Household Leverage, Housing Markets, and Macroeconomic Fluctuations

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Abstract

This paper examines implications of housing and household leverage on macroeconomic fluctuations under credit shocks in the presence of the zero lower bound (ZLB) on nominal interest rates. To this end, I incorporate housing in a standard deleveraging model and endogenize the debt limit of borrowers by tying it to the market value of collateral assets and credit market conditions. I find that, only with sufficiently high leverage, the nominal interest rate is more likely to hit the ZLB and macroeconomic fluctuations are more amplified under an adverse credit shock in the housing model with endogenous debt limits than in the nonhousing model with exogenous debt limits. In addition, the amplification is more pronounced in the housing model than in the nonhousing model when the ZLB is binding. Importantly, a binding ZLB plays a key role in generating a significant decline in the housing price under a particularly adverse credit shock.

JEL classification: E21, E31, E44, E58.

Keywords: Household leverage, Housing markets, House prices, ZLB, Deleveraging, Great Recession

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November 12, 2014

1. Introduction

There are three striking stylized facts from the last recession. First, there was a surge in household leverage, defined as a debt-to-income ratio, during the 2002-2006 period, as documented in Mian and Sufi (2011). Second, the recession was worse and the housing price fell more in the regions where household leverage had increased more. Third, the nominal interest rate has reached the ZLB since December 2008, worsening the recession because the convetional monetary policy is ineffective in reducing short-term nominal interest rates further to stimulate the economy.

Apparently, the high leverage, the housing market, and the ZLB play an important role in causing the worst recession that the U.S. has ever observed since the Great Depression. However, the standard deleveraging and ZLB literature that models debt limits as an exogenous stochastic process, including Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011), hereafter called the EK and GL models, have no implications of the deleveraging, the housing market, and the ZLB on asset prices and macroeconomic fluctuations under a credit shock.

This paper aims to fill the gap. To this end, I extend the standard deleveraging and ZLB model by incorporating a housing market and modeling the debt limit endogenously. Specifically, the debt limit of borrowers is tied to both exogenous credit market conditions and the endogenous market value of collateral assets, which are houses.[1] I then consider two cases, high leverage and normal leverage, with a more focus on the former case. In the case of high leverage, the housing model is calibrated to match the high debt-to-income ratio in the U.S. at the onset of the last recession. In the case of normal leverage, the model is calibrated to match the

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[1]We can generalize to any assets other than houses such as mortgage-backed securities and other financial assets.

normal debt-to-income ratio in 2002, right before the housing boom.

Without the high leverage characteristic, borrowers would use durable housing goods as a cushion to fight against adverse shocks to the credit market. As a result, the ZLB is less likely to bind and output is less volatile in the housing model with endogenous debt limit than the nonhousing model with exogenous debt limit, such as the EK model. This finding contradicts the widely held belief that adding houses and endogenizing the debt limit will always amplify output and inflation fluctuations under an adverse shock to the credit market because cutting durable good might cause the debt limit to fall more.

More salient is that, in the presence of high leverage, the housing model with endogenous debt limit generates a more powerful transmission mechanism than the nonhousing model does. The economic variables are more responsive to an adverse credit shock the ZLB is more likely to bind in the housing model. When the ZLB binds, a great recession emerges with a fall in output and the price level, mainly due to the impact of the Fisherian debt deflation and endogenous debt limits.

The intuition for the results is as follows: An adverse shock to the credit market lowers the debt limit and makes the borrowing constraint tighter given the other factors, so borrowers have to cut nondurable goods or sell some durable housing goods. If the initial debt-to-value ratio is small, selling a dollar of durable goods helps free up much of home equity that can be used to reduce pressure on cutting back more necessary non-durable goods. So the interest rate declines less in the housing model than in the nonhousing model. In this case, even though the level of debt is lower due to the reduction in collateral assets, the pressure on borrowing tightness can be reduced substantially.

However, in a world with high leverage, the initial debt-to-value ratio is very high. The home equity of borrowers is substantially low, even negative. Therefore,

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selling durable housing goods is not helpful in reducing the pressure on the borrowing tightness. Together with the fact that houses provide utility and adjusting houses is costly, the borrowers do not want to cut back their durable housing goods. However, because durable and non-durable goods are not perfectly substitutable, durable goods must be reduced when the nondurable goods consumption is cut back.

In both cases, with and without high leverage, we see the debt level declines due to two reasons. First, the adverse shock to the credit market lowers the debt limit and tightens the borrowing constraint. Second, the initial decline in the debt limit will lead to lower durable goods consumption that makes the debt limit fall more, and so on. This reinforcement generates a spiral decline in both durable goods consumption and the debt limit. However, only in the case of high leverage can the housing model generate a tighter borrowing constraint and a larger cut in the nominal interest rate, compared to the nonhousing model.

The monetary policy in this paper follows a simple Taylor rule, so the central bank cannot stabilize output and inflation perfectly under an adverse shock to the credit market. Therefore, output falls. In the framework of monopolistic competition, the price level falls, leading to a higher real debt burden of credit-constrained households, causing them to further reduce their consumption. This Fisherian debt deflation, associated with high leverage, is more likely to drive the economy to the ZLB. I show quantitatively that the Fisherian debt deflation is extremely powerful when the ZLB binds. It generates a fall in output and the price level.

Another important contribution of this paper is about the role of a binding ZLB condition in amplifying housing price fluctuations under a particularly adverse credit shock. Without the presence of the ZLB, the credit shock is not able to generate significant changes in the housing price. This result is in line with the common finding in the existing literature of housing and macroeconomic fluctuations. The

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reason is that the housing price is the expected present discounted value of housing utility flows. The credit shock is not able to alter the flows significantly without the presence of the ZLB because the central bank has some power to stabilize the economy, including the borrowers' housing consumption and the marginal utility of houses. As a result, the housing price does not move much under a credit shock, see Liu et al. (2013) for more detailed discussion.

However, this is not the case when the ZLB presents. In a deep recession with a binding ZLB due to an adverse credit shock, the housing price falls substantially. Specifically, under an adverse credit shock that causes the debt-to-value ratio to fall about 10 percent permanently, the decline in the housing price is three times greater in the model with the ZLB than in the model without the ZLB. Intuitively, due to the ZLB effect, the borrowers have to scale back their durable housing goods more, affecting the flow of housing services more. As a result, the housing price falls more when the ZLB condition is imposed.

The related literature on the ZLB has been inspired by seminal work by Krugman (1998). After his work, extensive research related to the ZLB has been conducted, including Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011) among others.[2].

Guerrieri and Lorenzoni (2011) model a debt limit and household heterogeneity in labor productivity. They show that a decline in the exogenous debt limit causes future consumption to be more volatile because with a lower debt limit, households will be less able to insure their consumption risks. Therefore, savers will save more and borrowers will borrow less due to precautionary savings, resulting in a sharp

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[2]A more extensive list includes Eggertsson and Woodford (2003), Adam and Billi (2006, 2007), Nakov (2008), Levin et al. (2010), Bodenstein et al. (2010), Werning (2011), Ngo (2014), Fernandez-Villaverde et al. (2012), Hall (2011), and Judd et al. (2011).

decrease in the nominal interest rate, causing the ZLB to bind.

Eggertsson and Krugman (2012) also model the debt limit and deleveraging as a key factor driving the nominal interest rate to the ZLB. In contrast to Guerrieri and Lorenzoni where savings come from precautionary behavior, Eggertsson and Krugman model savings based on the difference in the two types of representative households. One type is patient; the other is not. The patient representative household saves and lends his money to the impatient one. Like Guerrieri and Lorenzoni, Eggertson and Krugman model the debt limit as an exogenous process.

