Function of Two Variables: A **function of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted $f(x, y)$. The set D is the **domain** of f and its **range** is the set $\{f(x, y) \mid (x, y) \in D\}$. So, a function of two variables is a function whose domain is a subset of $\mathbb{R}^2$ and whose range is a subset of $\mathbb{R}$. We sometimes write $z = f(x, y)$ to make explicit the value that f takes at the general point (x, y)

EXAMPLE 1: Evaluate the following functions.

1. Let $f(x, y) = \dfrac{3x + y}{2y - x}$. Find $f(2, -1)$.

2. Let $f(x, y) = e^{7xy^2}$. Find $f(3, 2)$.

EXAMPLE 2: Find and sketch the domain of the following functions.

1. $f(x, y) = \dfrac{x - 3y}{x + 3y}$

2. $f(x, y) = f(x, y) = \sqrt{x^2 + y^2 - 1} + \ln(4 - x^2 - y^2)$

The **graph** of a function of two variables is the set $\{(x, y, z) \mid z = f(x, y) \text{ where } (x, y) \in D\}$. This means that the graph is a surface $z = f(x, y)$ in $\mathbb{R}^3$.

EXAMPLE 3: Sketch the graph of the following functions.

1. $f(x, y) = \cos x$

2. $f(x, y) = 2 + y^2$

Another method for visualizing functions is a contour map on which points of constant elevation are joined to form level curves. The **level curves** of a function f of two variables are the curves with equations $f(x, y) = k$, where k is a constant (in the range of f.) A level curve $f(x, y) = k$ is the set of all points in the domain of f at which f takes on a given value k. In other words, it shows were the graph of f has a height of k.

EXAMPLE 4: Draw a contour map of $f(x, y) = \sqrt{x^2 + y^2}$ showing several level curves.

Functions of three variables: A function of three variables is a rule that assigns to each ordered triple (x, y, z) in the domain D a subset of $\mathbb{R}^3$ a unique real number w denoted by $f(x, y, z)$. (Note that similar definitions exist of functions of n variables). The graph of such a function is defined to be $\{(x, y, z, w) \in \mathbb{R}^4 \mid w = f(x, y, z)\}$. However, drawing this is impossible since it exists in four-dimensional space. However, we can draw the **level surfaces** for such a function. These are the surfaces with equations $f(x, y, z) = k$ where k is a constant. If the point (x, y, z) moves along a level surface, the value of $f(x, y, z)$ remains fixed.

EXAMPLE 5: Let $f(x, y, z) = 3 - xy + yz^2$. Find $f(-2, 1, 5)$.

EXAMPLE 6: Find the level surfaces of the function $f(x, y, z) = x^2 + y^2 + z^2$.

Homework: pp 934–936; 7–25 odd, 29, 35, 39–43 odd, 59