The Method of Lagrange Multipliers is a technique used for solving certain kinds of optimization problems where we have to find the maximum or minimum value of a function $f(x, y, z)$ subject to a "constraint" equation $g(x, y, z) = k$.

Lagrange Multiplier: If $f(x, y, z)$ and $g(x, y, z)$ have continuous partial derivatives and (a, b, c) is a local extremum for f when restricted to $g(x, y, z) = k$, then there is a number λ called a **Lagrange Multiplier** such that

$$\nabla f(a, b, c) = \lambda \nabla g(a, b, c).$$

Method of Lagrange Multipliers: To find the maximum and minimum values of $f(x, y, z)$ subject to the constraint $g(x, y, z) = k$ (assuming these extremum values exist and $\nabla g \neq \mathbf{0}$ on the surface of $g(x, y, z) = k$);

1. Find all values of x, y, z, and λ such that

$$\nabla f(x, y, z) = \lambda \nabla g(x, y, z) \qquad \text{and} \qquad g(x, y, z) = k$$

2. Evaluate f at all points (x, y, z) that result from step 1. The largest of these values is the maximum value of f; the smallest of these values is the minimum value of f.

EXAMPLE 1: Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y) = 4x + 6y$ subject to the constraint $x^2 + y^2 = 13$.

EXAMPLE 2: