Jacobian: The Jacobian of the transformation $T$ given by $x = g(u, v)$ and $y = h(u, v)$ is

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x \, \partial y}{\partial u \, \partial v} - \frac{\partial x \, \partial y}{\partial v \, \partial u}.$$  

If the transformation $T$ is given by $x = g(u, v, w), \ y = h(u, v, w), \ z = k(u, v, w)$, then the Jacobian of $T$ is the following $3 \times 3$ determinant:

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\ \frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w} \end{vmatrix}$$

**Example 1:** Find the Jacobian of the transformation: $x = e^{u-v}, \ y = e^{u+v}, \ z = e^{u+v+w}$.

**Change of Variables in a Double Integral:** Suppose that $T$ is a $C^1$ transformation whose Jacobian is nonzero and that maps a region $S$ in the $uv$–plane onto a region $R$ in the $xy$–plane. Suppose that $f$ is continuous on $R$ and that $R$ and $S$ are type I or type II plane regions. Suppose also that $T$ is one-to-one, except perhaps on the boundary of $S$. Then

$$\int \int_{R} f(x, y) \, dA = \int \int_{S} f(x(u, v), y(u, v)) \left| \frac{\partial(x, y)}{\partial(u, v)} \right| \, du \, dv$$
EXAMPLE 2: Use the given transformation to evaluate the integral:

$$\iint_R (4x + 8y) \, dA$$

where $R$ is the parallelogram with vertices $(-1, 3), (1, -3), (3, -1), (1, 5)$ and $x = \frac{1}{4}(u + v)$, $y = \frac{1}{4}(v - 3u)$. 
EXAMPLE 3: Evaluate the integral \( \int\int_R \frac{e^{(x+y)/(x-y)}}{(x-y)} \, dA \), where \( R \) is the trapezoidal region with vertices \((1, 0), (2, 0), (0, -2), \) and \((0, -1)\).
**Triple Integrals:** Let $T$ be a transformation that maps the region $S$ in the $uvw$—space onto the region $R$ in $xyz$—space by means of the equations

\[ x = g(u, v, w), \quad y = h(u, v, w), \quad z = k(u, v, w). \]

The **Jacobian** of $T$ is the following $3 \times 3$ determinant:

\[
\frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix}
\frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} & \frac{\partial x}{\partial w} \\
\frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} & \frac{\partial y}{\partial w} \\
\frac{\partial z}{\partial u} & \frac{\partial z}{\partial v} & \frac{\partial z}{\partial w}
\end{vmatrix}
\]

Therefore, under similar hypotheses as in the previous theorem, we have

\[
\iiint_{R} f(x, y, z) \, dV = \iiint_{S} f(x(u, v, w), y(u, v, w), z(u, v, w)) \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| \, du \, dv \, dw
\]

**Homework:** pg 1084; #1–7 odd, 11, 13, 19, 21