Why Equal Wage Policies May Not Eliminate Wage Disparity

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ABSTRACT

This paper develops a search model with discrimination where some jobs are constrained by equal-wage policies. The model predicts a persistent aggregate wage gap, imperfect job segregation, and infrequent within-job wage disparity. The model also generates an intriguing, counter-intuitive relationship relevant for policy making: while a higher proportion of wage-equality jobs leads to a smaller short-run wage gap, markets with a higher proportion of wage-equality jobs can result in more steady-state discrimination and greater wage disparity. This results from endogenous job destruction; because wage-equality requires non-discriminators to sacrifice their competitive advantage, prejudiced jobs are not effectively cleared by market forces.

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1. Introduction

Most economic theories of discrimination have proven difficult to reconcile with empirical evidence. The original “taste-based” theories of discrimination developed by Becker (1957) predict that discrimination should be cleared by the market, and that persistent wage differentials will be eradicated by competition.\footnote{It should be noted that Becker’s original analysis treated this as one of many possible outcomes, and that the distribution of preferences, firms’ production functions, competition, and time are all important in determining the equilibrium level of discrimination in a market (Becker 1957 Ch. 3).} Statistical models of discrimination based on the original formulations of Aigner & Cain (1977) are also difficult to reconcile with the data. While Altonji & Pierret (2001) do find some evidence that employers learn about their employees over time, it is unclear how much evidence of statistical discrimination this provides (Altonji & Blank 1999). Additionally, most models of statistical discrimination cannot account for the evidence that employers learn quickly and that aggregate wage gaps are persistent even when accounting for skill (Lange 2007; Neal & Johnson 1996).

More recent models developed by Black (1995) and Lang et al. (2005) appear more consistent with the stylized facts of discrimination. Black (1995) develops an equilibrium search model in which there are some discriminatory firms that refuse to hire disfavored workers; the presence of even a few of these firms reduces the value of searching for disfavored workers, and lowers the reservation wages and equilibrium wage offers by all firms for these workers. However, Black’s model requires wage differentials within jobs and firms, a key prediction that is inconsistent with both U.S. labor laws and empirical evidence. Lang et al. (2005) produce a model with binding, posted wage offers, making wage-discrimination impossible. However, their model predicts near perfect job segregation, and hence few observations of within-firm wage equality; this too is inconsistent with most empirical findings. Still, the Black and Lang et al. models are amongst the few that can account for the persistent wage gap observed in the empirical discrimination literature with relatively few assumptions about skill distributions and/or industry structure.
In this paper I develop an equilibrium search model with discrimination that extends Black’s model by allowing for an additional type of firm, one that is truly “non-discriminating.” Non-discriminating firms not only hire workers of any type but, due to internal or governmental policies, pay identical wages to these workers. Using this framework I generate closed form equations for reservation wages, optimal wage offers, and the equilibrium wage gap. I then allow for dynamic market composition by introducing endogenous job destruction in response to random productivity shocks in order to generate steady-state predictions for the composition of job types.

As might be expected, the model predicts that firms adhering to wage-equality policies will indeed reduce the equilibrium wage-gap in a market with constant market composition. However, the model also predicts that long-run wage gaps will persist under any level of hiring discrimination, even when all unprejudiced firms adopt wage-equality policies. Additionally, the model predicts there will be some (but imperfect) by-job segregation, and can account for much higher levels of within-job wage equality than most existing models of discrimination. In perhaps the most interesting and surprising result, the model shows that wage-equality policies can actually increase the steady-state wage gap by allowing for a higher steady-state share of prejudiced firms. This results from endogenous job destruction, as wage-equality jobs effectively sacrifice their competitive advantage in the labor market, and become no more stable than prejudiced jobs. I show this outcome depends on the proportion of disfavored workers in the labor market, and I argue that this prediction could imply different optimal anti-discrimination policy targets for combating race-versus gender-based discrimination in the United States’ labor force.

2. Background

The model employed is very similar to the one proposed by Black (1995). Black’s model is a straightforward equilibrium search model with two types of firms (discriminatory and non-discriminatory) and two types of workers. “Discriminatory” firms (which I refer to as prejudiced) hire only favored workers, while “non-discriminatory” firms (which I refer to as unprejudiced) hire workers of any type, but strategically vary their wage offers based on a worker’s type. As disfavored
workers face fewer potential matches in the labor market, the value of searching (and hence their reservation wages) are lower, and in equilibrium, unprejudiced firms offer a lower wage to these workers. Since favored workers face more potential matches, all firms must offer higher wages to favored workers in order to ensure a match. Black closes his model with a discussion of firm entry, and finds that if owners have different entrepreneurial abilities, some prejudiced firms (with highly skilled owners) will be able to enter the market despite the competitive disadvantages they face in the labor market.

The model I develop here introduces truly “non-discriminatory” firms that offer both types of workers the same wage. Black’s model results in unprejudiced employers exploiting the reduced opportunities for disfavored workers and offering those workers a (lower) wage commensurate with their reservation wage (Black 1995 p.317). Just as is the case in Black’s model, a firm’s optimal strategic response in this model is to adjust the wage offer based on a worker’s type. However, it is easy to imagine unprejudiced firms being committed to an equal hiring policy–either by choice or by law–and thus constrained to offer a single wage to either worker type for a given job. Black himself notes this, and is careful to be specific that in his model the term “…‘unprejudiced employers’ is somewhat misleading…[as they] pay lower wages to the minority workers than to workers facing no discrimination.” (Black 1995 p.317). He also notes policies that offer different wages based on an applicant’s race or gender are illegal, and firms would likely only adopt such regimes in cases where they can segregate by job or occupation. While the implication is clear in his paper that some employers could choose not to exploit the discriminatory preferences of other market agents, the profit motives of employers are assumed to dominate any equity or legal compliance considerations.

Is it realistic to assume some firms might willingly forego profits to ensure equitable treatment of their employees? While this seems to contradict the principle of profit maximization, this type of assumption is actually quite common in the discrimination literature. Assuming that some customers, coworkers, owners, or firms could have a preference for equitable treatment large enough that it would impact their economic decisions is perhaps no more unrealistic than assuming these same agents would be willing to place themselves at a competitive disadvantage (often ensuring a
higher chance of being “weeded out” of the market) in order to indulge in discriminatory preferences. And, much like the case with prejudiced firms, in a labor market with search frictions the presence of these disadvantaged wage-equality firms is not eliminated by market forces.

Two alternative sources for equal-wage jobs do not require a taste for equal treatment. The first case is when a firm sets the wage for a particular job before the applicant types are known. While Lang et al. (2005) show that externally-posted can result in sorting of applicants, if the wages are internally-posted (i.e. determined before hiring but only revealed to applicants at the time a job offer is made), within-job wage equality could easily result. A final possible source for wage equality is compliance with external policy. United States law prohibits discrimination based on race, color, religion, sex, national origin, age, or disability. Firms may therefore avoid systematically paying unequal wages for equal work not because of any internal preference or policy, but in order to avoid the costly penalties for non-compliance.

Regardless of the source, there appears to be a good empirical foundation for exploring wage-equality jobs. While evidence of aggregate black-white and male-female wage gaps is ample (see Altonji & Blank (1999) for an overview of the literature), it is uncommon to see workers at the same firm (and/or in the same jobs) receiving different wages based on their type. In fact, observing wage equality within a firm and job has a surprisingly long history. The Equal Opportunity Employment Commission (EEOC) currently monitors claims of wage discrimination, and Heckman & Payner (1989) provide evidence that Federal antidiscrimination policies may be effective (as they had a wage-equalizing effect for many black workers in the 1960’s). But studies by Foote et al. (2003), Craig & Fearn (1993), and Higgs (1977) show that within-firm wage equality between black and white workers existed in some firms as far back as 1850, and may not have been uncommon by the early twentieth century. While it is unclear whether any within-job wage-equality we observe today is the result of a voluntary internal policy or due to governmental oversight, it is likely that both motivations are at play. Thus, a model that is able to explain some job segregation and a persistent aggregate wage gap without relying on within-job wage disparity or perfect segregation by job would fill a void in the current literature. I develop such a model in the next section.
3. The Basic Model with Constant Job Composition

The model I develop extends Black’s in a few important ways. Instead of focusing on discrimination at the firm level, I focus on discrimination at the job level. Next I allow for three types of jobs. The first two job types are similar to those in Black’s model: prejudiced jobs that are unwilling to hire disfavored workers, and strategic jobs that are willing to hire workers of any type, but use the labor market disparity facing workers to strategically pay lower wages to disfavored workers. The third job type is non-discriminating jobs; these jobs hire any worker type, and pay the same wage regardless of the worker type. Black’s model is effectively nested in this framework; if the share of unprejudiced firms covered by wage-equality policies is zero, than Black’s results are replicated. Finally, to account for dynamic market composition I introduce endogenous job destruction, which allows for endogenous evolution of the proportion of firm types.

