Jacobian: The **Jacobian** of the transformation T given by $x = g(u,v)$ and $y = h(u,v)$ is

$$\frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{vmatrix} = \frac{\partial x}{\partial u}\frac{\partial y}{\partial v} - \frac{\partial x}{\partial v}\frac{\partial y}{\partial u}.$$

If the transformation T is given by $x = g(u,v,w)$, $y = h(u,v,w)$, $z = k(u,v,w)$, then the **Jacobian** of T is the following 3×3 determinant:

$$\frac{\partial(x,y,z)}{\partial(u,v,w)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} & \dfrac{\partial x}{\partial w} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} & \dfrac{\partial y}{\partial w} \\ \dfrac{\partial z}{\partial u} & \dfrac{\partial z}{\partial v} & \dfrac{\partial z}{\partial w} \end{vmatrix}$$

EXAMPLE 1: Find the Jacobian of the transformation: $x = e^{u-v}$, $y = e^{u+v}$, $z = e^{u+v+w}$.

Change of Variables in a Double Integral: Suppose that T is a C^1 transformation whose Jacobian is nonzero and that maps a region S in the $uv-$plane onto a region R in the $xy-$plane. Suppose that f is continuous on R and that R and S are type I or type II plane regions. Suppose also that T is one-to-one, except perhaps on the boundary of S. Then

$$\boxed{\iint\limits_R f(x,y)\,dA = \iint\limits_S f(x(u,v), y(u,v)) \left|\frac{\partial(x,y)}{\partial(u,v)}\right| du\ dv}$$

EXAMPLE 2: Use the given transformation to evaluate the integral:

$$\iint\limits_R (4x + 8y)\, dA$$

where R is the parallelogram with vertices $(-1, 3)$, $(1, -3)$, $(3, -1)$, $(1, 5)$ and $x = \frac{1}{4}(u + v)$, $y = \frac{1}{4}(v - 3u)$.

EXAMPLE 3: Evaluate the integral $\iint_R e^{(x+y)/(x-y)} dA$, where R is the trapezoidal region with vertices $(1,0)$, $(2,0)$, $(0,-2)$, and $(0,-1)$.

Triple Integrals: Let T be a transformation that maps the region S in the $uvw-$space onto the region R in $xyz-$space by means of the equations

$$x = g(u, v, w), \quad y = h(u, v, w), \quad z = k(u, v, w).$$

The **Jacobian** of T is the following 3×3 determinant:

$$\frac{\partial(x, y, z)}{\partial(u, v, w)} = \begin{vmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} & \dfrac{\partial x}{\partial w} \\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} & \dfrac{\partial y}{\partial w} \\ \dfrac{\partial z}{\partial u} & \dfrac{\partial z}{\partial v} & \dfrac{\partial z}{\partial w} \end{vmatrix}$$

Therefore, under similar hypotheses as in the previous theorem, we have

$$\boxed{\iiint\limits_R f(x, y, z)\, dV = \iiint\limits_S f(x(u, v, w), y(u, v, w), z(u, v, w)) \left| \frac{\partial(x, y, z)}{\partial(u, v, w)} \right| du\ dv\ dw}$$

Homework: pg 1084; #1–7 odd, 11, 13, 19, 21