MATH 22005 HOMEWORK #4 (15 pts) SPRING 2006
SHOW ALL WORK FOR FULL CREDIT — PLEASE CIRCLE FINAL ANSWER

DUE: TUESDAY, FEBRUARY 28, 2006 AT THE BEGINNING OF CLASS

1. (2 pts) Let $a = f(u - v, v - u)$. Show that $\dfrac{\partial z}{\partial u} + \dfrac{\partial z}{\partial v} = 0$.

2. (2 pts each) Let $w = f(x, y, z) = e^{x/y} + e^{z/x}$ where $x = \dfrac{\ln u}{v}$, $y = \ln u$ and $z = \dfrac{\ln u}{uv}$.

   (a) Find $\dfrac{\partial w}{\partial u}$

   (b) Find $\dfrac{\partial w}{\partial v}$

3. (1 pt each) In the following problems, assume that the equation

$$yz^4 + x^2z^3 = e^{xyz} - 3\cos x^2yz$$

   defines z implicitly as a function of x and y.

   (a) Find $\dfrac{\partial z}{\partial x}$.

   (b) Find $\dfrac{\partial z}{\partial y}$.

4. (2 pts) Find the directional derivative of $f(x, y, z) = x^2y + x\sqrt{1 + z}$ at the point $(1, 2, 3)$ in the direction of $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$.

5. (1 pt each) Let $f(x, y, z) = ze^{xy}$.

   (a) Find the direction in which f increases most rapidly at th point $(0, 1, 2)$.

   (b) what is the maximum rate of increase?

6. (3 pts) Find the directions in which the directional derivative of $f(x, y) = x^2 + \sin xy$ at the point $(1, 0)$ has a value of 1.