3.1. Worker Behavior

I assume two types of workers, a and b. a workers are favored relative to b workers, and face no discrimination in the market. I allow for three types of jobs, prejudiced jobs denoted by p, and two types of unprejudiced jobs: non-discriminating jobs denoted u, and strategically discriminating jobs denoted s. p jobs are offered only to a workers, while u and s jobs are offered to workers regardless of type; however u jobs are constrained to offer a common wage to both types of workers. The proportion of p jobs is given by θ, the proportion of u jobs is γ, and the proportion of s jobs is (1 − θ − γ). In the most flexible specification, wages are allowed to vary by both worker and job type, with \( w^i_j \) representing the wages offered to workers of type i for jobs of type j, and \( g(w^i_j) \) the pdf of those wage offers. I assume a simple, constant rate of matching, so that any worker who searches for employment receives an independently drawn wage offer with probability \( π \).

\footnote{This is in contrast to Black (1995) who used prejudiced and unprejudiced firms. The result is that unprejudiced firms hire both worker types, but pay the favored types more for the same jobs. If the common assumption that each firm has but one job is retained, there is no difference between this and Black’s model.}
worker accepts any wage offer greater than his or her reservation wage \( w \), and earns that wage in perpetuity (this assumption will be relaxed in Section 4). If a worker fails to receive an offer, or if the worker receives an offer below his or her reservation wage, he or she must wait until the next period to search again. Future wages are discounted with a constant per-period discount factor \( \beta \).

In expectation, the value of search at time \( t \) for favored job seekers is denoted by \( S_t^a \), and can be written as:

\[
S_t^a = \pi \theta \int_{w^a}^{\infty} w^a g(w^a) dw^a + \pi \gamma \int_{w^a}^{\infty} w^a g(w^a) dw^a + \pi(1 - \theta - \gamma) \int_{w^a}^{\infty} w^a g(w^a) dw^a + [(1 - \pi) + \pi \theta G_p(w^a) + \pi \gamma G_u(w^a) + \pi(1 - \theta - \gamma) G_s(w^a)] \beta S_{t+1}^a. \tag{1}
\]

The first three terms represent the expected value of accepted wage offers from the various firm types. The coefficients multiplying \( \beta S_{t+1}^a \) are simply the chance no offer is received \((1 - \pi)\) and the chance that an offer is received but rejected, resulting in the worker searching again in the next period (which happens only when \( w^j < w^i \)). Assuming stationary search values, that is \( S_t = S_{t+1} = S \), and grouping terms yields:

\[
[1 - \beta](1 - \pi) + \pi \theta G_p(w^a) + \pi \gamma G_u(w^a) + \pi(1 - \theta - \gamma) G_s(w^a)] S^a = \\
\pi \theta \int_{w^a}^{\infty} w^a g(w^a) dw^a + \pi \gamma \int_{w^a}^{\infty} w^a g(w^a) dw^a + \pi(1 - \theta - \gamma) \int_{w^a}^{\infty} w^a g(w^a) dw^a.
\]

In an equilibrium with no idiosyncratic preferences, the reservation wage for type \( a \) workers, \( w^a \) is simply the reduced form value of a job search \( S^a \), which is:

\[
w^a = S^a = \frac{\pi \theta \int_{w^a}^{\infty} w^a g(w^a) dw^a + \pi \gamma \int_{w^a}^{\infty} w^a g(w^a) dw^a + \pi(1 - \theta - \gamma) \int_{w^a}^{\infty} w^a g(w^a) dw^a}{1 - \beta[(1 - \pi) + \pi \theta G_p(w^a) + \pi \gamma G_u(w^a) + \pi(1 - \theta - \gamma) G_s(w^a)]}. \tag{2}
\]

The derivation of a reduced form reservation wage equation for type \( b \) workers follows in much the same manner. Since prejudiced employers never offer disfavored workers a job, the value of

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3 See Appendix A.1 for a more complete development of the value to search.
searching a prejudiced job is simply zero \( 0 + S_{t+1} \). Taking offers below the workers’ reservation wages to also have value \( 0 + \beta S_{t+1} \) yields

\[
S_b^b = \pi \gamma \int \frac{w_b^b g(w_u^b) dw_u^b}{w_b^b} + \pi (1 - \theta - \gamma) \int \frac{w_s^b g(w_u^b) dw_u^b}{w_s^b} + [(1 - \pi) + \pi \theta + \pi G_u(w_b^b) + \pi G_s^b(w_b^b)] \beta S_{t+1}.
\]

(3)

Applying stationarity and solving for \( S_b^b \) results in the reduced form value of a job search by a type \( b \) worker. As with type \( a \) workers, this value is equal to a type \( b \) worker’s reservation wage absent an idiosyncratic preference, and is given by:

\[
\frac{w^b}{w^b} = \frac{\pi \gamma \int \frac{w_b^b g(w_u^b) dw_u^b}{w_b^b} + \pi (1 - \theta - \gamma) \int \frac{w_s^b g(w_u^b) dw_u^b}{w_s^b}}{1 - \beta [(1 - \pi) + \pi \theta + \pi G_u^b(w_b^b) + \pi G_s^b^b(w_b^b)]}.
\]

(4)

Note that Equations 2 and 4 imply that

\[
\begin{align*}
\frac{w^a}{w^b} &> \frac{w^b}{w^b} \quad \text{when } \theta > 0 \\
\frac{w^a}{w^b} &= \frac{w^b}{w^b} \quad \text{when } \theta = 0.
\end{align*}
\]

(5)

In words: the average reservation wages for type \( a \) workers are greater than those of type \( b \) workers, because of the increased labor market opportunities facing them. The reservation wages would be equal if there were no prejudiced jobs, so that there would be no disparity in the probability of successfully matching.

While this singleton reservation wage result make comparison between types easy, it will be worthwhile to induce a distribution of reservation wages. As in Black (1995), the case of linear utility is assumed, such that \( u = w + e \) so \( u = w + e \) where \( e \) is a worker-specific idiosyncratic preference that has a zero mean. Effectively this just “shocks” the reservation wages in Equations 2 and 4 with the random component \( e \), creating a distribution of worker-specific reservation wages around the values reported in those equations.
3.2. Employer Behavior

For simplicity’s sake, firms are assumed to have linear production functions, and to be identical in their technology, infinitely patient, and facing no advertising costs. As such, a firm chooses a wage $w$ to maximize

$$R = [1 - F(w - w)](MP - w).$$  \hfill (6)

where $R$ is the firm’s rent from hiring a worker, $MP$ is the worker’s marginal product, $f(.)$ is the pdf of the workers’ idiosyncratic utility terms, and $F(w - w)$ is the proportion of workers for whom $w \leq w$.

As, by assumption, prejudiced jobs hire only favored types, these firms maximize

$$R_p = [1 - F_p(w^b - w^p)](MP - w^p).$$  \hfill (7)

Strategic firms will hire either type, but may pay different wages to each. As such, the these firms have objective functions dependent on the worker type, the general form of which is

$$R^i_s = [1 - F^i_s(w^b - w^i_s)](MP - w^i_s).$$  \hfill (8)

The objective functions of non-discriminating firms are slightly more complicated, as they are constrained to offer a single wage to workers of any type. As such, their objective function depends on the relative frequencies of the worker types. The non-discriminating firm’s objective function is given by

$$R_u = [1 - F_u(w^b - w_u)](MP - w_u)B_{pop} + [1 - F_u(w^a - w_u)](MP - w_u)(1 - B_{pop}),$$ \hfill (9)

where $B_{pop} = \frac{b_{pop}}{a_{pop} + b_{pop}}$ is the share of type $b$ workers in the population of searchers (and $(1 - B_{pop}) = \frac{a_{pop}}{a_{pop} + b_{pop}}$ in that population). This objective function is simply the return from hiring a worker at

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$^4$These assumptions remove any differential wage offers motivated by different costs or patience levels. The assumption of perfect patience also avoids the problem of firms altering their wages when in negotiations with a particular worker, as patient firms are more willing to wait for a future match than to adjust their wage offer in the current period to ensure a match.

$^5$As in Black (1995), the source of the hiring prejudice is taken to be exogenous.
wage $w_u$ weighted by both the chance that a worker of a given type will accept the offer, and the chance that a worker of that type is matched to the firm in the search process.

Without an explicit distribution of idiosyncratic preferences that would create a distribution of $w$’s (i.e. all workers of a given type have a common reservation wage), the solutions for the optimal wage offer by prejudiced and strategic firms are trivial, and are simply $w^i$ (recalling that prejudiced firms will only make offers to type $a$ workers). This is clear to see, as any wage lower than $w^i$ would always be rejected by a worker of type $i$ (meaning $1 - F(w_i) = 0$), and any wage greater than $w^i$ would unnecessarily reduce the prejudiced firm’s profits (as it would decrease $MP - w_i$).

