Topic 1: Statements

statement: is a declarative sentence that is true or false but not both.

Examples:		Not Examples:	
	It is raining		What time is it?
	$(2)(3) = 6$		Ohio is the nicest state.
	$5 + 3 = 7$		The big dog
	Ohio is the largest state		This sentence is false.

Notation: statements are represented by lowercase letters.

negation: the negation of a statement p, denoted $\sim p$, is a statement with the opposite truth value of p. (i.e.– if p is true then $\sim p$ is false, and if p is false then $\sim p$ is true.)

LOGICAL CONNECTIVES:

- AND: The **conjunction** of p and q, denoted $p \wedge q$, is the statement "p and q".

- OR: The **disjunction** of p and q, denoted $p \vee q$, is the statement "p or q".

- IF–THEN: An **implication** (or **conditional statement**), denoted $p \to q$, is the statement "If p then q". (p is called the hypothesis and q is called the conclusion.)

- IF AND ONLY IF: The **biconditional statement**, denoted $p \leftrightarrow q$, is the statement "p if and only if q". ($p \leftrightarrow q$ is the conjunction of $p \to q$ and $q \to p$.)

Example 1: If p is false and q is true, find the truth values for each of the following:

(a) $p \wedge q$

(b) $p \vee q$

(c) $\sim p$

(d) $\sim (\sim p)$

(e) $\sim p \vee q$

(f) $p \wedge \sim q$

(g) $\sim (p \vee q)$

(h) $\sim (\sim p \wedge q)$

(i) $\sim q \wedge \sim p$

(j) $\sim p \rightarrow q$

(k) $\sim (p \rightarrow q)$

(l) $(p \vee q) \rightarrow (p \wedge q)$

(m) $(p \vee \sim p) \rightarrow p$

(n) $(p \vee q) \leftrightarrow (p \wedge q)$

The **converse** of $p \to q$ is $q \to p$

The **inverse** of $p \to q$ is $\sim p \to \sim q$

The **contrapositive** of $p \to q$ is $\sim q \to \sim p$

logically equivalent: Two statements are logically equivalent when they have the same truth tables.

Example 2: Determine whether $p \to q$ and $\sim q \to \sim p$ are logically equivalent.

Example 3: Determine whether $p \to q$ and $q \to p$ are logically equivalent.