Topic 1: Statements

**statement**: is a declarative sentence that is true or false but not both.

Examples: It is raining
(2)(3) = 6
5 + 3 = 7
Ohio is the largest state

Not Examples: What time is it?
Ohio is the nicest state.
The big dog

Notation: statements are represented by lowercase letters.

**negation**: the negation of a statement $p$, denoted $\sim p$, is a statement with the opposite truth value of $p$. (i.e.– if $p$ is true then $\sim p$ is false, and if $p$ is false then $\sim p$ is true.)

LOGICAL CONNECTIVES:

- **AND**: The conjunction of $p$ and $q$, denoted $p \land q$, is the statement “$p$ and $q$”.
• **OR**: The **disjunction** of \( p \) and \( q \), denoted \( p \lor q \), is the statement “\( p \) or \( q \)”.

• **IF–THEN**: An **implication** (or **conditional statement**), denoted \( p \rightarrow q \), is the statement “If \( p \) then \( q \)”. (\( p \) is called the hypothesis and \( q \) is called the conclusion.)

• **IF AND ONLY IF**: The **biconditional statement**, denoted \( p \leftrightarrow q \), is the statement “\( p \) if and only if \( q \)”. (\( p \leftrightarrow q \) is the conjunction of \( p \rightarrow q \) and \( q \rightarrow p \).)
Example 1: If $p$ is false and $q$ is true, find the truth values for each of the following:

(a) $p \land q$
(b) $p \lor q$
(c) $\sim p$
(d) $\sim (\sim p)$
(e) $\sim (p \lor q)$
(f) $p \land \sim q$
(g) $\sim (p \lor q)$
(h) $\sim (p \land q)$
(i) $q \land \sim p$
(j) $\sim p \rightarrow q$
(k) $\sim (p \rightarrow q)$
(l) $(p \lor q) \rightarrow (p \land q)$
(m) $(p \lor \sim p) \rightarrow p$
(n) $(p \lor q) \leftrightarrow (p \land q)$
The **converse** of \( p \to q \) is \( q \to p \)

The **inverse** of \( p \to q \) is \( \sim p \to \sim q \)

The **contrapositive** of \( p \to q \) is \( \sim q \to \sim p \)

**logically equivalent**: Two statements are logically equivalent when they have the same truth tables.

**Example 2**: Determine whether \( p \to q \) and \( \sim q \to \sim p \) are logically equivalent.

**Example 3**: Determine whether \( p \to q \) and \( q \to p \) are logically equivalent.