The optimal wage for wage-equality firms is derived as follows. Recalling that $w^a \geq w^b$, it should be apparent that firms would never offer wages greater than the maximum reservation wage ($w^a$) or less than the minimum reservation wage ($w^b$), as these offers would lead to reduced profits or universally rejected offers respectively. It should also be apparent that unprejudiced firms would not offer a wage such that $w^b \leq w_u \leq w^a$, as these wages would never be accepted by type $a$ workers, and would reduce the firms’ returns from hiring type $b$ workers. As such, the only rationalizable wage offers by unprejudiced firms are $w^b$ and $w^a$. In the scenario where type $b$ workers comprise a small portion of the labor market (i.e. $B_{pop}$ is small), and/or when jobs are not required to offer a wage that can appeal to workers of either type, the optimal wage offer will be $w^a$. In the alternative, when type $b$ workers make up a larger portion of a job’s potential labor market and/or when jobs must offer a wage that appeals to workers of either type, then the wage offer will be $w^b$, and non-discriminating jobs will only be filled by type $b$ workers.

While the assumption of uniform preferences within type allows for a simplified analysis and consistency with the findings of Lang et al. (2005), extending the firm response to a context of heterogeneous worker preferences may be more appropriate. If there is a distribution of idiosyncratic worker preferences that affect $w$, e.g in the case of linear utility, where $u = w + \epsilon$ so $w = u - \epsilon$, then employers adopt a slightly less straightforward wage offer strategy. In order to maximize profits,
prejudiced employers will choose \( w_p \) such that
\[
    w_p = MP - \frac{1 - F^a(w^a - w_p^a)}{f^a(w^a - w_p^a)},
\]
while strategic employers would choose wages conditional on a worker’s type such that
\[
    w_s^a = MP - \frac{1 - F^a(w^a - w_s^a)}{f^a(w^a - w_s^a)}
\]
and
\[
    w_s^b = MP - \frac{1 - F^b(w^b - w_s^b)}{f^b(w^a - w_s^b)},
\]
with \( w_s^a \geq w_s^b \). Non-discriminating employers that set a single wage for a job that will appeal to either type choose \( w_u \) such that
\[
    [1 - F^b(w^b - w_u)](MP - w_u)B_{pop} + [1 - F^a(w^a - w_u)](MP - w_u)(1 - B_{pop}),
\]
which yields:
\[
    w_u = MP - \frac{(1 - B_{pop})[1 - F^a(w^a - w_u)] + B_{pop}[1 - F^b(w^b - w_u)]}{(1 - B_{pop})f^a(w^a - w_u) + B_{pop}f^b(w^b - w_u)}. \tag{13}
\]
These conditions follow from setting the first derivatives of equations 7, 8, and 9 equal to 0 and rearranging (these derivatives are \( f_j(w^i - w)(MP - w_j) - [1 - F_j(w^i - w)] \) for \( p \) and \( s \) firms, and \((\frac{a}{a + b})[f_u(w^a - w_u)(MP - w_u) - (1 - F_u(w^a - w_u))] + (\frac{b}{a + b})[f_u(w^b - w_u)(MP - w_u) - (1 - F_u(w^b - w_u))]\) for \( u \) firms). Note that these conditions are the same as in the case where there is no distribution of preferences. However, in the case without heterogeneity within a type, \( f(w - w) \) has a mass point at the reservation wage, which makes the optimal solution trivial.

### 3.3. The Equilibrium Wage Gap

As mentioned in Section 3.2, when all workers of a given type have the same reservation wage, the optimal wage policy reduces to a singleton wage offer for each job type. In this case, a persistent wage gap will arise when there are \( s \)-type jobs, and/or when disfavored workers comprise a large enough proportion of the labor market that \( u \)-type jobs have a sufficiently high expectation of
matching with them. If disfavored workers are common \( w_u = w^b \), and there will be a large degree of segregation by job (with all \( u \) jobs worked by type \( b \) workers and all \( p \) jobs worked by type \( a \)’s), and a wage gap given simply by \( w^a - w^b \). If disfavored workers are scarce \( w_u = w^a \), and there will be less segregation by job (with \( p \) jobs worked by only type \( a \) workers), and a (weakly) smaller wage gap given by \((1 - \gamma)(w^a - w^b)\).

These persistent wage gaps are a useful outcome of the model, as they appear consistent with the empirical evidence of wage gaps. Additionally, the resulting policy analysis in this specification is quite straightforward. Any change in offer rates (\( \theta \)) has a direct impact on reservation wages, and reservation wages have a one-to-one correspondence with realized wages. Additionally, increasing \( \gamma \) directly reduces the wage gap through two avenues; an increased \( \gamma \) increases \( w^b \), and decreases the wage gap directly by reducing \((1 - \gamma)\). In essence, a policy maker has a two policy levers in \( \theta \) and \( \gamma \) that directly affect the wage gap. However, the assumption that there is no variation in the reservation wage within a type is a strong one. So, while the calculations are greatly simplified by assuming a “mass point distribution” within types, the usefulness and relevance of these results are limited.

When the reservation wages are no longer singletons, a firm’s objective function and wage offer calculation become more complex. For type \( a \) job seekers, the initial expected wage is given by

\[
E[w^a] = \theta(1 - G_p(w^a))w_p + \gamma(1 - G_u(w^a))w_u + (1 - \theta - \gamma)(1 - G_s(w^a))w_s^a, \tag{14}
\]

which reduces to

\[
E[w^a] = \gamma[(1 - G_u(w^a))w_u + G_u(w^a)w_p] + (1 - \gamma)(1 - G_p(w^a))w_p \tag{15}
\]

in the long-run.\(^6\) For type \( b \) workers the expected wage for an initial search is given by

\[
E[w^b] = \theta_0 + \gamma(1 - G_u(w^b))w_u + (1 - \theta - \gamma)(1 - G_s(w^b))w_s^b, \tag{16}
\]

\(^6\)Appendix A.2 details the calculations of all the expected wage and wage-gap conditions when the composition of firms is constant.
which reduces to

\[ E[w^b] = \gamma w_u + (1 - \gamma)[w_s^b + G_s^b(w^b)w_u] \]  

(17)

in the long-run. The long-run wage gap is simply the difference in the expected wages for type \(a\) and \(b\) workers, or

\[ E[w^a - w^b] = (1 - \gamma)w_p + \gamma[w_u + G_u(w^a)w_p] - \gamma w_u - (1 - \gamma)[w_s^b + G_s^b(w^b)w_u], \]  

(18)

which reduces to:

\[ E[w^a - w^b] = \gamma[G_u(w^a)(w_p - w_u)] + (1 - \gamma)[w_p - (G_s^b(w^b)w_u + (1 - G_s^b(w^b)))w_u^b]. \]  

(19)

Note that

\[ \frac{dE[w^a - w^b]}{d\gamma} < 0, \]  

(20)

so the greater the share of non-discriminating firms, the lower the long-run wage gap. Even though \(\theta\) does not enter Equation 19 directly, it does affect the wage gap. As

\[ \frac{dw_u}{d\theta}, \frac{dw_u^a}{d\theta} < 0, \]  

(21)

it must also be that

\[ \frac{dE[w^a - w^b]}{d\theta} > 0. \]  

(22)

Again, the greater the share of prejudiced firms in the market, the larger the long-run wage gap will be.

In this case, the policy maker again has two levers with which to affect the wage gap. By either increasing \(\gamma\) through a wage-equality policy, or decreasing \(\theta\) via an equal hiring policy, the wage gap can effectively be reduced. But policy makers do not have access to the simple levers that were available with the mass point distribution case. This is because the distributions of reservation wages, and hence \(F_s, F_p,\) and \(F_u\) depend on \(\theta\) and \(\gamma\) (see Equations 2 and 4). This, coupled with the fact that job types and any differential wage offers must be accurately identified (despite the possibility of overlapping wage distributions), makes policy adjustments in the general equilibrium case less straightforward. However, unlike the mass point case the qualitative setup
and requirements for this formulation seem more tenable, even if estimations and policy analyses are more difficult. But there is one other consideration that must be made before the total impact of $\theta$ and $\gamma$ can be understood: the steady-state composition of job types.

4. **Endogenous Job Destruction and the Steady-State Composition of Jobs**

In order to account for long-run market dynamic composition, Black’s model assumed prejudiced firms are able to enter a competitive market due to a high level of “entrepreneurial skill” that compensated for the competitive disadvantages they faced by having to pay higher average wages. In this model I introduce endogenous job destruction. This contrasts Black’s model, as matches are not always infinitely-lived, and can be destroyed in response to productivity shocks. As matches are destroyed and replaced, the composition of prejudiced and unprejudiced firms present in the market can evolve. For simplicity’s sake, I assume that the market is of a fixed size in the steady state (so each destroyed worker/job match is replaced one-for-one) and that new jobs are pulled at random from a population of jobs with a constant probability $\rho$ of being prejudiced jobs.\(^7\)

Formally, technology shocks are introduced by a Poisson process, arriving with chance $\lambda$, and delivering a random, wage-independent shock with value $\epsilon \sim (0, \sigma)$ with pdf $d(\epsilon)$.\(^8\) For simplicity’s sake, I assume that any job with $R^i_j < C$ is destroyed, and without loss of generality, let $C = 0$.\(^9\)

\(^7\)Allowing for endogenous market size, differential job creation rates, or type-specific market tightness would each be reasonable extensions of the model. However, each extension would add greater complexity to the model, and none is essential for developing the general results presented here.

\(^8\)Note that by introducing shocks in this manner, none of the reservation wage, wage offer, or wage gap calculations need to be adjusted. This is because these equations are all based on expectations, and that $E[\epsilon] = 0$ assures job matches are never expected to be destroyed.

\(^9\)In effect, this is simply a zero fixed cost assumption, and only impacts the speed of convergence to the steady state. In the presence of fixed costs some unprofitable matches could be preserved, and the value of $\epsilon$ needed to destroy a match would be more negative and would arrive less frequently. However, the relative destruction rates would be unaffected. So, while the assumption that jobs with negative profits are destroyed immediately is a strong one, it should not impact the long run predictions of the model.
Let $D(\epsilon)$ be the cdf of $\epsilon$, and $e^j_i$ be the value of $\epsilon$ sufficient to make jobs filled by type $i$ workers unprofitable. Note that the production technology is assumed to be linear, so $MP_p = MP_u = MP_s$, and as $w_p = w^a_s \geq w_u \geq w^b_s$, $R_p = R^a_s \leq R_u \leq R^b_s$. Thus prejudiced firms should have smaller rents, and be more sensitive to idiosyncratic technology shocks. From these conditions we have

$$D(e_p) = D(e^a_s) \geq D(e_u) \geq D(e^b_s);$$

that is, the proportion of jobs that are destroyed is inversely related to the profitability of those jobs.

### 4.1. The Steady-State Level of Prejudiced Jobs

At any time $t$ the share of prejudiced, non-discriminating, and strategic firms are given by $\theta_t$, $\gamma_t$, and $(1 - \theta_t - \gamma_t)$. As destroyed jobs are replaced at random from a population with $\rho$ prejudiced jobs, the share of prejudiced jobs in the market at time $t$ is given by:

$$\theta_t = \theta_{t-1} - \lambda D(e_p)\theta_{t-1} + \rho\lambda[D(e_p)\theta_{t-1} + D(e_u)\gamma_{t-1} + (1 - \theta_{t-1} - \gamma_{t-1})(B_{t-1}D(e^b_s) + (1 - B_{t-1})D(e^a_s))].$$

(24)

where $B_t$ is the proportion of strategic jobs held by type $b$ workers at time $t$. The system reaches a steady state when all the time dependent variables become stationary, e.g. $\theta_t = \theta_{t-1} = \theta^*$. Thus the steady state value of $\theta$ is given by

$$\theta^* = \theta^* - \lambda D(e_p)\theta^* + \rho\lambda[D(e_p)\theta^* + D(e_u)\gamma^* + (1 - \theta^* - \gamma^*)(B^*D(e^b_s) + (1 - B^*)D(e^a_s))].$$

(25)

Solving for $\theta^*$ gives the following reduced form equation for the steady-state level of prejudiced jobs in the market:

$$\theta^* = \rho \frac{[D(e_u)\gamma^* + (1 - \gamma^*)(B^*D(e^b_s) + (1 - B^*)D(e^a_s))]}{[(1 - \rho)D(e_p) + \rho(B^*D(e^b_s) + (1 - B^*)D(e^a_s))]}.$$

(26)

(see Appendix A.3 for the calculations of this condition). This implies $\theta^* \leq \rho$ for the feasible values of the parameters (i.e. $0 \leq \rho, D(e_j), \gamma^*, B^*, (\theta^* + \gamma^*), \leq 1$). In other words, the steady-state level of discrimination present in jobs in the market is weakly lower than the level of discrimination in the world of potential jobs (with equality only when $\rho = \{0, 1\}$).
4.2. The Impact of Wage Equality Jobs on the Steady-State Level of Prejudiced Jobs

To illustrate the impact of wage-equality firms on the market level of discrimination, I explore the two extreme cases: \( \gamma = 0 \) and \( \gamma = (1 - \theta) \). Setting \( \gamma = 0 \) replicates the results of Black’s model (with the addition of endogenous job destruction), i.e. all unprejudiced firms are strategically discriminating in their wage offers, and pay disfavored workers less than favored workers. As such, \( w_p = w^a_p, w^b_s = w^b, D(e_p) = D(e^a_s) = D(e^a) \) and \( D(e^b_s) = D(e^b) \). In this case, the steady state share of prejudiced firms in the market is

\[
\theta^{*\gamma=0} = \rho B^* D(e^b) + (1 - B^*) D(e^a) \left/ \left[ (1 - \rho) D(e^a) + \rho B^* D(e^b) + (1 - B^*) D(e^a) \right] \right.
\]

When \( \gamma = (1 - \theta) \) (or \( (1 - \theta - \gamma) = 0 \)) all unprejudiced firms pay the same wages to workers of either type, and there are no strategic firms. Under this assumption the steady state share of prejudiced firms in the market is:

\[
\theta^{*\gamma=(1-\theta)} = \rho \frac{D(e_u)}{(1 - \rho) D(e_p) + \rho D(e_u)}.
\]

Note that \( \theta_{\gamma=0} \) can be greater than or less than \( \theta_{\gamma=(1-\theta)} \) depending on the proportion of disfavored workers in the market, \( B \).\(^\text{10}\) Intuitively, it should be unsurprising that \( B \) is an important factor in determining the steady-state level of discrimination. Simply put, the greater the level of \( B \), the greater the disadvantage faced by a prejudiced job unwilling to hire type \( b \) workers. In this model, when \( B^* \) is large—that is when disfavored workers comprise a large portion of the labor market in the steady-state—then \( \theta_{\gamma=0} > \theta_{\gamma=(1-\theta)} \). This results from the relationship between \( B \) and \( w_u \), i.e. when \( B \) is large, \( w_u \) is relatively small. In this event, wage equality jobs are placing themselves at a relatively small competitive disadvantage compared to strategic jobs, as \( w_u \) is heavily weighted toward \( w^b_s \). Thus a relatively large share of non-discriminating jobs persist in the face of productivity shocks, and a larger share of the destroyed jobs are prejudiced jobs. So, when \( B^* \) is large enough,

\[
\frac{d\theta^*}{d\gamma^{*}} \leq 0
\]

\(^{10}\)See Appendix A.4 for a more detailed derivation and discussion of this finding.
(over the relevant ranges of the parameters).

But when $B^*$ is small, $\theta_\gamma=0 < \theta_{\gamma=(1-\theta)}$. Again, this is the result of the relative frequency of disfavored workers. In this case, disfavored workers are a small share of the market, which increases the weight non-discriminating firms must place on type $a$ wages to ensure matches. This places non-discriminating jobs that match with type $b$ workers at a relatively large competitive disadvantage compared to strategic jobs that match with type $b$ workers. As such, more non-discriminating jobs are destroyed in any given period, and prejudiced jobs have a greater opportunity to enter the market. So, when $B$ is small enough,

$$\theta_\gamma^* \geq \theta_{\gamma=(1-\theta)}.$$  

This implies that when disfavored workers constitute a small share of the labor market,

$$\frac{d\theta^*}{d\gamma^*} \geq 0$$  

(again, over the relevant range of the parameters). Overall, these results show that

$$\frac{d\theta^*}{d\gamma^*} \leq 0,$$  

or the larger the share of disfavored workers in a market, the more likely wage-equality policies will decrease the steady-state presence and impact of prejudiced jobs.

Equation 32 implies that when $B^*$ is large (e.g. female workers within the market of college-educated labor), wage-equality jobs will decrease the steady-state proportion of prejudiced jobs. Prejudiced jobs have smaller profit margins than unprejudiced jobs, which in turn have smaller profit margins than strategic jobs filled by type $b$ workers. As such, unprejudiced jobs are more sensitive to idiosyncratic technology shocks than strategic jobs, as they do not extract additional rents from matches with type $b$’s. When $B^*$ is large, prejudiced jobs are at a large competitive disadvantage compared to both strategic jobs that match with type $b$ workers and non-discriminating jobs. In this situation, prejudiced jobs are significantly more likely be destroyed by market forces than the other job types. Non-discriminating jobs that forgo their competitive advantage (by paying higher wages to type $b$ workers) may actually accelerate the “weeding out” mechanism by which
discrimination could be cleared from the market (by offsetting their competitive disadvantage in type $b$ wages with relatively low wages to type $a$ workers). So, in a high-$B$ environment, a higher $\gamma$ will reduce the steady-state level of $\theta^*$. This “equal wages decreases discrimination” relationship seems to be the implicit assumption behind governmentally-mandated wage-equality policies.

