Wealth and Taxes-The Constraint

\[ z = \frac{(1 - \tau_w)w_2}{(1 + (1 - \tau_r)r)} + \frac{(1 - \tau_w)w_3}{(1 + (1 - \tau_r)r)^2} \]

Assumptions
- People live four periods.
- Before-tax wage income of \( w_1, w_2, w_3, w_4 \)
- Before-tax interest rate of \( r \)
- Tax rates of \( \tau_w, \tau_r \)

The Basic Consumption Function

Logging our consumption function:

\[ c_i = \frac{1}{4} z_i \]

\[ c_2 = \frac{1}{3} z_2 \]

\[ c_3 = \frac{1}{2} z_3 \]

\[ c_4 = z_4 \]

\[ \log(c_1) + \log(c_2) + \log(c_3) + \log(c_4) \]

Assets

\[ a_1 = w_1(1-\tau_w) - c_1 \]
\[ a_2 = a_1[1+r(1-\tau_r)] + w_2(1-\tau_w) - c_2 \]
\[ a_3 = a_2[1+r(1-\tau_r)] + w_3(1-\tau_w) - c_3 \]
\[ a_4 = a_3[1+r(1-\tau_r)] + w_4(1-\tau_w) - c_4 \]
Assets

\[ a_1 = w_1(1-\tau) - c_1 \]
\[ a_2 = a_1[1+r(1- \tau)] + w_2(1-\tau) - c_2 \]
\[ a_3 = a_2[1+r(1- \tau)] + w_3(1-\tau) - c_3 \]
\[ a_4 = a_3[1+r(1- \tau)] + w_4(1-\tau) - c_4 \]

Manipulation-Hold

\[ a_1 = w_1(1-\tau) - c_1 \]
\[ a_2 = a_1[1+r(1- \tau)] + w_2(1-\tau) - c_2 \]
\[ a_3 = a_2[1+r(1- \tau)] + w_3(1-\tau) - c_3 \]
\[ a_4 = a_3[1+r(1- \tau)] + w_4(1-\tau) - c_4 \]

Manipulation

\[ a_4[1+r(1- \tau)] + w_4(1-\tau) - c_4 = 0 \]
\[ a_3 = a_2[1+r(1- \tau)] + w_3(1-\tau) - c_3 \]
\[ \{ a_2[1+r(1- \tau)] + w_3(1-\tau) - c_3 \} \]
\[ [1+r(1- \tau)] + w_4(1-\tau) - c_4 = 0 \]

Manipulation

\[ \{ a_3[1+r(1- \tau)] + w_3(1-\tau) - c_3 \} \]
\[ [1+r(1- \tau)] + w_4(1-\tau) - c_4 = 0 \]
\[ a_4 = a_3[1+r(1- \tau)] + w_4(1-\tau) - c_4 \]
Manipulation

\[ \{a_1[1+r(1- \tau r)] + w_2(1- \tau w) - c_2\}\] 
\[ + w_3(1- \tau w) - c_3\] 
\[ + w_4(1- \tau w) - c_4 = 0 \]

\[ a_1 = w_1(1- \tau w) - c_1 \]

\[ \{(w_1(1- \tau w) - c_1)[1+r(1- \tau r)] + w_2(1- \tau w) - c_2\}\] 
\[ + w_3(1- \tau w) - c_3\] 
\[ + w_4(1- \tau w) - c_4 = 0 \]

Divide through by 
\[ [1+r(1- \tau r)]^3 \]

Present Value

\[ c_1 + \frac{c_2}{(1+r(1- \tau r))} + \frac{c_3}{(1+r(1- \tau r))} + \frac{c_4}{(1+r(1- \tau r))} = \]
\[ \frac{w_1(1- \tau w)}{(1+r(1- \tau r))} + \frac{w_2(1- \tau w)}{(1+r(1- \tau r))} + \frac{w_3(1- \tau w)}{(1+r(1- \tau r))} + \frac{w_4(1- \tau w)}{(1+r(1- \tau r))} \]

The Wealth Formula

\[ z = \frac{(1- \tau w)w_2}{(1+(1- \tau r)r)} + \frac{(1- \tau w)w_3}{(1+(1- \tau r)r)^2} \]
Modifications are possible

- Set up the equations for $a_1$, $a_2$, $a_3$, $a_4$
- $a_4=0$
- Add different taxes, transfer payments.
- Adjust for different interest rates and tax rates for different periods.
- Derive $z$. 

End

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