The Chain Rule (Case 1): Suppose that $z = f(x, y)$ is a differentiable function of x and y, where $x = g(t)$ and $y = h(t)$ are both differentiable functions of t. Then z is a differentiable function of t and

$$\boxed{\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}}$$

EXAMPLE 1: Use the chain rule to find $\dfrac{dz}{dt}$ where $z = \sqrt{x^2 + y^2}$, $x = e^{2t}$, $y = e^{-2t}$.

EXAMPLE 2: Suppose that $z = 4x^2y^3$ where $x = \sin t$ and $y = \cos t$. Find $\dfrac{dz}{dt}$ when $t = \pi$.

The Chain Rule (Case 2): Suppose that $z = f(x, y)$ is a differentiable function of x and y, where $x = g(s, t)$ and $y = h(s, t)$ are differentiable functions of s and t. Then

$$\boxed{\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \qquad \frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}}$$

EXAMPLE 3: Use the chain rule to find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ where $z = e^{xy}\tan y$, $x = s + 2t$, $y = s/t$.

The Chain Rule (Generalized Version): Suppose that u is a differentiable function of n variables x_1, x_2, $\ldots$, x_n and each x_i is a differentiable function of the m variables t_1, t_2, $\ldots$, t_m. Then u is a function of t_1, t_2, $\ldots$, t_m and

$$\boxed{\frac{\partial u}{\partial t_i} = \frac{\partial u}{\partial x_1}\frac{\partial x_1}{\partial t_i} + \frac{\partial u}{\partial x_2}\frac{\partial x_2}{\partial t_i} + \cdots + \frac{\partial u}{\partial x_n}\frac{\partial x_n}{\partial t_i}}$$

for each $t = 1, 2, \ldots, m$.

EXAMPLE 4: If $u = x^4 y + y^2 z^3$, where $x = rse^t$, $\quad y = rs^2 e^{-t}$, and $z = r^2 s \sin t$, find the value of $\dfrac{\partial u}{\partial s}$ when $r = 2, \quad s = 1, \quad t = 0$.

The Chain Rule (Implicit Differentiation): If the equation $F(x, y) = 0$ defines y implicitly as a differentiable function of x, then

$$\boxed{\frac{dy}{dx} = -\frac{F_x(x, y)}{F_y(x, y)}, \quad F_y(x, y) \neq 0}$$

If the equation $F(x, y, z) = 0$ defines z implicitly as a differentiable function of x and y, then

$$\boxed{\frac{\partial z}{\partial x} = -\frac{F_x(x, y, z)}{F_z(x, y, z)} \qquad \text{and} \qquad \frac{\partial z}{\partial y} = -\frac{F_y(x, y, z)}{F_z(x, y, z)}, \quad F_z(x, y, z) \neq 0}$$

EXAMPLE 5: Find $\dfrac{dy}{dx}$ given that $y^5 + x^2y^3 = 1 - ye^{x^2}$.

EXAMPLE 6: Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ given that $3x^2z - x^2y^2 + 2z^3 + 3yz - 5 = 0$.

Homework: pp 974–975; 1–13 odd, 19–33 odd, 43, 47