However, Equation 32 also implies a counter-intuitive condition, specifically that when disfavored workers represent a small share of the labor market (e.g. black workers within the market of college-educated labor), then wage-equality jobs can actually increase the relative prevalence of prejudiced jobs in the long run. In the case of a small $B^*$, the scarcity of disfavored workers means that non-discriminating jobs must pay wages nearly as high as prejudiced jobs, simply because there are so few type $b$ workers to match with. Wage-equality firms must forgo a great deal of their competitive advantage in hiring type $b$ workers, since they must pay relatively high wages to retain the services of enough workers in expectation. These wage-equality jobs are now much more likely to be destroyed than their strategic counterparts that are filled by type $b$'s. As such, a larger share of destroyed jobs are non-discriminating jobs, meaning that prejudiced jobs have both a wider avenue through which to enter the market, and a lower destruction rate relative to their wage-equality counterparts. So the long-run level of discrimination would actually be higher than in the case where no wage-equality firms were in the market. This relationship seems to be at odds with the assumption that wage-equality is necessarily a prejudice-reducing policy.

How large must $B^*$ be in order for wage-equity jobs to reduce the steady-state level of prejudiced jobs? Appendix A.4 uses Equations 27 and 28 to solve for this condition, yielding:

$$B_c > \frac{(D(e_u) - [(1 - \rho)D(e^a_u) + \rho D(e_u)])D(e^a_u)}{(D(e^b_u) - D(e^a_u))[1 - \rho)D(e^a_u) + \rho D(e_u)] - \rho D(e_u)(D(e^b_u) - D(e^a_u))}.$$

(33)

This “critical $B$” is an important measure, as it represents a cross-over point in the impact of wage-equality jobs. That is, when $B^* > B_c$, more the wage-equality jobs will decrease the steady-state level of prejudiced jobs, but when $B^* < B_c$, more the wage-equality jobs will increase the steady-state level of prejudiced jobs. Of note is that this critical $B^*$ depends not on the absolute magnitude of the $D(e)$ parameters, but on their relative magnitude in comparison to one another. In other words, $B^*$ is not determined by the actual job destruction rates, but the ratios of destroyed
4.3. The Steady-State Level of Wage-Equality Jobs

While it has been shown that \( \gamma \) can have an impact on the steady-state level of \( \theta \), it has yet to be shown that the steady-state level of \( \gamma \) is non-zero. The proportion of \( \gamma \) firms in a market at any time \( t \) where \( \Gamma \) is the proportion of potential entrants that are non-discriminatory is given by

\[
\gamma_t = \gamma_{t-1} - D(e_u)\gamma_{t-1} + \Gamma[D(e_p)\theta_{t-1} + D(e_u)\gamma_{t-1} + (1-\theta_{t-1}-\gamma_{t-1})][(1-B)D(e^a_s + BD(e^b_s))].
\] (34)

This equation of motion is very similar to the one for \( \theta \) given in Equation 35. Likewise, the reduced form equation for the steady-state level of \( \gamma \) is very similar to one for \( \theta \) given in Equation 26. This steady-state level is given by

\[
\gamma^* = \Gamma \frac{\theta^* D(e_p) + (1-\theta)[(1-B)D(e^a_s + BD(e^b_s))]}{(1-\Gamma)D(e_u) + \Gamma[(1-B)D(e^a_s + BD(e^b_s))]}.
\] (35)

(the derivation of this steady-state equation can be found in Appendix A.5). Thus for any positive levels of \( \Gamma \) and \( D(e_j) \), \( \gamma^* \) is strictly positive, despite the competitive disadvantage non-discriminating firms place themselves in relative to their strategic unprejudiced counterparts. As \( \gamma^* \) can be both positive and persistent, the \( \frac{d\theta^*}{d\gamma} \) findings are relevant.

5. Possible Extensions

As with most search models, the general framework proposed here is easily extensible. Some of the most obvious extensions would be to make allowances for selective job creation, search intensity, concave production functions, and market tightness.

As prejudiced jobs are less profitable than unprejudiced jobs, it is reasonable to think they would be created at a slower rate. Black (1995) makes this observation and offers an explanation for the presence of prejudiced jobs based on “entrepreneurial skill,” where owners face differing levels of fixed costs based on a random draw of ability. The result is that only those prejudiced jobs
with “skilled” owners are created. Accordingly, a smaller proportion of the potential prejudiced jobs are created, decreasing the level of prejudiced jobs entering the market. Selective job creation would yield a similar result here; if the proportion of prejudiced jobs looking to enter the market decreased, this would simply decrease $\rho$ in the model, but would not drive $\theta$ to 0 (unless $\rho = 0$). As such, selective job creation would not alter the model’s main results, unless no prejudiced jobs were able to enter the market.

Search intensity has the potential to narrow the wage gap if disfavored workers searched with greater intensity or efficacy. However, modeling search intensity typically results in greater intensity by workers with higher values of search (Pissarides 2000 Ch. 5.2). In this model that would imply that favored workers are more willing and able to search with greater intensity, increasing their expected wages. As such, a typical modeling of search intensity would not affect the model’s main predictions. However, if disfavored workers were able to identify and avoid searching prejudiced jobs, or had more developed networks to identify opportunities, it could result in a relative decrease to the cost of searching and the wage gap. Without a compelling reason to believe disfavored workers face relatively low search costs, the inclusion of search intensity in the model would only exacerbate the model’s wage gap results.

There are two additional extensions that have the potential to substantially extend the model’s results, and provide a more complete picture of the economic welfare implications of discrimination in a search model by improving on the analysis of unemployment offered here. Both extensions deal with the relative demand for type $a$ and type $b$ labor in equilibrium. The first allows for non-linear production functions, the second allows the relative demand and supply for types of labor to affect matching rates.

If firms face concave production functions (as opposed to the linear production functions assumed here), the result could be a higher quantity demanded of type $b$ labor. That is, if firms face a decreasing marginal productivity of labor schedule, the lower wages commanded by disfavored workers could result in the creation of jobs that would only be viable when filled by type $b$ workers. This could effectively change the matching parameter ($\pi$) so that it is different for type $a$ and $b$
workers. This would result in relatively low wages, but high employment for disfavored workers (which is not generally consistent with the empirical findings for female or black workers). In this case, the entrance of wage-equality jobs would increase the wages, but decrease the employment levels for disfavored workers (by increasing the reservation wages, but decreasing the quantity of viable jobs at the type \( b \) wage). This extension would likely not affect the wage gap results of the model, but could change the both the welfare implications of wage-equality jobs, and the steady-state level of \( B \).

A similar extension would be to allow the matching parameter and the proportion of strategic jobs held by type \( b \) workers to adjust based on market tightness in the general equilibrium results. The model developed here focuses on the presence and persistence of wage gaps in a search framework with discrimination. In so doing, I assume a constant rate of matching (given by \( \pi \)) and do not model the steady-state level of \( B \) in order to simplify the analysis and isolate the impact of prejudiced firms. Allowing \( \pi \) or \( B \) to vary based on the ratio of unemployment to vacancies could allow the model more flexibility to “self-regulate” and minimize the importance of prejudiced jobs. If the demand for type \( a \) workers is higher (the presence of any prejudiced jobs means this is true), then the unemployment rates for type \( a \) workers would be lower. This would make the creation of additional prejudiced jobs more costly, as the likelihood of finding an available type \( a \) worker to match with would be decreased. As such, the selective creation of jobs would likely vary in accordance with the market tightness for different worker types, and would serve to somewhat attenuate the wage gap while increasing the steady-state level of \( B \) (for a given level of \( B_{\text{pop}} \)).

So, while modeling endogenous rates of matching and worker composition (through concave production functions and/or market tightness considerations) could be an interesting course for further research, the main results of this analysis (the existence of wage gap and the impacts of wage-equality policies) should not be overly sensitive to such extensions.
6. Policy Implications

The fact that traditional theories of economic discrimination seem disconnected from the empirical evidence on wage gaps and segregation patterns makes policy prescription difficult. In an environment of coworker discrimination, targeting equal hiring rates at firms would reduce the segregation that typically results from coworker discrimination, but may exacerbate the wage gap between workers (as favored workers will demand higher wages in the newly-integrated environment). Targeting equal wages may minimize the wage gap for workers in environments with customer or statistical discrimination, but would dramatically decrease the hiring rates in industries affected by most forms of discrimination. In short, different types of discrimination have different optimal policy solutions, yet few of the traditional theories of discrimination have proven consistent enough with the data to allow for confident policy prescription.

The model developed here has direct implications for optimal policy efforts, as it predicts the short- and long-run wage and employment impacts of differential hiring and compensation. Obviously a policy that effectively and immediately equated the hiring rates of workers regardless of type would be optimal for narrowing disparity, as this would equalize all the opportunities and outcomes described by this type of search model (e.g. wages, unemployment spells and durations). In fact, effectively targeting hiring discrimination is the first-best policy implied by this model. But “perfect” policy prescription is often irrelevant, as it assumes an efficacy that it infeasible. Instead, the implications for the most effective policy on the margin will be explored; that is, the most “bang for the buck” that could be achieved through an imperfectly effective policy.