There are several recent independent working papers that incorporate and study housing in a general equilibrium model. Among these papers, Justiano et al. (2014) and Guerrieri and Iacoviello (2014) are closer to this paper. They also extend the standard deleveraging model, such as the EK and GL models, by tying the debt limit to the credit market condition (or the loan-to-value ratio) and the market value of collateral housing assets.

Like this paper, Justiano et al. (2014) also cast doubt on whether credit shocks could account for the behavior of home prices and mortgage debt during the housing boom period. However, they do not study the implication of high leverage. More importantly, they are not able to discuss the impact of the ZLB on home prices because the ZLB is not binding in their paper.

Guerrieri and Iacoviello (2014) also study the role of housing and collateral constraints on generating asymmetric macroeconomic fluctuations in a DSGE model where debt limits are endogenously linked to the market value of houses. Both their paper and this paper take into account the ZLB. They also show that a negative housing demand shock can push the economy into the ZLB and can generate large declines in inflation and output. The main difference is that they focus on housing demand shock whereas I focus on the shock to the credit market, or the debt-to-income

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ratio shock.

Recent work by Midrigan and Philippon (2011) and Favilukas et al. (2013) also incorporate housing into a DSGE model with credit shocks. However, this paper is more focused on whether the housing margin amplifies or dampens credit shocks in the presence of the ZLB. In their papers, the ZLB does not present.

The remainder of this paper is organized as follows. Section 2 presents the structure of the economy. Section 3 shows how key parameters are calibrated to match some facts in the U.S., and Section 4 reports main results and intuition based on the two-period assumption. Section 5 reports dynamic results under a permanent shock to the credit market without relying on the two-period assumption, and Section 6 shows some sensitivity analyses. Section 7 concludes.

2. Model

The model in this paper is a standard two-representative agent model, as found in Eggertsson and Krugman (2012) and Iacoviello (2005). There are two types of households: credit-constrained households (or borrowers) of mass χ_b, and unconstrained households (or savers) of mass $\chi_s = 1 - \chi_b$. The borrowers are impatient while the savers are patient and act as the lenders. The households consume nondurable goods and enjoy housing service from owning houses that have a fixed supply.

Houses play two roles in the model. First, they provide housing services to the households. Second, they can be used as collateral assets for borrowing. One of the two key features in our model is an endogenous debt limit that is determined by both the endogenous market value of houses and exogenous financial market conditions (determined by a debt-to-value ratio), as in Kiyotaki and Moore (1997) and Iacoviello (2005). This feature distinguishes this paper from the current literature of

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deleveraging and the ZLB, as in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2011), who model debt limits as an exogenous process.

2.1. The borrowing-constrained household's problem

The representative borrowing-constrained household chooses the path of non-durable goods, durable housing goods, real debts, and labor to maximize his expected present discounted lifetime utility subject to his budget constraint and borrowing constraint. His problem can be described mathematically as follows:

$$\max \quad E_0 \sum_{t=0}^{\infty} \beta_b^t U_{bt}(C_{bt}, H_{bt}, N_{bt}) \tag{1}$$

subject to the budget constraint:

$$C_{bt} + D_{bt-1}(1 + r_{t-1}) + q_t(H_{bt} - H_{bt-1}) + \frac{\phi_H}{2}\left(\frac{H_{bt}}{H_{bt-1}} - 1\right)^2 q_t H_{bt} = w_t N_{bt} + D_{bt} \tag{2}$$

and the borrowing constraint:

$$D_{bt}(1 + r_t) \le \xi_t E_t[q_{t+1} H_{bt}] \tag{3}$$

where

$$1 + r_t = \frac{1 + i_t}{1 + \pi_{t+1}}; \tag{4}$$

U is per-period utility; C, D_b, H_b, N_b are composite non-housing goods, real debts, housing quantity, and labor supply by the borrower respectively; i, π, r, q denote the nominal interest rate, the inflation rate, the real interest rate, and the real price of a house, respectively; ϕ_H is the housing adjustment cost parameter, showing how costly to adjust houses; and ξ reflects the credit market conditions. The credit

market shock follows an AR(1) process:

$$\ln\left(\xi_{t+1}\right) = \left(1 - \rho_\xi\right)\ln\left(\overline{\xi}\right) + \rho_\xi \ln\left(\xi_t\right) + \varepsilon_{\xi,t+1} \tag{5}$$

where $\epsilon_{\xi,t}$ is is independently and identically distributed with the mean 0 and variance σ_ξ^2; ρ_ξ presents the persistence of the credit shock. In this paper, I will investigate the case of permanent shocks, where $\rho_\xi = 1$.

Because the credit market condition ξ is a very important parameter, I would like to clarify two issues that could potentially arise. First, I interpret the parameter as a debt-to-market value of collateral assets ratio, or debt-to-value (DTV) ratio. The debt-to-value ratio can be interpreted as a cumulative loan-to-value ratio instead of a normal loan-to-value ratio. Using a cumulative loan-to-value-ratio allows us to capture high leverage, measured as the high debt-to-income ratio that we observed at the onset of the recent housing bust.

Second, I am not going to model why the parameter exists and why it is too high at some point. The rationale for the existence of the parameter could be an asymmetric information problem, and the rationale for why it is too high at some point comes from lenders' overly optimism about the likelihood of getting their money back. This overly optimism is grounded on an extended period of steady economic growth and/or rising asset prices, such as housing prices.

The endogenous debt limit in the paper is the value in the right-hand side of equation (3). It is a certain proportion ξ_t, which is exogenous, of the expected market value of collateral assets $E_t[q_{t+1}H_{bt}]$, which is endogenously determined in the model. In contrast, Eggertsson and Krugman (2012) model the debt limit exogenously. In their model, the deleveraging shock occurs when there is an exogenous downward revision of the debt limit due to a change in lenders' point of view toward the risk

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of borrowers or toward the collateral asset values. The sudden downward revision is called the Minsky moment. The main reason for using an exogenous debt limit in their model is to find out the closed form solution of two-period models.

In reality, it is hard to imagine how the debt limit is exogenously determined by lenders because most of loans in the U.S. are actually collateralized debts. What is reasonably exogenous is the proportion of the market value of collateral assets. An exogenous change in the proportion can trigger deleveraging that could potentially affect the market value of the collateral assets, leading to another round of deleveraging, and so on. In this paper, the time when the sudden change in the proportion ξ_t happens is considered as a Minsky moment.

