Section 9.1: Rational Numbers

- **Rational numbers:** are of the form $\frac{a}{b}$ where a and b are both integers and $b \neq 0$.

$$\mathbb{Q} = \left\{ \frac{a}{b} \mid a \text{ and } b \text{ are integers}, b \neq 0 \right\}.$$

  NOTE: Every integer, whole number, and fraction is a rational number.

- Let $\frac{a}{b}$ be any rational number and n be a nonzero integer. Then

$$\frac{a}{b} = \frac{na}{nb} = \frac{an}{bn}.$$

- Let $\frac{a}{b}$ be any rational number. Then

$$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}.$$

- $\frac{a}{b}$ is a positive rational number when either a and b are both positive or when a and b are both negative.
- $\frac{a}{b}$ is a negative rational number when a and b have different signs (one negative and one positive).

Properties of Rational Number Addition

- **Closure Property**: Rational number + Rational number = Rational number.
- **Commutative Property**: $\frac{a}{b} + \frac{c}{d} = \frac{c}{d} + \frac{a}{b}$.
- **Associative Property**: $\frac{a}{b} + \left(\frac{c}{d} + \frac{e}{f}\right) = \left(\frac{a}{b} + \frac{c}{d}\right) + \frac{e}{f}$.
- **Identity Property**: $\frac{a}{b} + 0 = \frac{a}{b} = 0 + \frac{a}{b}$.
- **Additive Inverse Property**: For every rational number $\frac{a}{b}$, there exists a unique rational number $-\frac{a}{b}$ such that

$$\frac{a}{b} + -\frac{a}{b} = 0 = -\frac{a}{b} + \frac{a}{b}.$$

  $-\frac{a}{b}$ is called the **additive inverse**.

Properties of Rational Number Multiplication

- **Closure Property**: Rational number · Rational number = Rational number.
- **Commutative Property**: $\frac{a}{b} \cdot \frac{c}{d} = \frac{c}{d} \cdot \frac{a}{b}$.
- **Associative Property**: $\frac{a}{b} \cdot \left(\frac{c}{d} \cdot \frac{e}{f}\right) = \left(\frac{a}{b} \cdot \frac{c}{d}\right) \cdot \frac{e}{f}$.
- **Identity Property**: $\frac{a}{b} \cdot 1 = \frac{a}{b} = 1 \cdot \frac{a}{b}$.
- **Multiplicative Inverse Property**: For every nonzero rational number $\frac{a}{b}$, there exists a unique rational number $\frac{b}{a}$ such that

$$\frac{a}{b} \cdot \frac{b}{a} = 1 = \frac{b}{a} \cdot \frac{a}{b}.$$

  $\frac{b}{a}$ is called the **multiplicative inverse** or **reciprocal**.
- **Distributive Property**:

$$\frac{a}{b}\left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \cdot \frac{c}{d} + \frac{a}{b} \cdot \frac{e}{f}$$

Cross Multiplication of Rational Number Inequality: Let $\frac{a}{b}$ and $\frac{c}{d}$ be rational numbers with $b > 0$ and $d > 0$. Then

$$\frac{a}{b} < \frac{c}{d} \qquad \text{if and only if} \qquad ad < bc.$$

NOTE: BE CAREFUL!!! Both denominators must be positive in order to use this. DO NOT use if one of the denominators is negative unless you first the rational number using $\frac{a}{-b} = \frac{-a}{b}$.

EXAMPLES: Put the appropriate sign ($<$, $=$, $>$) between each pair of rational numbers to make a true statement.

1. $-\frac{1}{3}$ $\qquad$ $\frac{5}{4}$

2. $\frac{-5}{6}$ $\qquad$ $\frac{-11}{12}$

3. $\frac{-12}{15}$ $\qquad$ $\frac{36}{-45}$

4. $-\frac{3}{12}$ $\qquad$ $\frac{4}{-20}$