Under Title VII of the 1964 Civil Rights Act it is unlawful “to fail or refuse to hire or to discharge any individual, or otherwise to discriminate against any individual with respect to his compensation, terms, conditions, or privileges of employment, because of such individual’s race, color, religion, sex, or national origin.” Currently the Equal Employment Opportunity Commission targets employers who engage in differential hiring, firing, and/or compensation practices. Over the period from 1997 to 2007 the EEOC received an average of more than 28,000 charges of race-based discrimination, and more than 24,000 charges of sex-based discrimination (with more than
1,000 additional charges specific to gender-based unequal pay for equal work filed under the Equal Pay Act of 1963) each year (EEOC 2007). It is reasonable to think that if they were able to strategically target those cases that could have the most positive impact on the labor market, policy makers/enforcers could both give more attention to each case, and potentially have a more significant impact on the targeted outcomes (e.g. wage/hiring discrimination).

One policy implication of the model is that the effectiveness of a wage-equality policy depends on the proportion of disfavored workers in a particular market. If disfavored workers are relatively common, targeting wage-equality should be both an effective wage-equalization policy, and a discrimination-reducing policy in the long run. This is because when disfavored workers are a large share of the labor market, increasing the proportion of non-discriminating firms can decrease the steady-state level of prejudiced jobs (see Equation 32), in addition to providing a smaller short-run wage gap (see Equation 19). However, when disfavored workers are uncommon, targeting hiring discrimination is the only effective discrimination-reducing policy in the long run, as enforcement of wage-equality policies would actually increase the steady-state level of prejudiced jobs (and the corresponding steady-state wage gap). From a practical perspective, this finding could yield different optimal anti-discrimination policy targets for black and female workers.

In 2007 female workers constituted nearly 44% of the employed U.S. labor force, while black workers constituted just 12% of the labor force (BLS 2007). In the context of this model it is reasonable to think that the optimal policies for narrowing male-female and white-black disparities could differ. Specifically, the model implies that long-run gender disparity could best be targeted via equal wage enforcement, while long-run race disparity could best be targeted via narrowing hiring disparity. Unfortunately, if these policies are targeted only at long-run outcomes, they may have significant short-run welfare trade-offs. In the gender case, even a “perfect” equal-wage policy would result in longer unemployment spells and potentially lower employment levels for female workers in the short run (as a higher share of prejudiced firms could continue to engage in differential hiring). In the case of race, targeting hiring equality could lead to short-run wage gaps that could be lifetime welfare-decreasing for current workers (since strategic firms would be able
to engage in differential wage policies that would decrease the short-run wages for black workers). So, while there might be different optimal policies for combating race and gender discrimination, each of these policies is imperfect in the short run.

If such short-run tradeoffs for optimal long-run outcomes seems unsavory, a “magic bullet” solution may exist. If wage-equality stems less from oversight and more from preferences, government oversight of wage-equality may be non-binding. In this case, specializing government oversight to target the identification and elimination of jobs that exhibit hiring bias could be optimal in both the short- and long-runs. In other words, while the reasons for wage-equality jobs to exist was not considered in the model presented here (and have no impact on the outcomes of the model), the sources of wage-equality policies could be relevant for optimal policy prescription.

At this point it should be noted that wage-equality policies would only have this differential effectiveness if the proportion of black and female workers fall on different sides of the critical $B$ in Equation 33, a condition that is not at all obvious. In order to provide some sense of whether the magnitude of the “critical $B$” identified in Equation 33 could reasonably lie between the proportion of female and black workers, Table 8 uses example levels of the $\rho$ and $D(e_j)$ to show the critical $B$’s that would result. In many cases it appears that the critical value for $B$ could indeed fall between the proportion of female and black workers in most labor markets. Only in the cases where $\rho$ is very high or there is a dramatic difference between $\frac{D(e_u)}{D(e_u)}$ and $\frac{D(e_u)}{D(e_u)}$ would the critical value for $B$ lie below the proportion of black workers in the labor market. And only when $\frac{D(e_u)}{D(e_u)}$ is low does the critical $B$ lie above the proportion of female workers in the labor market. As such, it is quite possible that the optimal anti-discrimination policy targets for black and female workers implied by this model are indeed different. However, a more through empirical analysis of relative job destruction rates and hiring discrimination would be required before any assertion that this possibility is probable could fairly be made.
7. Comparison to Similar Models

The model I develop in this paper is similar to the one created by Black (1995). In fact, Black’s model is effectively nested here; if the share of unprejudiced firms covered by wage-equality policies is zero (i.e. $\gamma = 0$), than Black’s model results. However, the addition of firms that are bound by a wage-equality constraint creates unique predictions, although the equal-pay mechanism is not unique. Lang et al. (2005) develop a model that similarly avoids the possibility of a firm paying different wages to different types of workers in the same job. In that paper the authors consider a model with posted wage offers, which serves as a formal commitment mechanism to ensure firms do not adjust wages in response to an applicant’s type. As such, their formulation mitigates the concerns about the strategic sustainability of the applicant-independent wage offers used here. Similar to Black’s model and the model presented here, the presence of even small amount of discrimination can lead to decreased expectations and wages for disfavored workers. The result is that firms offer wages that either appeal to disfavored workers, or a wage high enough that only favored workers apply. This yields a persistent wage gap with perfect segregation; this qualitative result is similar to this paper’s model where unprejudiced firms offer a wage that only appeals to disfavored job seekers, while remaining consistent with one of the wage differential mechanisms suggested by Black (1995, p.318).

The models in both Black (1995) and Lang et al. (2005) also predict persistent aggregate wage gaps. Each paper achieves this through a framework that requires (at least) some segregation, and pervasive wage discrimination (i.e. all of the disfavored types are paid lower wages). But support for these predictions is mixed. Evidence of sex and race segregation in the workplace is ample. Hellerstein & Neumark (2008) find a substantial level of racial segregation in the U.S., most of which cannot be accounted for by differences in education or skill. Likewise, Carrington & Troske (1998) find a great deal of gender segregation, even within a single industry sector. However, neither find the type of “perfect” segregation implied in Lang et al. (2005) (though their model’s segregation prediction could be lessened if firms posted different wages for similar jobs). This would result in universal within-job wage disparity, which is also a key prediction in Black’s model.
The model developed here has a number of predictions that are more consistent with empirical findings than previous work. Like Black (1995) my models predicts persistent race and gender wage gaps as a result of any prejudice in hiring; but my model can also account for the within-job wage equality that is often observed in the labor market. Like Lang et al. (2005), this model allows for race- and/or gender-independent wage setting, but my model results in neither perfect job sorting, nor non-overlapping wage distributions across worker types. As such, the predictions of this seem to mesh well with the data, as empirical studies of the labor market generally show an aggregate wage gap, strong but imperfect segregation, and relatively common of within firm and job wage disparity.

The unique predictions of this model are achieved by allowing for not only prejudiced jobs that hire only favored workers and jobs that strategically vary their wage offers based on a worker’s type (as in Black (1995)), but also jobs that pay a single wage for a particular job (as in Lang et al. (2005)), and hire workers of any type. These “non-discriminating” firms may adopt such a wage equality policy as a result of governmental oversight, an internal policy for equal treatment of their workers, or simply as a result of payroll departments that are type-independent in their wage setting practices (as in the cases described by Heckman & Payner (1989), Foote et al. (2003), and Lang et al. (2005), respectively). Regardless of the source, these wage-equality jobs are consistent with most empirical explorations of within-job wage gaps, and have significant impacts on the wage outcomes and long-run dynamics of the model.

8. Summary and Conclusion

In this paper I develop an equilibrium search model with discrimination that allows for truly “non-discriminating” firms, and dynamic job composition through endogenous job destruction. The model predicts long-run wage gaps that persist with any hiring discrimination, even when all unprejudiced firms adopt wage-equality policies. Additionally, the model predicts the imperfect segregation and potentially high levels within-job wage equality that are both consistent with empirical findings, and absent in most existing models of discrimination.
As expected, an increase in the share of prejudiced jobs increases the equilibrium wage gap (in both the short and long runs), while an increase in the share of non-discriminating jobs decreases the short-run wage gap. What is both intriguing and perhaps unexpected is that when disfavored workers constitute a small portion of the labor market (e.g. black workers in the U.S.), then as the share of non-discriminating jobs increases, so does the steady-state level of prejudiced jobs. This means that in environments of discrimination against workers who comprise a small minority, wage-equality policies can actually increase the steady-state wage gap.

This intriguing result stems from the manner in which non-discriminating firms must set their wages so as to appeal to applicants of either type. When disfavored workers are a small minority, firms that pay equal wages to all workers must set a higher wage (as they are relatively more concerned with appealing to favored workers). In so doing, they forgo a greater competitive advantage by not exploiting the relatively low reservation wages of disfavored workers. As such, wage-equality jobs are less profitable relative to strategically discriminating jobs (those that vary their wage offers based on a worker’s type) when there are few disfavored workers. Since more profitable worker-job matches survive at a greater rate than less profitable matches in an environment with endogenous job destruction, the presence of wage-equality jobs can actually minimize the competitive disadvantage faced by prejudiced employers, and lead to higher levels of long-run discrimination.