Let λ_{bt}, ϕ_{bt} be the Lagrange multipliers with respect to the borrower's budget constraint and debt constraint. The optimal choices must satisfy the following conditions:

$$U_{bt,C} - \lambda_{bt} = 0 \tag{6}$$

$$\frac{-U_{bt,N}}{U_{bt,C}} = w_t \tag{7}$$

$$\lambda_{bt} - \phi_{bt} E_t \left[1 + r_t\right] = \beta_b E_t \left[\lambda_{bt+1} \left(1 + r_t\right)\right] \tag{8}$$

$$U_{bt,H} + \xi_t \phi_{bt} E_t \left[q_{t+1}\right] + \beta_b E_t \left[\lambda_{bt+1} q_{t+1} + \lambda_{bt+1} q_{t+1} \phi_H \left(\frac{H_{bt+1}}{H_{bt}} - 1\right) \left(\frac{H_{bt+1}}{H_{bt}}\right)^2\right]$$
$$= \lambda_{bt} q_t + \lambda_{bt} q_t \frac{\phi_H}{2} \left(\frac{H_{bt}}{H_{bt-1}} - 1\right)^2 + \lambda_{bt} q_t \phi_H \left(\frac{H_{bt}}{H_{bt-1}} - 1\right) \frac{H_{bt}}{H_{bt-1}} \tag{9}$$

$$\min \left\{\xi_t E_t \left[q_{t+1} H_{bt}\right] - D_{bt} \left(1 + r_t\right), \phi_{b,t}\right\} = 0 \tag{10}$$

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$$\phi_{bt} \geq 0 \qquad (11)$$

Equation (6) shows the marginal utility derived from consuming the composite non-durable goods. Equation (7) presents the intra-temporal trade-off between consumption and labor at the margin. Equation (8) is the Euler equation for the borrower, which is the inter-temporal trade-off between today's consumption and tomorrow's consumption. If the credit-constrained household consumes one unit of non-durable goods, he would receive utility from his consumption. In addition, he would put $(1 + r_t)$ more pressure on the collateral constraint that costs him $\phi_{bt}(1 + r_t)$ in terms of utility. Therefore, the left-hand side of the equation is the marginal benefit of consuming today, while the right-hand side is the marginal utility he has to forgo due to not saving.

The marginal trade-off between non-durable goods and durable housing goods is illustrated in equation (9). The left-hand side of the equation shows the marginal benefit of buying one more unit of houses. The marginal benefit includes: (i) housing services; (ii) the value of the debt limit that he would get by relaxing the collateral constraint due to owning more houses; and (iii) the next period's value of the houses in terms of utility and the housing adjustment cost saved due to having more houses today. The right-hand side of the equation is the marginal cost of buying houses. The borrowing constraint is rewritten as in equation (10). This equation is the combination of the collateral constraint and the non-negativity of the shadow value of debt, ϕ_{bt}.

2.2. The unconstrained household's problem

The representative unconstrained household never faces a borrowing constraint. He saves and lends to the credit-constrained households. He also owns intermediate-

goods firms. His problem is as follows:

$$\max \quad E \sum_{t=0}^{\infty} \beta_s^t U_{st} \left(C_{st}, H_{st}, N_{st} \right) \tag{12}$$

subject to the budget constraint:

$$C_{st} + D_{st-1} \left(1 + r_{t-1} \right) + q_t \left(H_{st} - H_{st-1} \right)$$
$$+ \frac{\phi_H}{2} \left(\frac{H_{st}}{H_{st-1}} - 1 \right)^2 q_t H_{st} = w_t N_{st} + \int_{i=0}^{1} Z_{it} di + D_{st} \tag{13}$$

where U is per-period utility; C, D_s, H_s, N_b are composite non-housing goods, real debts, housing quantity, and labor supply by the saver respectively; i, π, r, q denote the nominal interest rate, the inflation rate, the real interest rate, and the real price of a house, respectively; ϕ_H is the housing adjustment cost parameter, showing how costly to adjust houses; and Z denotes nominal profits from the ith intermediate-goods firms that are owned by the savers only.

Let λ_{st} be the Lagrange multiplier with respect to the budget constraint of the saver. The optimal choices of the saver must satisfy the following condition:

$$U_{st,C} - \lambda_{st} = 0 \tag{14}$$

$$\frac{-U_{st,N}}{U_{st,C}} = w_t \tag{15}$$

$$\lambda_{st} - \beta_s E_t \left[\lambda_{st+1} \left(1 + r_t \right) \right] = 0 \tag{16}$$

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$$U_{st,H} + \beta_s E_t \left[\lambda_{st+1} q_{t+1} + \lambda_{st+1} q_{t+1} \phi_H \left(\frac{H_{st+1}}{H_{st}} - 1 \right) \left(\frac{H_{st+1}}{H_{st}} \right)^2 \right]$$

$$= \lambda_{st} q_t + \lambda_{st} q_t \frac{\phi_H}{2} \left(\frac{H_{st}}{H_{st-1}} - 1 \right)^2 + \lambda_{st} q_t \phi_H \left(\frac{H_{st}}{H_{st-1}} - 1 \right) \frac{H_{st}}{H_{st-1}} \quad (17)$$

Equation (14) shows the saver's marginal utility derived from non-durable goods consumption. Equation (15) presents his marginal trade-off between consumption and labor. Equation (16) is the Euler equation for the saver, which is the intertemporal trade-off between today's consumption and tomorrow's consumption. The marginal trade-off between non-durable goods consumption and housing goods is illustrated in equation (17).

2.3. Final goods producers

There is a mass 1 of final goods producers who operate in a perfectly competitive market. Each final goods producer produces the consumption goods by aggregating a variety of differentiated goods using a CES technology. His problem is to maximize his contemporaneous profit:

$$\max P_t Y_t - \int P_t(i) Y_t(i) \, di \quad (18)$$

subject to

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}} \quad (19)$$

where y_{it} is the input of intermediate goods $i \in [0,1]$ and ε is the elasticity of substitution between differentiated goods.

The optimal decision of the final goods producer gives rise to the demand for the ith intermediate goods:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t \quad (20)$$

where P_t is the price level:

$$P_t = \left(\int P_t\left(i\right)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} \tag{21}$$

2.4. Intermediate goods producers

There is a mass 1 of intermediate goods firms. These firms are owned by the savers and are operated in a monopolistically competitive market. A firm's objective is to maximize its expected present discounted flows of profit. The firms adjust their prices according to a quadratic adjustment cost of Rotemberg's type. Firm i's problem is given below:

$$\max_{P_{it}, N_{it}} E_t^i \sum_{j=0}^{\infty} Q_{st,t+j} Z_{it+j} \tag{22}$$

subject to its demand function (Eq.20), profit

$$Z_{it} = \frac{P_{it}}{P_t} Y_{it} - w_t N_{it} - \frac{\phi_P}{2} \left(\frac{P_{it}}{P_{it-1}} - (1 + \theta\pi^*) \right)^2 Y_t, \tag{23}$$

a linear production technology

$$Y_{it} = N_{it}, \tag{24}$$

and the same initial price

$$P_{i0} = P_0; \tag{25}$$

where π^* is the target inflation; θ is the degree of price indexation; and ϕ_P is the price adjustment cost parameter.

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The optimality conditions give rise to the following condition:

$$\left(1 - \varepsilon + \varepsilon \frac{w_t}{A_t} - \phi_P \left(\pi_t - \theta \pi^*\right) \left(1 + \pi_t\right)\right) Y_t$$
$$+ \phi_P Q_{st,t+1} E_t \left[\left(\pi_{t+1} - \theta \pi^*\right) \left(1 + \pi_{t+1}\right) Y_{t+1}\right] = 0 \qquad (26)$$

where

$$Q_{st,t+1} = \beta_s \frac{U_{st+1,C}}{U_{st,C}} \qquad (27)$$

is the stochastic discount factor.

2.5. Monetary policy

The central bank conducts monetary policy using a simple Taylor rule as follows:

$$\left(\frac{1 + i_t}{1 + i}\right) = \left(\frac{Y_t}{Y^*}\right)^{\phi_y} \left(\frac{1 + \pi_t}{1 + \pi^*}\right)^{\phi_\pi} \qquad (28)$$

$$i_t \geq 0 \qquad (29)$$

where π^* and Y^* are the target inflation and output respectively. Equation (29) implies that the nominal interest rate is not allowed to be negative. This is the key condition in the literature of deleveraging and ZLB.

