In this section we wish to consider double integrals $\iint_D f(x,y)\,dA$ where D is a bounded region more general than a rectangle.

TYPE I: A plane region D is said to be of **type I** if it lies between the graphs of two continuous functions of x, that is

$$D = \{(x,y) \mid a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$$

where $g_1(x)$ and $g_2(x)$ are continuous on $[a,b]$.

Theorem: If f is continuous on a type I region D, such that

$$D = \{(x,y) \mid a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$$

then

$$\iint_D f(x,y)\,dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\,dy\,dx.$$

EXAMPLE 1: Evaluate $\iint_D \frac{4y}{x^3+2}\,dA$ where $D = \{(x,y) \mid 1 \le x \le 2,\ 0 \le y \le 2x\}$.

TYPE II: A plane region D is said to be of **type II** if it lies between the graphs of two continuous functions of y, that is

$$D = \{(x, y) \mid c \leq y \leq d,\ h_1(y) \leq x \leq h_2(y)\}$$

where $h_1(y)$ and $h_2(y)$ are continuous on $[c, d]$.

Theorem: If f is continuous on a type II region D, such that

$$D = \{(x, y) \mid c \leq y \leq d,\ h_1(y) \leq x \leq h_2(y)\}$$

then

$$\iint\limits_D f(x, y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y)\, dx\, dy.$$

EXAMPLE 2: Evaluate $\displaystyle\iint\limits_D x\sqrt{y^2 - x^2}\, dA$ where $D = \{(x, y) \mid 0 \leq y \leq 1,\ 0 \leq x \leq y\}$.

EXAMPLE 3: Find the volume of the solid under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$.

EXAMPLE 4: Find the volume of the solid bounded by the planes $z = x,\ y = x,\ x + y = 2$, and $z = 0$.

EXAMPLE 5: Evaluate $\int_0^1 \int_x^1 \sin y^2 \, dy \, dx$

EXAMPLE 6: Evaluate the integral by reversing the order of integration.

$$\int_0^8 \int_{\sqrt[3]{y}}^2 e^{x^4} \, dx \, dy.$$

Homework: pp 1038–1039; 7–27 every other odd (eoo), 37–47 odd