While neither the equilibrium level of discrimination, nor the composition of job types in the market is empirically explored, it is quite possible that the optimal long-run policy targets for combating race- and gender-based discrimination might differ. Since U.S. workers who face worse labor market conditions because of race often constitute a smaller minority than those who face worse conditions based on gender, it could be the case that wage-equality policies would have contradictory impacts on the labor marker outcomes of those workers. In other words, wage-equality policies aimed at narrowing the black-white wage gap may be less effective than those aimed at narrowing the male-female wage gap, regardless of the cause or level of the underlying discrimination.
Table 1. Calculations of the Minimum Proportion of Dis advantaged Workers Necessary for an Increase in Wage Equality Job to Decrease the Steady-State Level of Prejudiced Jobs, Using Equation 33 and Example Values of $D(e_a)$, $D(e_b^*)$, and $\rho$.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$\frac{D(e_b)}{D(e_a)} = .90$</th>
<th>$\frac{D(e_b)}{D(e_a)} = .75$</th>
<th>$\frac{D(e_b)}{D(e_a)} = .50$</th>
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<td>0.094</td>
<td>0.082</td>
</tr>
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</table>

Note. — The values in the table are the minimum proportion of type $b$ workers necessary for an increase in the proportion of wage-equality jobs to decrease the steady-state level of prejudiced jobs, and are based on parameters offered only for illustrative purposes (and not meant to imply the actual levels of the parameters). In 2007 female workers constituted nearly 44% of the employed U.S. labor force, while black workers constituted just 12% of the labor force (BLS 2007); this means half of the values above would imply different optimal policy targets for reducing race- and gender-based discrimination.
A. Appendices

A.1. Construction of Search Value Equations

In expectation, the value of a one-shot search at time $t$ for favored job seekers is denoted by $S^a_t$, and can be written as:

$$S^a_t = \pi \theta \int_0^\infty w^a_p g(w^a_p) dw^a_p + \pi \gamma \int_0^\infty w^a_u g(w^a_u) dw^a_u + \pi (1 - \theta - \gamma) \int_0^\infty w^a_s g(w^a_s) dw^a_s$$

Where $\int_0^\infty w^i_j g(w^i_j) dw^i_j$ is the expected wage offered to a seeker of type $i$ by a job of type $j$, with the wage equality constraint on non-discriminating firms requiring $\int_0^\infty w^a_u g(w^a_u) dw^a_u = \int_0^\infty w^b_u g(w^b_u) dw^b_u$. If an acceptable wage offer is not received, then the worker earns 0 for the current period and searches again in the next period with discounted expected value $\beta S^{a}_{t+1}$.

Note that by splitting the limits of the integrals at the reservation wage, the equation can be rewritten as:

$$S^a_t = \pi \theta \int_0^{W^a} w^a_p g(w^a_p) dw^a_p + \pi \gamma \int_0^{W^a} w^a_u g(w^a_u) dw^a_u + \pi (1 - \theta - \gamma) \int_0^{W^a} w^a_s g(w^a_s) dw^a_s$$

$$+ \pi \theta \int_0^{W^a} w^a_p g(w^a_p) dw^a_p + \pi \gamma \int_0^{W^a} w^a_u g(w^a_u) dw^a_u + \pi (1 - \theta - \gamma) \int_0^{W^a} w^a_s g(w^a_s) dw^a_s.$$ 

As a searcher that does not receive an offer will search next period, the total value of searching is:

$$S^a_t = \pi \theta G_p(w^a) + \pi \gamma G_u(w^a) + \pi (1 - \theta - \gamma) G_s(w^a) + \pi \theta \int_{W^a}^{\infty} w^a_p g(w^a_p) dw^a_p$$

$$+ \pi \gamma \int_{W^a}^{\infty} w^a_u g(w^a_u) dw^a_u + \pi (1 - \theta - \gamma) \int_{W^a}^{\infty} w^a_s g(w^a_s) dw^a_s + (1 - \pi) \beta S^{a}_{t+1}.$$ 

The first three terms represent the expected value of wage offers below the worker’s reservation wages from the various firm types. As these wages will be rejected and result in the worker searching again in the next period, the actual value of these wages are $(0 + \beta S^{a}_{t+1})$. Thus the above can be
written as:

\[
S_t^a = \pi \theta \int_0^{\infty} \frac{w_p^a g(w_p^a)}{W^a} dw_p^a + \pi \gamma \int_0^{\infty} \frac{w_u^a g(w_u^a)}{W^a} dw_u^a + \pi (1 - \theta - \gamma) \int_0^{\infty} \frac{w_s^a g(w_s^a)}{W^a} dw_s^a
+ [(1 - \pi) + \pi \theta G_p(w^a) + \pi \gamma G_u(w^a) + \pi (1 - \theta - \gamma) G_s(w^a)] \beta S_{t+1}^a,
\]

which is presented as Equation 1. A similar derivation can be made for type b workers by noting that prejudiced firms will never make an offer to type b’s.

### A.2. Wage Gap Calculations With Constant Market Composition

For type a job seekers, the initial expected wage is given in Equation 14, which is

\[
E[w^a] = \theta (1 - G_p(w^a)) w_p + \gamma (1 - G_u(w^a)) w_u + (1 - \theta - \gamma) (1 - G_s^a(w^a)) w_s^a.
\]

Noting that strategic firms offer wages identical to those of prejudiced firms and expanding yields

\[
E[w^a] = \theta (1 - G_p(w^a)) w_p + \gamma (1 - G_u(w^a)) w_u + (1 - \theta - \gamma) (1 - G_p(w^a)) w_p - \theta (1 - G_p(w^a)) w_p - \gamma (1 - G_p(w^a)) w_p.
\]

Grouping terms gives

\[
E[w^a] = \gamma (1 - G_u(w^a)) w_u + (1 - \gamma) (1 - G_p(w^a)) w_p.
\]

But in the steady-state, searchers with \( w^a > w_p \) will discontinue their searches, meaning \( G_p(w^a) = 0 \). Additionally, those searchers with \( w_u < w^a \leq w_p \) who initially attempted to match with unprejudiced firms will re-search until they match with a prejudiced firm. Thus the expected/average \( w^a \) in the long run becomes

\[
E[w^a] = \theta w_p + \gamma (1 - G_u(w^a)) w_u + \gamma (G_u(w^a) - G_p(w^a)) w_p + (1 - \theta - \gamma) (1 - G_p(w^a)) w_p.
\]

This reduces to simply

\[
E[w^a] = \gamma [(1 - G_u(w^a)) w_u + G_u(w^a) w_p] + (1 - \gamma) (1 - G_p(w^a)) w_p,
\]

which is the result reported in Equation 15.
For type $b$ workers the expected wage for an initial search is given in Equation 16

$$E[w^b] = \theta_0 + \gamma(1 - G_a(w^b))w_u + (1 - \theta - \gamma)(1 - G_b(w^b))w_s,$$

or simply

$$E[w^b] = \gamma(1 - G_u(w^b))w_u + (1 - \theta - \gamma)(1 - G_b(w^b))w_s.$$

Similar to the case of type $a$ workers, type $b$ workers with $w^b > w_u$ will discontinue searching, as they will never find a wage high enough to meet their reservation wage. Also, those searchers with $w_u \geq w^b > w^b_s$ who initially attempted to match with strategic firms will re-search until they match with an unprejudiced firm. Accordingly, the expected wage for a type $b$ worker in the long-run is

$$E[w^b] = \gamma(1 - G_u(w^b))w_u + (1 - \theta - \gamma)(1 - G_b(w^b))w_s + (1 - \theta - \gamma)(1 - G_b(w^b))(G_b(w^b) - G_u(w^b))w_s.$$

Noting that if workers incapable of matching discontinue searching, $F_u(w^b) = 0$ leads to

$$E[w^b] = \gamma w_u + (1 - \gamma)[w^b_s + G_b(w^b)w_u],$$

which is the result reported in Equation 17.

The long-run wage gap is simply the difference in the expected wages for type $a$ and $b$ workers, given in Equation 18 as

$$E[w^a - w^b] = (1 - \gamma)w_p + \gamma[w_u + G_a(w^a)w_p] - \gamma w_u - (1 - \gamma)[w^b_s + G_b(w^b)w_u].$$

Grouping $\gamma$-related terms gives

$$E[w^a - w^b] = \gamma G_a(w^a)(w_p - w_u) + (1 - \gamma)[w_p - w^b_s - G_b(w^b)w_u],$$

which reduces to the long run wage gap, given in Equation 19

$$E[w^a - w^b] = \gamma(G_a(w^a)(w_p - w_u)) + (1 - \gamma)[w_p - (G_b(w^b)w_u + (1 - G_b(w^b))w^b_s].$$

Note that

$$\frac{dE[w^a - w^b]}{d\gamma} < 0,$$
so the greater the share of unprejudiced firms, the lower the long-run wage gap. Even though \( \theta \) does not enter this equation directly, it does affect the wage gap. As

\[
\frac{d w_u}{d \theta} + \frac{d w^u}{d \theta} < 0,
\]

it must also be that

\[
\frac{d E[w^a - w^b]}{d \theta} > 0.
\]

So, the greater the share of prejudiced firms in the market, the larger the long-run wage gap will be.