2.6. Aggregate conditions

In equilibrium, all the markets are cleared:

$$\chi_b H_{bt} + \chi_s H_{st} = \overline{H} \qquad (30)$$

$$\chi_b N_{bt} + \chi_s N_{st} = N_t \qquad (31)$$

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$$\chi_b D_{bt} + \chi_s D_{st} = 0 \tag{32}$$

$$\chi_b C_{bt} + \chi_s C_{st} + \chi_b \frac{\phi_H}{2} \left(\frac{H_{bt}}{H_{bt-1}} - 1 \right)^2 q_t H_{bt} + \chi_s \frac{\phi_H}{2} \left(\frac{H_{st}}{H_{st-1}} - 1 \right)^2 q_t H_{st}$$
$$= (A_t N_t) \left(1 - \frac{\phi_P}{2} (\pi_t - \theta \pi^*)^2 \right) \tag{33}$$

Equation (30) shows that the total demand for houses equals the total fixed housing supply. Equations (31), (32), and (33) present the market clearing conditions for labor, debts, and the non-housing composite goods markets respectively.

2.7. Equilibrium

Definition 1. *An equilibrium consists of the path of prices* $\{i_t, w_t, \pi_t, q_t\}_{t=0}^{\infty}$ *and the path of allocation* $\{C_{bt}, N_{bt}, D_{bt}, H_{bt}, C_{st}, N_{st}, D_{st}, H_{st}, C_t, N_t, Y_t, \phi_b\}_{t=0}^{\infty}$ *that satisfies the following conditions:*

(1) The borrowers' and savers' optimization conditions.

(2) The firms' optimization conditions.

(3) The aggregate conditions.

(4) The Taylor rule and the ZLB.

(5) The balanced government budget condition.

(6) The motion equations for the exogenous shocks.

Table 1: Benchmark Parameterization

Symbol	Description	Value
χ_b	Fraction of borrowers	0.60
χ_s	Fraction of savers	0.40
β_s	Subjective discount of savers	0.99
β_b	Subjective discount of borrowers	0.96
$\overline{H}$	Fixed stock of housing supply	1
γ_b, γ_s	CRRA parameters	1
ψ_b, ψ_s	Housing utility parameters	1
$\overline{\xi}$	Debt-to-value (DTV) ratio	0.91
j_s	Borrower's housing utility parameter	0.02
j_b	Saver's housing utility parameter	0.03
η_b	Borrowers' labor disutility parameter	1.02
η_s	Savers' labor disutility parameter	0.86
ϕ	Inverse labor supply elasticity	1/2
ϕ_P	Price adjustment cost paremeter	168
ε	Elasticity of substitution among differentiated goods	20
ϕ_H	Housing adjustment cost parameter	0.1
θ	Inflation indexation parameter	0
π^*	Target inflation rate of 2% per year	0.05
ϕ_π	Weight of inflation in the Taylor rule	1.5
ϕ_y	Weight of output in the Taylor rule	0.5

3. Calibration

The per-period utility functions for the borrowers and savers are specified as below:

$$U_{bt} = \frac{C_{bt}^{1-\gamma_b}}{1-\gamma_b} + j_b\frac{H_{bt}^{1-\psi_b}}{1-\psi_b} - \eta_b\frac{N_{bt}^{1+\phi}}{1+\phi}$$

$$U_{st} = \frac{C_{st}^{1-\gamma_s}}{1-\gamma_s} + j_s\frac{H_{st}^{1-\psi_s}}{1-\psi_s} - \eta_s\frac{N_{st}^{1+\phi}}{1+\phi}$$

I calibrate the parameters based on the observed data in the U.S. and on the other

studies. The key parameters are presented in Table 1. The subjective discount factor for the savers, β_s, is 0.99, corresponding to the real interest rate of 4% per year. The subjective discount factor for the borrower, β_b, is 0.96. The fraction of borrowers, χ_b, is 0.60, corresponding to 60% of households are borrowing constrained. These numbers are close to the values reported by empirical studies, and they are in the range used in the existing literature of housing and macroeconomic fluctuation, see Iacoviello (2005) for more detailed discussion.

The constant relative risk aversion parameters for the borrower and the saver are calibrated to be 1, or $\gamma_b = \gamma_s = 1$, corresponding to log utility function with respect to composite nondurable consumption goods. The housing demand elasticities for the borrower and the saver are 1, or $\psi_b = \psi_s = 1$. These parameters are commonly used in the existing literature, see Iacoviello (2005). The labor supply elasticity with respect to wages is chosen to be 2, or $\phi = 1/2$, which is in the range commonly used in the literature of ZLB and deleveraging, such as Eggertsson and Krugman (2012).

The borrower labor disutility parameter, η_b, and the saver labor disutility parameter, η_s, are calibrated to match the steady state labor supply that is normalized to 1 for both the borrower and the saver. The fixed stock of housing supply, $\overline{H}$, is also normalized to 1. These parameters can change the scale of some steady state values, but they are almost irrelevant to the results of the paper.

I calibrate the initial debt-to-value ratio, $\overline{\xi}$, to match three facts: (i) the total debt to income ratio is 1.32 in the fourth quarter of 2006 - the onset of the housing burst. The total debt is measured by the total liabilities of households and nonprofit organizations, while the total income is measured by the personal disposable income of households and nonprofit organization; (ii) the total housing wealth to income ratio is around 2.45. The total housing wealth is measured by the households and nonprofit organization real estate at market value. The values of total debt, income,

and housing wealth are from the Flows of Funds Tables reported by the Federal Reserve System; and (iii) on average, each household owns a house, or H_b/H_s is 1.

These facts result in the debt-to-value ratio, $\bar{\xi}$, of around 0.91 that I consider as high leverage.[3] This value is smaller than 0.95, the maximal value of cumulative loan-to-value ratio for the first-time home buyers in the U.S. reported by Duca et al. (2011). However, it is larger than some values estimated in the existing literature, i.e. 0.89 in Iacoviello (2005). These facts are also used to uniquely determine the values of the housing utility parameters for the saver and the borrower, j_s and j_b.

In addition to using the high debt-to-income ratio of 1.32, I investigate the case when the debt-to-income ratio is 1, the value that we observed in the U.S. in the first quarter of 2002, right before the housing boom period. This lower value of the debt-to-income ratio results in the debt-to-value ratio, $\bar{\xi}$, or 0.68 that I consider as normal leverage.

For the Taylor rule, I choose the inflation target to be 2 percent per year, or $\bar{\pi} = 0.005$. The weight of inflation, ϕ_π, and the weight of output, ϕ_y are set at the conventional values, 1.5 and 0.5, respectively. Note that the conventional value, $\phi_\pi = 1.5$, is used when the ZLB is not imposed. In the presence of the ZLB and in this framework, using this conventional value is likely to produce very unstable results if prices are very sticky, i.e. when the average duration of keeping prices unchanged is 4 quarters or longer.

The demand elasticity for differentiated goods is calibrated to be 20, corresponding to a net markup of 5%, which is similar to the value used in Iacoviello (2005). The benchmark inflation indexation parameter, θ, is set at 0. The price adjustment

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[3]I use the debt-to-income ratio instead of the debt-to-GDP ratio to measure the leverage because income captures the flow of wealth more precisely.

cost parameter, ϕ_P, is calibrated to be 168, corresponding to keeping a price unchanged for 3 quarters on average.[4] The housing adjustment cost parameter, ϕ_H, is set at 0.1, which corresponds to a realtors fee of 5% of the total transaction value for selling and buying a housing of the size at the steady state. This housing adjustment cost parameter is important in the paper. I will conduct some analyses regarding the sensitivity of the results with respect to different values of the housing adjustment cost parameter in a separate section.

