Iterated Integral: An **iterated integral** is an expression of the following form:

$$\int_a^b \int_c^d f(x,y)\,dx\,dy \quad \text{or} \quad \int_c^d \int_a^b f(x,y)\,dy\,dx.$$

We interpret these as

$$\int_a^b \int_c^d f(x,y)\,dx\,dy = \int_a^b \left(\int_c^d f(x,y)\,dx \right)\,dy.$$

That is, you do the inside integral first and then the outside integral. In the above example, for the inside integral you fix y to be a constant and integrate with respect to x. So, the inside integral when it is evaluated will be a function of y. Then this is the integrand in the outside integral which you then integrate with respect to y.

EXAMPLE 1: Evaluate $\displaystyle\int_0^3 \int_0^2 \left(x^2y + 4xy^3\right)\,dy\,dx$.

EXAMPLE 2: Evaluate $\displaystyle\int_0^2 \int_0^3 \left(x^2y + 4xy^3\right)\,dx\,dy$.

Fubini's Theorem: If f is continuous on the rectangle $R = \{(x,y) \mid a \le x \le b,\ c \le y \le d\ \}$, then

$$\iint\limits_R f(x,y)\,dA = \int_a^b \int_c^d f(x,y)\,dy\,dx = \int_c^d \int_a^b f(x,y)\,dx\,dy.$$

More generally, this is true if we assume that f is bounded on R, f is discontinuous only on a finite number of smooth curves, and the iterated integrals exist.

EXAMPLE 3: Evaluate $\displaystyle\int_1^2 \int_0^\pi y\sin(xy)\,dy\,dx$

EXAMPLE 4: Calculate the double integral $\displaystyle\iint\limits_R \frac{x}{x^2+y^2}\,dA$ where $R = [1,2] \times [0,1]$.

EXAMPLE 5: Find the volume of the solid that lies under the hyperbolic paraboloid $z = 4+x^2-y^2$ and above the square $R = [-1, 1] \times [0, 2]$.

EXAMPLE 6: Find the volume of the solid enclosed by the surface $z = 1+e^x \sin y$ and the planes $x = \pm 1,\ y = 0,\ y = \pi$, and $z = 0$.

Homework: pp 1030–1031; 1–29 odd, 33