The Method of Lagrange Multipliers is a technique used for solving certain kinds of optimization problems where we have to find the maximum or minimum value of a function $f(x, y, z)$ subject to a “constraint” equation $g(x, y, z) = k$.

**Lagrange Multiplier:** If $f(x, y, z)$ and $g(x, y, z)$ have continuous partial derivatives and $(a, b, c)$ is a local extremum for $f$ when restricted to $g(x, y, z) = k$, then there is a number $\lambda$ called a Lagrange Multiplier such that

$$\nabla f(a, b, c) = \lambda \nabla g(a, b, c).$$

**Method of Lagrange Multipliers:** To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$ (assuming these extremum values exist and $\nabla g \neq 0$ on the surface of $g(x, y, z) = k$);

1. Find all values of $x$, $y$, $z$, and $\lambda$ such that

   $$\nabla f(x, y, z) = \lambda \nabla g(x, y, z) \quad \text{and} \quad g(x, y, z) = k.$$ 

2. Evaluate $f$ at all points $(x, y, z)$ that result from step 1. The largest of these values is the maximum value of $f$; the smallest of these values is the minimum value of $f$.

**Example 1:** Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = 4x + 6y$ subject to the constraint $x^2 + y^2 = 13$. 

**Example 2:**