Section 9.1: Rational Numbers

- **Rational numbers**: are of the form \( \frac{a}{b} \) where \( a \) and \( b \) are both integers and \( b \neq 0 \).

\[
\mathbb{Q} = \left\{ \frac{a}{b} \mid a \text{ and } b \text{ are integers}, b \neq 0 \right\}.
\]

NOTE: Every integer, whole number, and fraction is a rational number.

- Let \( \frac{a}{b} \) be any rational number and \( n \) be a nonzero integer. Then
  \[
  \frac{a}{b} = \frac{na}{nb} = \frac{an}{bn}.
  \]

- Let \( \frac{a}{b} \) be any rational number. Then
  \[
  -\frac{a}{b} = -\frac{a}{b} = \frac{a}{-b}.
  \]

- \( \frac{a}{b} \) is a positive rational number when either \( a \) and \( b \) are both positive or when \( a \) and \( b \) are both negative.

- \( \frac{a}{b} \) is a negative rational number when \( a \) and \( b \) have different signs (one negative and one positive).

**Properties of Rational Number Addition**

- **Closure Property**: Rational number + Rational number = Rational number.

- **Commutative Property**: \( \frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b} \).

- **Associative Property**: \( \frac{a}{b} + \left( \frac{c}{a} + \frac{e}{f} \right) = \left( \frac{a}{b} + \frac{c}{a} \right) + \frac{e}{f} \).

- **Identity Property**: \( \frac{a}{b} + 0 = \frac{a}{b} = 0 + \frac{a}{b} \).

- **Additive Inverse Property**: For every rational number \( \frac{a}{b} \), there exists a unique rational number \( -\frac{a}{b} \) such that
  \[
  \frac{a}{b} + -\frac{a}{b} = 0 = -\frac{a}{b} + \frac{a}{b}.
  \]

\( -\frac{a}{b} \) is called the **additive inverse**.
**Properties of Rational Number Multiplication**

- **Closure Property**: Rational number \( \cdot \) Rational number = Rational number.

- **Commutative Property**: \( \frac{a}{b} \cdot \frac{c}{d} = \frac{c}{d} \cdot \frac{a}{b} \).

- **Associative Property**: \( \frac{a}{b} \cdot \left( \frac{c}{d} \cdot \frac{e}{f} \right) = \left( \frac{a}{b} \cdot \frac{c}{d} \right) \cdot \frac{e}{f} \).

- **Identity Property**: \( \frac{a}{b} \cdot 1 = \frac{a}{b} = 1 \cdot \frac{a}{b} \).

- **Multiplicative Inverse Property**: For every nonzero rational number \( \frac{a}{b} \), there exists a unique rational number \( \frac{b}{a} \) such that
  \[
  \frac{a}{b} \cdot \frac{b}{a} = 1 = \frac{b}{a} \cdot \frac{a}{b}.
  \]

  \( \frac{b}{a} \) is called the **multiplicative inverse** or **reciprocal**.

- **Distributive Property**:
  \[
  \frac{a}{b} \left( \frac{c}{d} + \frac{e}{f} \right) = \frac{a}{b} \cdot \frac{c}{d} + \frac{a}{b} \cdot \frac{e}{f}
  \]

**Cross Multiplication of Rational Number Inequality**: Let \( \frac{a}{b} \) and \( \frac{c}{d} \) be rational numbers with \( b > 0 \) and \( d > 0 \). Then

\[
\frac{a}{b} < \frac{c}{d} \quad \text{if and only if} \quad ad < bc.
\]

**NOTE**: BE CAREFUL!!! Both denominators must be positive in order to use this. DO NOT use if one of the denominators is negative unless you first the rational number using \( \frac{a}{b} = \frac{-a}{b} \).

**EXAMPLES**: Put the appropriate sign \( (<, =, >) \) between each pair of rational numbers to make a true statement.

1. \(-\frac{1}{3} < \frac{5}{4}\)
2. \(-\frac{5}{6} < \frac{-11}{12}\)
3. \(-\frac{12}{15} < \frac{36}{45}\)
4. \(-\frac{3}{12} < \frac{4}{20}\)