4. Results: A two-period deleveraging model

To provide intuition about the mechanism of the model, in this section, I work with a simple two-period deleveraging model. The timing of the model is similar to the one in Eggertsson and Krugman (2012). At time 0, the economy stands at an initial steady state associated with a certain debt-to-value ratio $\xi = \bar{\xi}$. Then a permanent shock to the credit market occurs at time 1, so ξ changes to ξ'. The representative households choose new debts, housing quantities, nondurable consumption goods, and labor at time 1. The economy returns to the new steady state associated with the new value of the debt-to-value ratio, ξ', at time 2.[5]

4.1. The case without the ZLB

To understand the role of the ZLB in the following section, in this section I provide the results from the case with high leverage and without the ZLB, as presented in Figure 1. The x-axis shows a permanent annualized percentage change in the credit

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[4]See Miao and Ngo (2014) for how to calculate this duration.

[5]See Appendix D for more details on how to solve multi-period models, including two-period models. Note that, for a two-period model, the economy is forced to return to the new steady state in period 2 as in Eggertsson and Krugman (2012). However, for a multi-period model, the economy is allowed to gradually return to the new steady state.
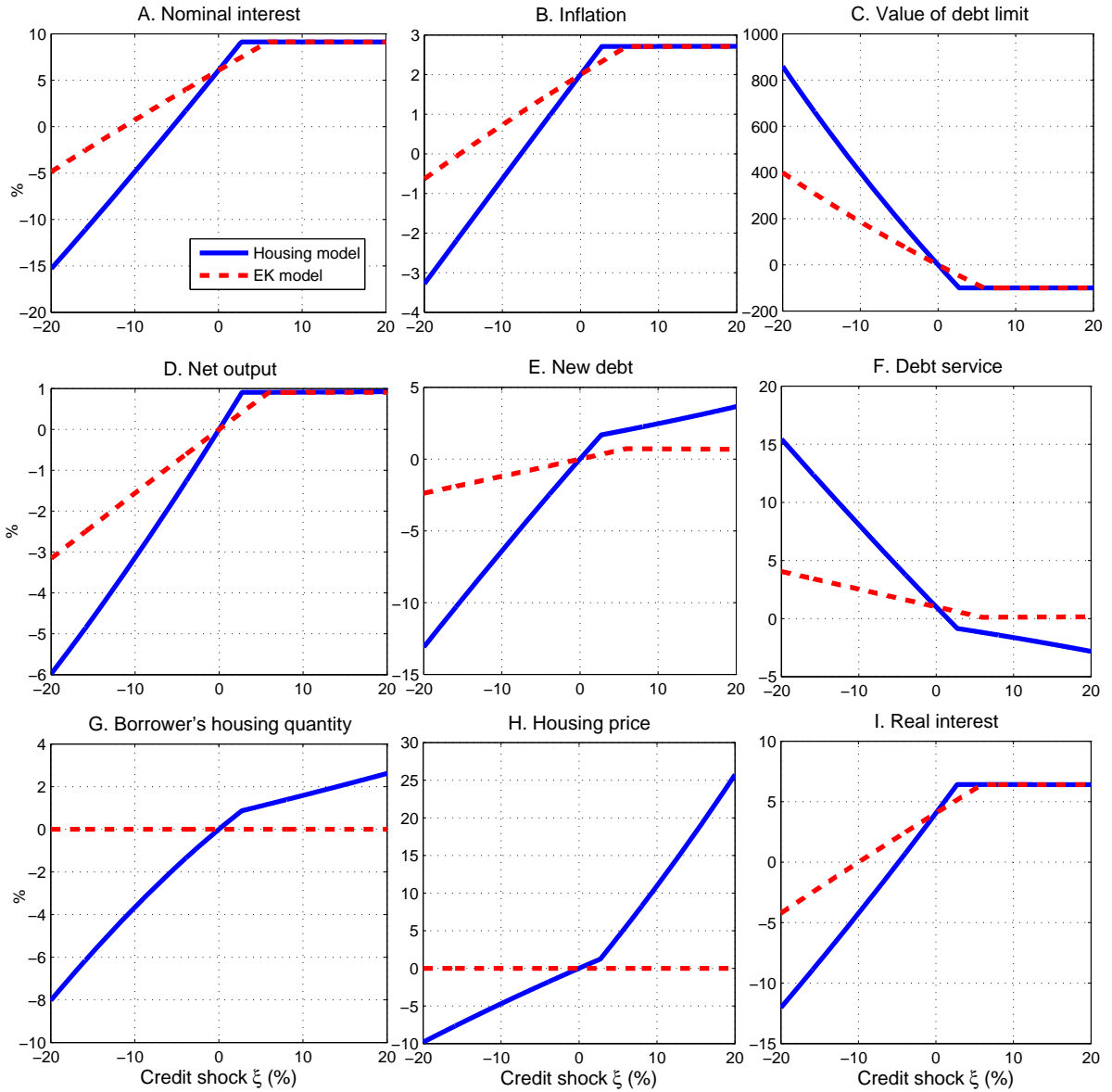
Figure 1: Responses of selected economic variables in period 1 under a permanent credit shock (ξ). The case with high leverage and without the ZLB. Values are percentage deviations from the initial steady state and normalized to yearly responses for the nominal interest rate (i), the inflation rate (π), the real interest rate (r), the housing price (q), and the credit shock (ξ). The EK model is the nonhousing model with exogenous debt limits.

market parameter (ξ) from the initial steady state value. The short-run responses of selected macro economic variables in period 1 are presented in the y-axis.

The solid blue lines present the results from the housing model with endogenous debt limit, while the dashed red lines show the results from the nonhousing model with exogenous debt limit, or the EK model. The intersections between the solid blue lines and the dashed red lines presents the initial steady state values. In the EK model, the housing price and housing quantities for both borrowers and the savers are kept unchanged.

Panels D, E, G, and H of Figure 1 show the percentage change of net output, new debt, housing quantity of the borrowers, and housing price from the initial steady state values respectively. The net output is the total output minus the price adjustment cost. Panel F of Figure 1 shows debt service as a percentage of the initial debt, where debt service is computed as debt payment minus new debt. A decline in debt services means that the borrowers can borrow more than the amount of debt payment.

Under a positive shock to the debt-to-value ratio, from Figure 1 we see that, in the housing model, the value of debt limit and debt service decline, while borrowers' new debt and new housing quantity, total output, inflation, and interest rates all increase in equilibrium. Intuitively, when the debt-to-value ratio increases initially, the debt limit faced by the borrowers increases, and they are allowed to borrow more given the other factors. As a result, the shadow value of the debt limit decreases and the borrowers borrow more. Hence, both borrowers' nondurable and durable housing consumptions increase.

The increase in the housing quantity leads to another increase in the debt limit, encouraging borrowers to borrow and spend more, and creating another round of expansion. The debt service declines because the borrowers are not only able to roll

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over their debt, but are also able to borrow more. In the monopolistic framework, inflation rises when output increases. Under a simple Taylor rule, the nominal interest rate increases.

However, up to a certain value, a further increase in the debt-to-value ratio would not alter the responses of some macroeconomic variables. After that threshold, borrowers are allowed to borrow up to the amount they want. In other words, the borrowers no longer face a credit constraint. Therefore, the shadow value of the debt limit is zero. The inflation and nominal interest rate hit the upper bounds of around 2.8% and 10.0% per year respectively.

The opposite mechanism occurs under a negative shock to the credit market. In this circumstance, borrowers are not able to borrow as much as before given the other factors. The debt limit decreases, while the shadow value of the debt limit rises. Therefore, new debt falls and debt service increases. As a result, the borrowers' nondurable and housing consumption fall due to deleveraging. Because the monetary policy is a simple Taylor rule, it is not powerful enough to stabilize output and inflation. Hence, output declines and, as a result, disinflation occurs. The responses are also amplified by the collateral effect and debt deflation effect.

