In section 9.3 we found the center of mass of a thin, flat plate of material, called a **lamina**, with uniform density. (For a lamina density is a measure of mass per unit of area.)

Moments and Center of Mass of a Lamina with uniform density: Let f and g be continuous functions such that $f(x) \geq g(x)$ on $[a, b]$ and consider the planar lamina of uniform density ρ bounded by the graphs of $y = f(x)$ and $y = g(x)$, and $a \leq x \leq b$.

- The **moments about the $x-$axis and $y-$axis** are

$$M_x = \int_a^b \frac{\rho}{2}\left(f^2(x) - g^2(x)\right)\, dx$$
$$M_y = \int_a^b \rho x\left(f(x) - g(x)\right)\, dx$$

- The **center of mass** $(\overline{x}, \overline{y})$ is given by

$$\overline{x} = \frac{M_y}{m} \qquad \text{and} \qquad \overline{y} = \frac{M_x}{m},$$

where $m = \int_a^b \rho\left(f(x) - g(x)\right)\, dx$ is the mass of the region.

We now consider a lamina with variable density given by $\rho(x, y)$. We define the moment of a particle about an axis as the product of its mass and its directed distance from the axis.

Moments and Center of Mass of a Lamina with variable density: Let ρ be a continuous density function on the lamina D. The **moment about the $x-$axis** is given by

$$M_x = \iint\limits_D y\rho(x, y)\, dA$$

and the **moment about the $y-$axis** is given by

$$M_y = \iint\limits_D x\rho(x, y)\, dA$$

The coordinates $(\overline{x}, \overline{y})$ of the **center of mass** of the lamina are

$$\overline{x} = \frac{M_y}{m} \qquad \overline{y} = \frac{M_x}{m}$$

where the mass m is given by

$$m = \iint\limits_D \rho(x, y)\, dA.$$

EXAMPLE 1: Find the mass and the center of mass of the lamina that occupies the triangular region D with vertices $(0,0)$, $(1,1)$, $(4,0)$ and has density function $\rho(x,y) = x$.

Homework: pg 1054; 3, 5, 7, 9