A.3. Derivation of Steady State Level of Prejudice Jobs

Starting with Equation 25, or

\[
\theta^* = \theta^* - \lambda D(e_p)\theta^* + \rho \lambda [D(e_p)\theta^* + D(e_u)\gamma^* + (1 - \theta^* - \gamma^*)(B^* D(e^b_s) + (1 - B^*)D(e^a_s))],
\]

and canceling terms yields

\[
D(e_p)\theta^* = \rho [D(e_p)\theta^* + D(e_u)\gamma^* + (1 - \theta^* - \gamma^*)(B^* D(e^b_s) + (1 - B^*)D(e^a_s))].
\]

By rearranging and expanding terms we get

\[
(1 - \rho)D(e_p)\theta^* = \\
\rho D(e_u)\gamma^* + \rho (B^* D(e^b_s) + (1 - B^*)D(e^a_s)) - \rho \theta^* (B^* D(e^b_s) + (1 - B^*)D(e^a_s)) - \rho \gamma^* (B^* D(e^b_s) + (1 - B^*)D(e^a_s)).
\]

Grouping terms yields

\[
\rho (B^* D(e^b_s) + (1 - B^*)D(e^a_s)) \theta^* + (1 - \rho)D(e_p)\theta^* = \\
\rho D(e_u)\gamma^* + \rho (B^* D(e^b_s) + (1 - B^*)D(e^a_s)) - \gamma^* (B^* D(e^b_s) + (1 - B^*)D(e^a_s)),
\]

which simplifies to

\[
[(1 - \rho)D(e_p) + \rho (B^* D(e^b_s) + (1 - B^*)D(e^a_s))] \theta^* = \rho [D(e_u)\gamma^* + (1 - \gamma^*)(B^* D(e^b_s) + (1 - B^*)D(e^a_s))].
\]
Solving for $\theta^*$ gives the reduced form equation for the steady-state level of prejudiced jobs in the market in Equation 26:

$$\theta^* = \rho \frac{[D(e_u)\gamma^* + (1 - \gamma^*)(B^*D(e^b_u) + (1 - B^*)D(e^a_u))] - \rho (1 - \rho)D(e_p) + \rho (B^*D(e^b_u) + (1 - B^*)D(e^a_u))}{[(1 - \rho)D(e_p) + \rho (B^*D(e^b_u) + (1 - B^*)D(e^a_u))]}.$$

### A.4. Determining When Wage Equality Firms Decrease the Share of Prejudiced Jobs

In order for the proportion of non-discriminating jobs to decrease the steady-state level of prejudiced jobs, it must be that $\theta^*_\gamma = 0 > \theta^*_\gamma = (1 - \theta)$. From Equations 27 and 28, this condition is equivalent to

$$BD(e^b_u) + (1 - B)D(e^a_u) > \frac{D(e_u)[(1 - \rho)D(e^a_u) + \rho (B^*D(e^b_u) + (1 - B^*)D(e^a_u))]}{(1 - \rho)D(e^a_u) + \rho D(e_u)}.$$

Canceling $\rho$’s and multiplying by the left side’s denominator yields

$$BD(e^b_u) + D(e^a_u) - BD(e^a_u) > \frac{D(e_u)[(1 - \rho)D(e^a_u) + \rho (B^*D(e^b_u) + (1 - B^*)D(e^a_u))]}{(1 - \rho)D(e^a_u) + \rho D(e_u)}.$$

Grouping terms involving $B$, we have

$$BD(e^b_u) - BD(e^a_u) - B\rho D(e_u) - \frac{B(e^b_u) + D(e^a_u)}{(1 - \rho)D(e^a_u) + \rho D(e_u)} > \frac{(1 - \rho)D(e^a_u)D(e_u) + \rho D(e^a_u)D(e_u)}{(1 - \rho)D(e^a_u) + \rho D(e_u)} - D(e^a_u).$$

Isolating $B$ gives

$$[D(e^b_u) - D(e^a_u) - \rho D(e_u) - \frac{D(e^b_u) + D(e^a_u)}{(1 - \rho)D(e^a_u) + \rho D(e_u)}]B > \frac{(1 - \rho)D(e^a_u)D(e_u) + \rho D(e^a_u)D(e_u)}{(1 - \rho)D(e^a_u) + \rho D(e_u)} - D(e^a_u).$$

From here it should be apparent that both the left and right sides of the inequality are (weakly) negative. This follows from the observations that $0 \leq B, D(e^b_u), D(e_u), D(e^a_u), \rho \leq 1$ and $D(e^b_u) \leq D(e_u) \leq D(e^a_u)$. These observations imply $D(e^b_u) \leq D(e^a_u)$ (making the left side negative), and $D(e_u)D(e^a_u) \leq (1 - \rho)D(e^a_u) + \rho D(e_u) \leq D(e^a_u)$ (ensuring the right side is negative). Solving for $B$ gives

$$B > \frac{(1 - \rho)D(e^a_u)D(e_u) + \rho D(e^a_u)D(e_u)}{D(e^b_u) - D(e^a_u) - \rho D(e_u) - \frac{D(e^b_u) + D(e^a_u)}{(1 - \rho)D(e^a_u) + \rho D(e_u)}}.$$
which is (weakly) positive, as both the numerator and denominator are (weakly) negative. Multi-
plying both sides of the inequality by 1, and exploiting the fact that $1 = \frac{(1-\rho)D(e_a^u) + \rho D(e_u)}{(1-\rho)D(e_a^u) + \rho D(e_u)}$ for the
right side gives

$$B > \frac{(D(e_u) - [(1-\rho)D(e_a^u) + \rho D(e_u)])D(e_a^u)}{(D(e_u^0) - D(e_a^0))[(1-\rho)D(e_a^u) + \rho D(e_u)] - \rho D(e_u)(D(e_b^0) - D(e_a^0))}.$$  

This is the condition under which an increase in the share of wage-equality firms will decrease the
steady-state level of prejudiced firms. Formally, when this inequality holds, $\frac{d\theta}{d\gamma} < 0$. Note that an
increase in $D(e_b^0)$ or $D(e_a^0)$ increases value of the “critical $B$,” as does a decrease in $D(e_u)$ or $\rho$.

### A.5. Determining The Steady-State Share of Non-Discriminating Jobs

Letting $\Gamma$ represent the proportion of non-discriminating firms in the population of potential
firms, $\gamma_t$ (the proportion of non-discriminating firms in the market at any time $t$) is given by
Equation 34.

$$\gamma_t = \gamma_{t-1} - D(e_u)\gamma_{t-1} + \Gamma[D(e_p)\theta_{t-1} + D(e_u)\gamma_{t-1} + (1 - \theta_{t-1} - \gamma_{t-1})][1 - B)D(e_a^0) + BD(e_a^b)]].$$

In the steady state, each of the time-dependent variables assumes a time-consistent level (e.g.
$\gamma_{t-1} = \gamma_t = \gamma^*$). As such, the above becomes

$$D(e_u)\gamma^* = \Gamma[D(e_p)\theta^* + D(e_u)\gamma^* + (1 - \theta^* - \gamma^*)[(1 - B)D(e_a^0) + BD(e_a^b)]].$$

Expanding and rearranging yields

$$D(e_u)\gamma^* - \Gamma D(e_u)\gamma^* = \Gamma [D(e_p)\theta^* + \Gamma[(1 - B)D(e_a^0) + BD(e_a^b)]]
- \Gamma \theta^*[(1 - B)D(e_a^0) + BD(e_a^b)] - \Gamma \gamma^*[(1 - B)D(e_a^0) + BD(e_a^b)].$$

Collecting $\gamma^*$ terms gives

$$\frac{(D(e_u) - \Gamma D(e_u) + \Gamma[(1 - B)D(e_a^0) + BD(e_a^b)])\gamma^* = \Gamma D(e_p)\theta^* + \Gamma[(1 - B)D(e_a^0) + BD(e_a^b)]
- \Gamma \theta^*[(1 - B)D(e_a^0) + BD(e_a^b)]}$$
Solving for the steady-state proportion of non-discriminatory firms,

\[ \gamma^* = \Gamma \frac{\theta^* D(e_p) + (1 - \theta)[(1 - B)D(e_a^u) + BD(e_b^h)]}{(1 - \Gamma)D(e_u) + \Gamma[(1 - B)D(e_a^u) + BD(e_b^h)]}. \]

This is the result in Equation 35, and is highly similar to the steady-state result for prejudiced firms given in Equation 26, as is the interpretation. As non-discriminating firms are not profit-maximizing, they are subject to the same market forces that prejudiced firms are. As such, the steady state level of non-discriminatory firms in the market is less than the level of non-discriminatory firms in the larger population of potential firms.
REFERENCES