The most interesting and important feature is that the model with housing generates more amplified responses of macroeconomic variables to a credit market shock compared to the EK model. The finding contradicts the belief that adding durable goods will help to mitigate the likelihood of the nominal interest rate hitting the ZLB, and, as a result, reducing macroeconomic fluctuations.

Let us look at the scenario under a negative shock. Panel A in Figure 1 shows that the nominal interest rate in the housing model falls more than in the EK model. Although we do not study the impact of the ZLB in this section, it is worth noting that, compared to the EK model, the nominal interest rate reaches zero and below

more frequently in the housing model with high leverage. In other words, to drive the nominal interest rate to zero and below, it requires a smaller negative credit shock in the housing model than in the EK model.

In addition, the housing model produces a bigger decline in inflation and output, as in Panels B and D of Figure 1. The borrowers suffer a tighter collateral constraint in the housing model than in the EK model, as shown in Panel C where the shadow value of debt limit is higher in the housing model than in the EK model.

The more amplified transmission mechanism of the housing model can be found only when we allow for high leverage. As explained above, by high leverage, I calibrate the parameter $\bar{\xi}$ to match the high debt-to-income ratio at the onset of the housing bubble burst. The ratio is substantially high: around 1.32. Without the high leverage, we cannot generate such results.

To demonstrate the role of high leverage, I report the results from the case with normal leverage and without the ZLB in Figure 2. In the case of normal leverage, I calibrate the initial debt-to-value of collateral asset to match with the debt-to-income ratio in the first quarter of 2002, right before the 2002-2006 period of housing and credit boom. As documented by the Federal Reserve System, in the Flows of Funds Tables, the debt-to-income ratio was around 1 in the first quarter of 2002, instead of 1.32 as in the benchmark calibration. This results in a lower debt-to-value ratio that is around 0.68, smaller than 0.91 as in the case of high leverage.

As in Panel A of Figure 2, under a negative demand shock, the decrease in the nominal interest rate in the housing model is not as big as in the EK model. Compared to the EK model, the nominal interest rate reaches zero and below less frequently. In other words, to drive the nominal interest rate to zero and below, it requires a larger credit shock in the housing model than in the EK model.

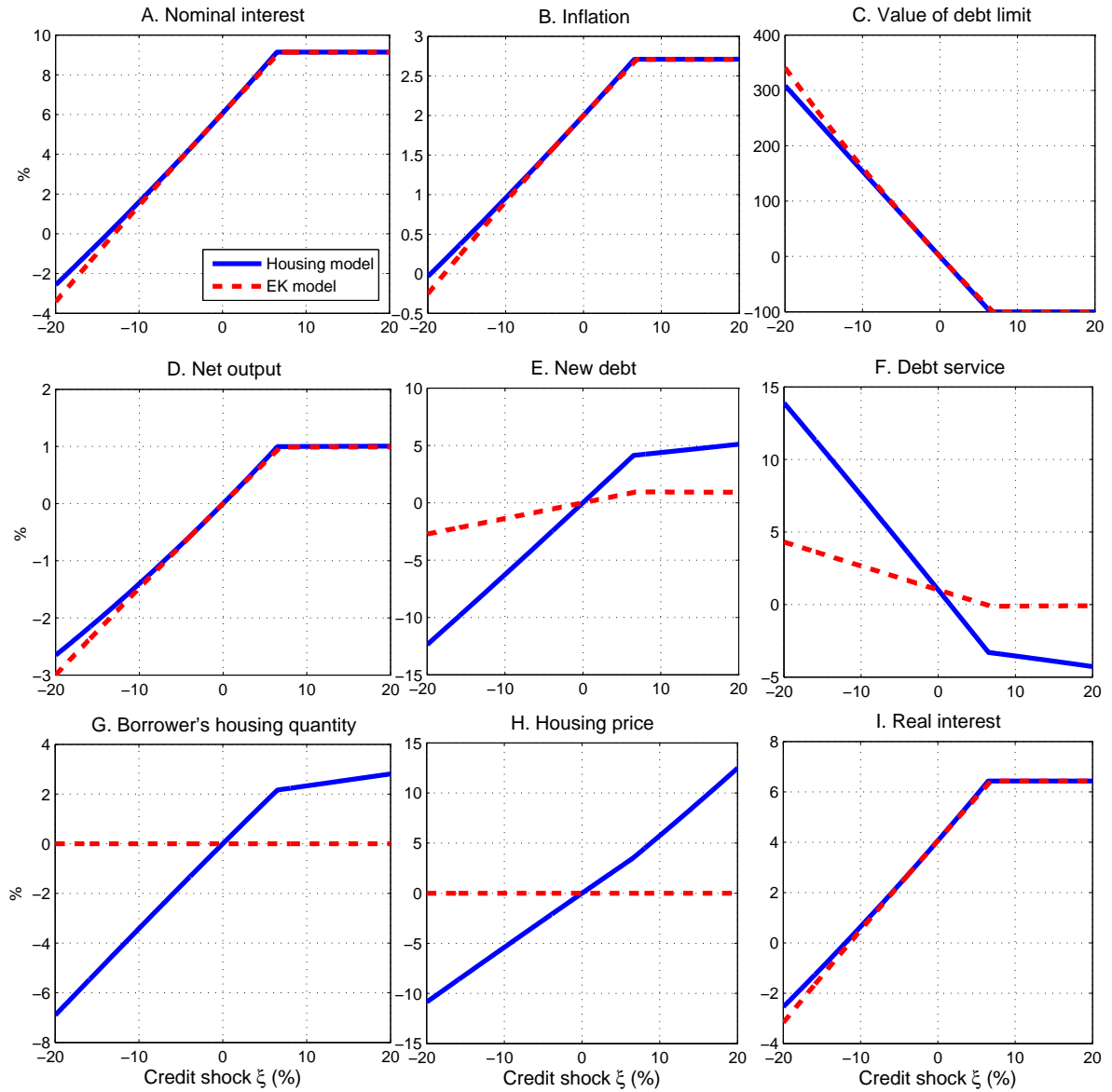Inflation and output fluctuate less in the housing model with normal leverage than

Figure 2: Responses of selected economic variables in period 1 under a permanent credit shock (ξ). The case with normal leverage and without the ZLB. Values are percentage deviations from the initial steady state and normalized to yearly responses for the nominal interest rate (i), the inflation rate (π), the real interest rate (r), the housing price (q), and the credit shock (ξ). The EK model is the nonhousing model with exogenous debt limits.

in the EK model. Although, we see a higher debt service and a smaller new debt in the housing model, the shadow value of debt limit is actually smaller in this model compared to the EK model. This means that the slackness of the collateral constraint is smaller in the housing model. The result contradicts to the common belief that the endogenous debt limit would always amplify macroeconomic fluctuations under shocks.

The intuition for the results is as follows: An adverse shock to the credit market lowers the debt limit initially and makes the borrowing constraint tighter given the other factors. So the borrowers have to cut nondurable goods or sell some durable housing goods. If the initial debt-to-value ratio is small, selling a dollar of durable goods helps free up much of home equity that can be used to reduce pressure on cutting back the more necessary non-durable goods. In this case, even though the level of debt is lower due to the reduction in collateral assets, the pressure on borrowing tightness can be reduced substantially compared to the EK model.

