MATH 22005 Motion in space: Velocity and Acceleration SECTION 14.4

Velocity and Acceleration: Suppose that a particle moves through space so that its position vector at any time t is given by $\mathbf{r}(t)$. Then the velocity vector $\mathbf{v}(t)$ at time t is given by

$$\mathbf{v}(t) = \lim_{h \to 0} = \frac{\mathbf{r}(t+h) - \mathbf{r}(t)}{h}.$$

The velocity vector is also the tangent vector and points in the direction of the tangent line. The magnitude of the velocity vector, $\|\mathbf{v}(t)\|$, is the **speed** of the particle at time t. The **acceleration** os the particle is defined as the derivative of the velocity. Therefore,

$$\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t).$$

From the vector integrals in section 14.2, we have

$$\mathbf{r}(t) = \int \mathbf{v}(t)\, dt \qquad \text{and} \qquad \mathbf{v}(t) = \int \mathbf{a}(t)\, dt.$$

EXAMPLE 1: Find the velocity, acceleration, and speed of a particle with position vector $\mathbf{r}(t) = \langle 2 - 5,\, 4\sqrt{t} \rangle$. Sketch the path of the particle and draw the velocity and acceleration vectors for $t = 1$.

EXAMPLE 2: Find the velocity, acceleration, and speed of a particle with position function $\mathbf{r}(t) = t \sin t\mathbf{i} + t \cos t\mathbf{j} + t^2\mathbf{k}$.

EXAMPLE 3: Find the velocity and position vectors of a particle that has acceleration vector $\mathbf{a}(t) = t\mathbf{i} + t^2\mathbf{j} + \cos 2t\mathbf{k}$ and initial velocity $\mathbf{v}(0) = \mathbf{i} + \mathbf{k}$ and initial position $\mathbf{r}(0) = 2\mathbf{i} + 3\mathbf{j}$.

Tangential and Normal Components of Acceleration:
We will now resolve the acceleration into two components – one in the direction of the tangent and one in the direction of the normal. If we let $v = \|\mathbf{v}(t)\|$, the **tangential component of acceleration**, denoted a_T, is given by

$$\boxed{a_T = v' = \frac{\mathbf{v} \cdot \mathbf{a}}{v} = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{\|\mathbf{r}'(t)\|}}$$

The **normal component of acceleration**, denoted a_N, is given by

$$\boxed{a_N = \kappa v^2 = \frac{\|\mathbf{r}'(t) \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|}}$$

EXAMPLE 4: A particle moves with position function $\mathbf{r}(t) = t\mathbf{i} + t^2\mathbf{j} + 3t\mathbf{k}$. Find the tangential and normal components of acceleration.

Homework: pp 914–915; 3–15 odd, 19, 22, 31–35 odd