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## Section 3.3: Exponents

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### Definition:

- An **whole number exponent** is a number that tells how many times a factor is repeated in a product. For example, in the problem  $2^4$ , 2 is called the base and 4 is the exponent.

$$2^4 = \underbrace{2 \cdot 2 \cdot 2 \cdot 2}_{4 \text{ times}} = 16.$$

### Exponent Rules:

- **Product Rule:** For any whole numbers  $m$  and  $n$ ,

$$a^m \cdot a^n = a^{m+n}.$$

When multiplying like bases, we add the exponents.

- **Quotient Rule:** For any nonzero number  $a$  and any whole numbers  $m$  and  $n$ ,

$$\frac{a^m}{a^n} = a^{m-n}.$$

When we divide like bases, we subtract the exponents.

- **Power Rule:** For any whole numbers  $m$  and  $n$ ,

$$(a^m)^n = a^{mn}.$$

When we raise a power to another power, we multiply the exponents.

- For any whole number  $m$ ,

$$(ab)^m = a^m \cdot b^m.$$

When we have a product raised to a power, we raise each factor to the power.

- For any whole number  $m$ ,

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}.$$

When we have a quotient raised to a power, we raise both the numerator and denominator to the power.

- **Zero Exponent Rule:** For any nonzero real number  $a$ ,

$$\boxed{a^0 = 1.}$$

**Common Mistakes to Avoid:**

- When using the product rule, the bases MUST be the same. If they are not, then the expressions cannot be combined. Also, remember to keep the bases the same and only add the exponents. For example,

$$3^2 \cdot 3^4 = 3^{2+4} = 3^6 \qquad 3^2 \cdot 3^4 \neq 9^6.$$

- When using the quotient rule, the bases MUST be the same. If they are not, then the expressions cannot be combined. Also, remember to keep the bases the same and only subtract the exponents. For example,

$$\frac{4^5}{4^3} = 4^2 = 16.$$

- The Power Rule and Quotient Rule do NOT hold for sums and differences. In other words,

$$(a + b)^m \neq a^m + b^m \qquad \text{and} \qquad (a - b)^m \neq a^m - b^m.$$

**Example 1:** Using properties of exponents, determine the larger of the following pairs. Explain your reasoning.

(a)  $4^{14}$  or  $8^{10}$

(b)  $6^{10}$  or  $3^{20}$

(c)  $8^{23}$  or  $8^{22} + 8^{22} + 8^{22} + 8^{22}$

**Example 2:** Find  $x$ .

(a)  $5^2 \cdot 5^x = 5^{19}$

(b)  $49^x \cdot 7^5 = (7^3)^5$

**Example 3:** Rewrite the following using a single exponent.

(a)  $25^9 \div 5^4$

(d)  $2^4 \cdot (32^2 \div 8^2) \cdot 4^3$

(b)  $8^6 \cdot (2^4 \cdot 2^3)^3$

(e)  $27^6 \div 3^8 \cdot 9^4 \cdot 81$

(c)  $16^7 \cdot 4^8 \cdot 8^3 \div 2^{12}$

(f)  $125^2 \cdot 9^4 \cdot 27^2 \cdot 25^4$