Applying the Monopoly Model

An Application

• Let's do some simple applications, first mathematically and then using a spreadsheet.

The Demand Functions

\[ Q = 100 - 2P \]

\[ MC = 5 \]

Step One

\[ Q = 100 - 2P \]

\[ MC = 5 \]

• Find where

\[ MR = MC \]

Finding MR

\[ R = PQ \]
Finding MR

\[ R = PQ \]
\[ Q = 100 - 2P \]
\[ P = 50 - \frac{1}{2}Q \]
\[ R = [50 - \frac{1}{2}Q]Q \]

An Application

\[ MR = \frac{dR}{dQ} \]

• We must find the derivative of our equation
\[ R = 50Q - \frac{1}{2}Q^2 \]

Derivative Review

• The derivative of
\[ ax^3 + bx + c \]
is
\[ 2ax + b \]
Derivative Review

• The derivative of $ax^2 + bx + c$ is $2ax + b$

$50Q - (1/2)Q^2$

Set MR = MC

$MR = 50 - Q$

Step Two

• What price will the monopolist charge?
  Remember the inverse demand function
  $P = 50 - (1/2)Q$

Finding the Price

• What price will the monopolist charge?
  Remember the inverse demand function
  $P = 50 - (1/2)Q$
  $P = 50 - (1/2)(45)$
Finding the Price

• What price will the monopolist charge?
  Remember the inverse demand function
  \[ P = 50 - \frac{1}{2}Q \]
  \[ P = 50 - \frac{1}{2}(45) \]
  \[ P = 27.5 \]

Finding Price

• Working from the demand function
  \[ Q = 100 - 2P \]
  \[ 45 = 100 - 2P \]
  \[ P = 27.5 \]

Last Steps

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Price</th>
<th>Revenue</th>
<th>Cost</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
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</table>

Revenue

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Price</th>
<th>Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>[ = P \times Q ]</td>
</tr>
<tr>
<td></td>
<td></td>
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We Know

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Price</th>
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<tbody>
<tr>
<td>45</td>
<td>$27.50</td>
</tr>
</tbody>
</table>
### Applying the Monopoly Model

#### Revenue

<table>
<thead>
<tr>
<th>Quantity</th>
<th>45</th>
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</thead>
<tbody>
<tr>
<td>Price</td>
<td>$27.50</td>
</tr>
<tr>
<td>Revenue</td>
<td>$(27.5)(45)</td>
</tr>
<tr>
<td>Cost</td>
<td></td>
</tr>
<tr>
<td>Profit</td>
<td></td>
</tr>
</tbody>
</table>

#### Revenue

<table>
<thead>
<tr>
<th>Quantity</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$27.50</td>
</tr>
<tr>
<td>Revenue</td>
<td>$(27.5)(45) = $1237.50</td>
</tr>
<tr>
<td>Cost</td>
<td></td>
</tr>
<tr>
<td>Profit</td>
<td></td>
</tr>
</tbody>
</table>

#### Total Cost

<table>
<thead>
<tr>
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<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$27.50</td>
</tr>
<tr>
<td>Revenue</td>
<td>$1237.50</td>
</tr>
<tr>
<td>Cost</td>
<td>$5Q = 5(45) = $225</td>
</tr>
<tr>
<td>Profit</td>
<td></td>
</tr>
</tbody>
</table>

#### Profit

\[ \pi = \text{Revenue} - \text{Cost} = $1227.50 - $225 = $1012.50 \]

#### An Application

- Find the value of Q at which MR = MC
Review

• Find MC

• Find MC
  • Find MR
    – The Revenue Function is PQ

• Find MC
  • Find MR
    – The Revenue Function is PQ
    – Solve for the inverse demand function
    – Substitute for P into the revenue function
## An Application

- Find the value at which MR = MC
- Find the marginal cost (MC)
- Find the marginal revenue (MR)
  - Solve for the inverse demand function
  - Substitute for P into the revenue function
  - Find the derivative

\[
MR = \frac{dR}{dQ}
\]

\[
MC = \frac{dC}{dQ}
\]

## A spreadsheet approach

- An alternative means of doing the problem is to build a spreadsheet. Let's work through that approach.

## End

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