Topic 2: Congruence Modulo

- Recall that the clock number is the additive identity (or zero). So, we can associate the integers with a particular clock by "wrapping" the integer number line around the clock. Therefore, there are infinitely many integers associated with each clock number. We express this symbolically as follows:

  Congruence Modulo m: Let a, b, and m be integers with $m \geq 2$. Then

  $$a \equiv b \pmod{m} \qquad \text{if and only if} \qquad m|\ (a-b).$$

  NOTE: To do this we need an extended version of "divides" which holds for integers. Namely,

  $$a|\ b \quad (a \neq 0) \text{ if there exists an integer } c \text{ such that } a \cdot c = b.$$

Example 1: True or False

(a) $8 \equiv 3 \pmod{5}$

(b) $7 \equiv 2 \pmod{6}$

(c) $14 \equiv 2 \pmod{6}$

(d) $25 \equiv 3 \pmod{13}$

Example 2: Describe all integers n, where $-20 \leq n \leq 20$, which make each of the following congruences true.

(a) $n \equiv 2 \pmod{9}$

(b) $5 \equiv n \pmod{4}$

(c) $12 \equiv 4 \pmod{n}$