Section 5.2: GCF and LCM

- **Greatest Common Factor (GCF)**: of two (or more) nonzero whole numbers is the largest whole number that is a factor of both (or all) of the numbers. The GCF of a and b is denoted GCF(a, b).

  1. **Set Intersection Method**: List all factors for a in one set. List all factors of b in another set. Take the intersection of these two sets. The GCF is the largest number in the intersection.

  2. **Prime Factorization Method**: Express the prime factorization of each number. The GCF is the product of the <u>common</u> primes to their <u>smallest</u> exponent.

Example 1. Find the GCF(291060, 858000).

Theorem 1 *If a and b are whole numbers, with $a \geq b$, then*

$$GCF(a, b) = GCF(a - b, b).$$

- **Least Common Multiple (LCM)**: of two (or more) nonzero whole numbers is the smallest nonzero whole number that is a multiple of each (or all) of the numbers. The LCM of a and b is denoted LCM(a, b).

  1. **Set Intersection Method**: List the first several nonzero multiples of a in one set. List the first several nonzero multiples of b in another set. Take the intersection of these sets. The LCM is the smallest number in the intersection.

  2. **Prime Factorization Method**: Express the prime factorization of each number. The LCM is the product of all primes appearing in the factorizations to their highest exponent.

Example 2. Find the LCM(291060, 858000).

Theorem 2 *Let a and b be any two whole numbers. Then*

$$GCF(a, b) \cdot LCM(a, b) = a \cdot b.$$

Example 3. If $a = 2^3 \cdot 3^2 \cdot 5^4 \cdot 7^3$, $GCF(a, b) = 2^2 \cdot 3^2 \cdot 7^3$, and $LCM(a, b) = 2^3 \cdot 3^8 \cdot 5^4 \cdot 7^4 \cdot 11$, then find b.

Theorem 3 *Suppose that a counting number n is expressed as a product of distinct primes with their respective exponents; say,*

$$n = p_1^{n_1} \cdot p_2^{n_2} \cdots p_m^{n_m}$$

Then the number of factors of n is the product

$$(n_1 + 1)(n_2 + 1) \cdots (n_m + 1).$$

Example 4. How many factors does 173250 have?