Vector-Valued Functions: A vector-valued function $r(t) = \langle f(t), g(t), h(t)\rangle$ is a function whose domain is the set of all real numbers and whose range is a set of vectors. Therefore, it assigns to every real number a vector $r(t)$. We write

$$r(t) = \langle f(t), g(t), h(t)\rangle = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$$

where $f(t)$, $g(t)$, and $h(t)$ are called the **component functions** of the vector function.

Limit of a vector-valued function: The **limit** of a vector-valued function $\mathbf{r}$ is defined by taking the limits of its component functions. In other words, if $\mathbf{r}(t) = \langle f(t), g(t), h(t)\rangle$, then

$$\lim_{t\to a} \mathbf{r}(t) = \left\langle \lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t) \right\rangle$$

provided the limits of the component functions exist.

Continuous at a: A vector function $\mathbf{r}$ is **continuous** at a if

$$\lim_{t\to a} \mathbf{r}(t) = \mathbf{r}(a).$$

Therefore, $\mathbf{r}$ is continuous at a if and only if its component functions f, g, and h are continuous at a.

EXAMPLE 1: Find $\displaystyle\lim_{t\to 0} \left\langle \frac{e^t - 1}{t}, \frac{\sqrt{1+t} - 1}{t}, \frac{3}{1+t} \right\rangle$.

EXAMPLE 2: Find a vector equation and parametric equations for the line segment the joins the point $P = (1, 0, 1)$ and $Q = (2, 3, 1)$.

EXAMPLE 3: Find a vector function that represents the curve of intersection of the two surfaces: The cylinder $x^2 + y^2 = 4$ and the surface $z = xy$.

Homework: pp 891-892; 1–7 odd, 11, 15, 17, 19, 33, 35