Conservative Vector Field: A vector field $\mathbf{F}$ is called a **conservative vector field** if there exists a differentiable function f such that $\mathbf{F} = \nabla f$. The function f is called the **potential function** for $\mathbf{F}$.

Test for Conservative Vector Field in the Plane: Let $P(x, y)$ and $Q(x, y)$ have continuous first partial derivatives on an open disk R. The vector field given by $\mathbf{F}(x, y) = P(x, y)\mathbf{i} + Q(x, y)\mathbf{j}$ is conservative if and only if

$$\boxed{\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}}$$

EXAMPLE 1: Show that $\mathbf{F} = 2xy\mathbf{i} + (x^2 - y)\mathbf{j}$ is a conservative vector field and then find the potential function of $\mathbf{F}$.

EXAMPLE 2: If $\mathbf{F}(x,y,z) = y^2\mathbf{i} + (2xy + e^{3z})\mathbf{j} + 3ye^{3z}\mathbf{k}$, find a function f such that $\nabla f = \mathbf{F}$.

Fundamental Theorem of Calculus for Line Integral: Let C be a piecewise smooth curve lying in an open region R and given by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j},\ \ a \le t \le b$. If $\mathbf{F}(x,y) = P\mathbf{i} + Q\mathbf{j}$ is conservative in R, and P and Q are continuous in R, then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_C \nabla f \cdot d\mathbf{r} = f(x(b), y(b)) - f(x(a), y(a))$$

where f is the potential function of $\mathbf{F}$; that is, $\mathbf{F}(x,y) = \nabla f(x,y)$.

NOTE: Line integrals of *conservative* vector field are independent of path. This means that the line integral only depends on the initial and terminal point.

EXAMPLE 3: Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where

$$\mathbf{F}(x, y, z) = (2xz + y^2)\mathbf{i} + 2xy\mathbf{j} + (x^2 + 3z^2)\mathbf{k}$$

where C is given by $x = t^2,\ y = t + 1,\ z = 2t - 1;\quad 0 \leq t \leq 1.$

Homework: pp 1117–1118; 3–9 odd, 13–17 odd