ ? = 2 \ \ (6 \text{ clock})$$

  3. **Adding the opposite**: The opposite of a is the number which we add to a in order to get the clock number.

$$6 \ominus 9 \ \ (11 \text{ clock}) =$$
$$3 \ominus 12 \ \ (15 \text{ clock}) =$$
$$3 \ominus 4 \ \ (6 \text{ clock}) =$$

- **Multiplication**:

  1. **Repeated Addition**:

$$3 \otimes 2 \ \ (4 \text{ clock}) = 2 \oplus 2 \oplus 2 = 6 - 4 = 2$$
$$5 \otimes 4 \ \ (6 \text{ clock}) = 4 \oplus 4 \oplus 4 \oplus 4 \oplus 4 = 2$$

  2. **Alternative Method**: To find $a \otimes b$ in any clock number, first multiply a and b as whole numbers. If this product exceeds the clock number, divide by the clock number. The remainder will be the clock product.

- **Division**:

  1. **Missing Factor Approach**:

$$4 \div 3 = ? \ \ (5 \text{ clock}) \quad \text{if and only if} \quad 3 \ \otimes \ ? = 4 \ \ (5 \text{ clock})$$

  2. **Multiplying by the reciprocal**: The reciprocal of a is the number we need to multiply to a in order to get 1.

     For example, in a 5-clock, 2 is the reciprocal of 3; and 4 is the reciprocal of 4.

$$2 \div 4 \ \ (5 \text{ clock}) = 2 \otimes 4 \ \ (5 \text{ clock}) = 3$$
$$3 \div 2 \ \ (5 \text{ clock}) = 3 \otimes 3 \ \ (5 \text{ clock}) = 4$$

     NOTE 1: Not every clock number has a reciprocal.
     NOTE 2: In composite number clocks, some divisions are impossible.