Section 9.2: Real Numbers

- **Irrational Numbers**: Set of numbers which have an infinite non-repeating decimal approximation. An irrational number cannot be written as a fraction.
- **Real Numbers**: are the union of rational and irrational numbers.
- **Square root**: Let a be a nonnegative real number. Then the **square root** of a (**principal square root**), denoted $\sqrt{a}$, is defined as

$$\sqrt{a} = b \quad \text{where} \quad b^2 = a, \text{ and } b \geq 0.$$

- **n-th roots**: Let a be a real number and n be a positive integer.

1. If $a \geq 0$, then $\sqrt[n]{a} = b$ if and only if $b^n = a$ and $b \geq 0$.
2. If $a < 0$ and n is odd, then $\sqrt[n]{a} = b$ if and only if $b^n = a$.
3. If $a < 0$ and n is even, then $\sqrt[n]{a}$ is undefined.

- **Rational exponents and Radicals**: Let a be any real number and $\frac{m}{n}$ be a rational number is simplest form. Then

$$a^{m/n} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m.$$

NOTE: In a rational exponent, the numerator indicates the power and the denominator indicates the root.

Examples: Simplify each expression

1. $8^{2/3} =$
2. $16^{3/4} =$
3. $27^{1/3} =$
4. $(-64)^{2/3} =$

NOTE: The properties that held for integer exponents also hold for rational exponents.

Solving Inequalities

- **Addition Property of Inequality:** If a, b, and c are real numbers, then

$$a < b \quad \text{and} \quad a + c < b + c$$

are equivalent. (That is, you can add or subtract the same quantity on both sides of the inequality without changing the solution.)

- **Multiplication Property of Inequality:** For all real numbers a, b, and c, with $c \neq 0$,

  1. $a < b$ and $ac < bc$ are equivalent if $c > 0$.

  2. $a < b$ and $ac > bc$ are equivalent if $c < 0$.

  (That is, whenever you multiply or divide by a negative number you must reverse or flip the inequality.)

Examples: Solve the following inequalities

1) $x - \frac{2}{3} > \frac{5}{6}$

2) $-2x + 4 \leq 11$

3) $3x + 5 \geq 6x - 7$

4) $\frac{3}{2}x - 3 < \frac{5}{6}x + \frac{1}{3}$