However, in a world with high leverage, the initial debt-to-value ratio is very high. The home equity of borrowers is substantially low, even negative. Therefore, selling durable housing goods is not helpful in reducing the pressure on the borrowing tightness. Together with the fact that houses provide utility and adjusting houses is costly, the borrowers do not want to cut back their durable housing consumption. However, because durable and non-durable goods are not perfectly substitutable, durable goods must be reduced when nondurable goods consumption is cut back.

In both cases, with leverage and with high leverage, we see the debt level decline due to two reasons. First, the adverse shock to the credit market lowers the debt limit initially and tightens the borrowing constraint. Second, the initial decline in the debt limit leads to lower durable goods consumption that makes the debt limit fall more, and so on. This reinforcement generates a spiral decline in both durable

goods consumption and the debt limit. However, only in the case of high leverage can the model generate a tighter borrowing constraint, compared to the standard EK model.

There is another way to explain why the endogenous debt model with high leverage can generate more amplified responses of economic variables to credit market shocks. In general, through the budget constraint (2), a one-dollar decrease in durable goods consumption would help to lower the current debt by one dollar. As a result, from the collateral constraint (3), the reduction relaxes the collateral constraint by R_t dollar, where R_t is the gross real interest rate. However, by reducing one dollar of durable housing goods, the borrowers have to give up the utility from the housing service and incur housing adjustment costs.

More importantly, reducing one dollar of durable goods will lead to a fall of the debt limit, which depends on the expected housing price and financial market conditions, as in equation (2). In the model with high leverage, the initial debt-to-value ratio is high. Therefore, it is more costly to cut durable housing goods because it will put more pressure on the collateral constraint due to the reduction of collateral assets. This additional pressure is more than the relaxation thanks to a lower debt, which results from cutting back durable goods. Hence, the borrowers do not want to cut durable housing goods. However, because durable and non-durable goods are not perfectly substitutable, durable goods must be reduced when nondurable goods consumption is cut.

In contrast, in the model with normal leverage, the initial debt-to-value is low. It is not too costly to cut durable goods because the additional pressure on the collateral constraint due to the reduction of the collateral asset can be offset by a lower level of debt resulting from scaling back durable good consumption. Therefore, cutting back durable good consumption is desirable. In both cases, we see a spiral decline

in both durable goods consumption and the debt limit. However, only in the case of high leverage can the model generate a tighter borrowing constraint compared to the standard EK model.

4.2. The case with the ZLB

In this section, I study the impact of an adverse shock to the credit market in the case of high leverage and in the presence of the ZLB. The results are presented in Figures 3. The transmission mechanism becomes more powerful when the ZLB binds. The total output and inflation in the economy drop under a shock that cause the ZLB to bind, as seen in Panels B and D of Figure 3.

The fall in output and inflation result from two main channels. First, from the collateral constraint, as in equation (3), when there is an adverse shock to the credit market conditions ξ_t, both the nominal interest rate i_t and the new debt D_{bt} fall. When the nominal interest hits the zero bound, more downward pressure will be put on the new debt, leading to a larger deleveraging compared to the case without the ZLB. In other words, a binding ZLB amplifies the collateral effect.

Second, compared to the model without the ZLB, the real interest payment is higher in the ZLB model due to the inability of the central bank to set a negative nominal interest rate. From Panel I of Figure 3, the real interest rate soars up when the economy is in a deep recession with a binding ZLB. This occurs because the central bank is not able to lower the nominal interest rate to the desired level that it would in the absence of the ZLB.

Specifically, when the economy is hit by an adverse credit shock that causes the debt-to-value to be 10% permanently lower than the initial value, the nominal interest reaches the ZLB and the real interest rate increases to around 10% per year. This real interest rate is much higher than the desired level of around -4%, as in
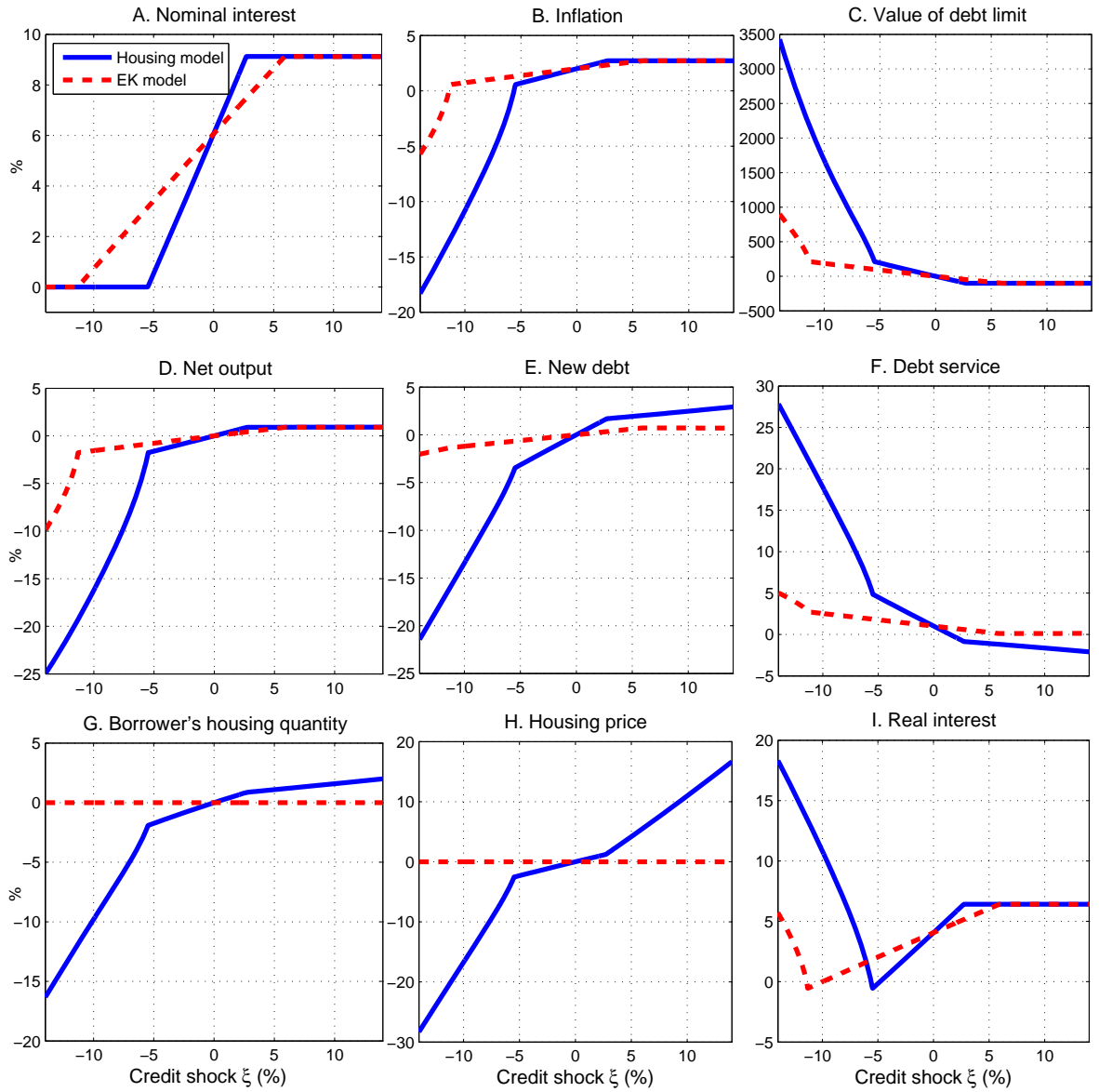
Figure 3: Responses of selected economic variables in period 1 under a permanent credit shock (ξ). The case with high leverage and with the ZLB. Values are percentage deviations from the initial steady state and normalized to yearly responses for the nominal interest rate (i), the inflation rate (π), the real interest rate (r), the housing price (q), and the credit shock (ξ). The EK model is the nonhousing model with exogenous debt limits.

