Local Extrema: A function of two variables has a **local maximum** at (a,b) if $f(x,y) \le f(a,b)$ for all points (x,y) in some disk with center (a,b). The number f(a,b) is called a **local maximum value**. If $f(x,y) \ge f(a,b)$ for all points (x,y) in some disk with center (a,b), then f(a,b) is a **local minimum value**.

Absolute Extrema: If $f(a,b) \ge f(x,y)$ for all (x,y) in the domain of f, then f has an **absolute maximum** at (a,b). Likewise, if $f(a,b) \le f(x,y)$ for all (x,y) in the domain of f, then f has an **absolute minimum** at (a,b).

Theorem: If f has a local maximum or minimum at (a, b) and the first-order partial derivatives of f exist there, then $f_x(a, b) = 0$ and $f_y(a, b) = 0$.

Critical Point: A point (a, b) is called a critical point of f if $f_x(a, b) = 0$ and $f_y(a, b) = 0$, or if one of these partial derivatives does not exist. Remember that not all critical points will lead to local extrema.

Second Derivative Test: Suppose the second partial derivatives of f are continuous on a disk with center (a, b), and suppose that (a, b) is a critical point of f. Let

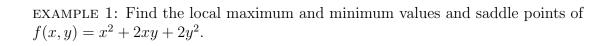
$$D = D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^{2}$$

- If D > 0 and $f_{xx}(a, b) > 0$, then f(a, b) is a local minimum.
- If D > 0 and $f_{xx}(a, b) < 0$, then f(a, b) is a local maximum.
- If D < 0, then f(a, b) is not a local maximum or minimum. (In this case, (a, b) is called a saddle point of f.)

NOTES:

- 1. If D=0 then the second derivative test gives no information.
- 2. The formula for D is the determinant of the matrix

$$\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$$



EXAMPLE 2: Find and classify all critical points of $f(x,y) = (2x - x^2)(2y - y^2)$.

EXAMPLE 3: Find the shortest distance from the point (1, 0, -2) to the plane x + 2y + z = 4.

Closed set: A closed set in \mathbb{R}^2 is one that contains all its boundary points.

Bounded set: A **bounded set** is \mathbb{R}^2 is one that is contained within some disk.

Extreme Value Theorem for functions of two variables: If f is continuous on a closed, bounded set D in \mathbb{R}^2 , then f attains an absolute maximum value $f(x_1, y_1)$ and an absolute minimum value $f(x_2, y_2)$ for some points (x_1, y_1) and (x_2, y_2) in D.

Guidelines to find absolute max and absolute min of a continuous function on a closed, bounded set D:

- 1. Find the critical points of f in D.
- 2. Evaluate f at each one of the critical points of D.
- 3. Find the extreme values of f on the boundary of D.
- 4. The largest of the values in steps 2 and 3 is the absolute maximum value on D; the smallest of the values in steps 2 and 3 is the absolute minimum value of D.

EXAMPLE 4: Find the absolute maximum and minimum values of $f(x,y) = 4x + 6y - x^2 - y^2$ on the set $D = \{ (x,y) \mid 0 \le x \le 4, \ 0 \le y \le 5 \}$.

HOMEWORK: pp 997–998; #5–15 odd, 27–31 odd, 37–41 odd, 47