MATH 22005 HOMEWORK #1 (15 pts) SPRING 2006
SHOW ALL WORK FOR FULL CREDIT — PLEASE CIRCLE FINAL ANSWER

DUE: TUESDAY, JANUARY 31, 2006 AT THE BEGINNING OF CLASS

1. (1 pt) Describe the curve $\mathbf{r}(t) = t\mathbf{i} + t\mathbf{j} + \cos t\mathbf{k}$.

2. (2 pts) Find parametric equations for the tangent line to the curve $\mathbf{r}(t) = \langle \ln t,\, 2\sqrt{t},\, t^2 \rangle$ at the point $(0,\, 2,\, 1)$.

3. (2 pts) Find the length of the curve $\mathbf{r}(t) = 12t\mathbf{i} + 8t^{3/2}\mathbf{j} + 3t^2\mathbf{k}$ for $0 \le t \le 1$.

4. Given $\mathbf{r}(t) = \langle t^2,\, 2t,\, \ln t \rangle$.

   (a) (1 pt) Find the unit tangent vector $\mathbf{T}(t)$.

   (b) (1 pt) Find the unit normal vector $\mathbf{N}(t)$.

5. (1 pt) Find the antiderivative of $\mathbf{r}'(t) = \cos 2t\mathbf{i} - 2\sin t\mathbf{j} + \dfrac{1}{1+t^2}\mathbf{k}$ that satisfies the initial condition $\mathbf{r}(0) = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$.

6. (2 pts) Evaluate $\displaystyle\int \left(e^{-t}\sin t\mathbf{i} + e^{-t}\cos t\mathbf{j} + \frac{1}{\sqrt{1-t^2}}\mathbf{k} \right) dt$

7. (1 pt) Find $\mathbf{r}(t)$ if $\mathbf{r}'(t) = te^{-t^2}\mathbf{i} - e^{-t}\mathbf{j} + \mathbf{k}$, $\quad \mathbf{r}(0) = \dfrac{1}{2}\mathbf{i} - \mathbf{j} + \mathbf{k}$.

8. (2 pts) Find the curvature of the curve given by $\mathbf{r}(t) = 2t\mathbf{i} + t^2\mathbf{j} - \dfrac{1}{3}t^3\mathbf{k}$.

9. (1 pt each) Given $\mathbf{r}(t) = \dfrac{1}{t}\mathbf{i} - 2\mathbf{j} + \ln t\mathbf{k}$ and $\mathbf{u}(t) = t^2\mathbf{i} - 2t\mathbf{j} + 5t\mathbf{k}$.

   (a) Find $\dfrac{d}{dt}\left[\mathbf{r}(t) \cdot \mathbf{u}(t)\right]$

   (b) Find $\dfrac{d}{dt}\left[\mathbf{u}(t) \times \mathbf{u}'(t)\right]$