Panel I of Figure 1, when the ZLB condition is not imposed. The increase in the real interest rate causes the debt burden to rise, making the borrowers' budget for consumption shrink substantially. This channel is called the Fisherian debt deflation.

To illustrate this powerful amplification mechanism of Fisherian debt deflation, I provide a comparison between the model with and without a Fisherian debt deflation channel in Figure 4. In the case without Fisherian debt deflation, the nominal interest rates are indexed by inflation. Hence, the real debt is constant no matter how large inflation or deflation is.

The solid blue lines show the results from the model with Fisherian debt deflation, while the dashed red lines present the results from the model without Fisherian debt deflation. Including the Fisherian debt deflation not only makes the ZLB to bind more easily, as in Panel A of Figure 4, but also makes a recession worse when the ZLB binds, as in Panels B and D of Figure 4.

From Panel A of Figure 1, the nominal interest rate reaches zero and below more frequently in the housing model than in the EK model. Given that the ZLB and the endogenous debt limit are key ingredients of the analysis and given that comparison with the EK model is an important theme of this paper, we would like to see how the endogenous debt limit with high leverage affects macroeconomic variables at the ZLB, relative to the EK model.

As seen in Panel A of Figure 3, now it is not surprising that the nominal interest is more likely to hit the ZLB in the housing model with high leverage than in the EK model. When the ZLB binds in both models, the housing model with high leverage generates more severe recessions than the EK model does. Interestingly, the two models generate very similar results during booms due to positive shocks.

Apparently, the results contradict to the prediction by Eggertsson and Krugman (2012) that adding durable housing goods in the standard deleveraging model would
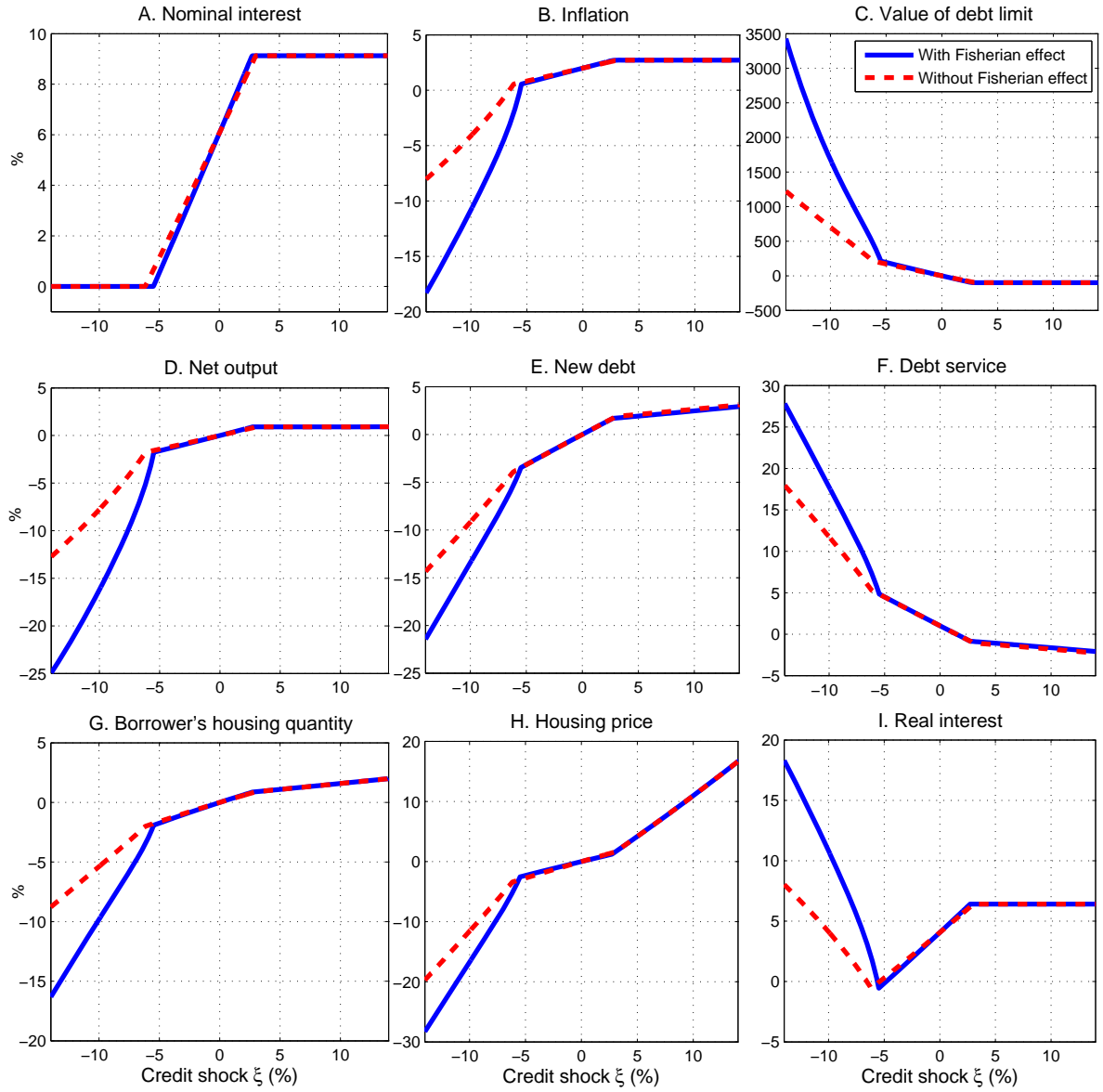
Figure 4: Responses of selected economic variables in period 1 under a permanent credit shock (ξ). The case with housing, high leverage, and the ZLB. Values are percentage deviations from the initial steady state and normalized to yearly responses for the nominal interest rate (i), the inflation rate (π), the real interest rate (r), the housing price (q), and the credit shock (ξ).

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help the economy to fight against a deep recession with a binding ZLB. It is because households would sell the durable housing goods to prevent the fall in the nondurable goods consumption. As explained in the previous section, the prediction is not right when the leverage is high. The equity in the housing durable goods is low, even negative due to the sharp decline in the housing price.

Now it is time for us to discuss the role of the credit shock in explaining the housing price fluctuations. It seems to be a consensus in the literature of housing and macroeconomic fluctuations that the credit shock is not able to generate a sharp decline in the housing price. It would be interesting to see if the point of view is supported in the analysis.

In the case without the ZLB. Panel H of Figure 1 shows that a 10% permanent decline in the debt-to-value ratio causes the housing price to decrease only about 5% per year. This result is in line with the existing literature, see more discussion in Liu et al. (2013). The intuition is that the housing price is the present discounted flow of housing services. Without the ZLB, the central bank is able to mitigate the fluctuation of the economy, including the fluctuation of the shadow value of debt limit and the borrowers' housing consumption, by adjusting the nominal interest rate freely, even to a negative level.

However, in the presence of the ZLB, the central bank is not able to lower the nominal interest rate below zero under an adverse credit shock. As a result, the borrowers have to deleverage by scaling back durable housing goods substantially. In the presence of the high leverage, the deleveraging causes the shadow value of debt limit to increase and the marginal benefit of owning houses to decrease. Consequently, the housing price drops. Particularly, under the negative 10% permanent credit shock, the housing price falls more than 15% per year, three times higher than the case without the ZLB.
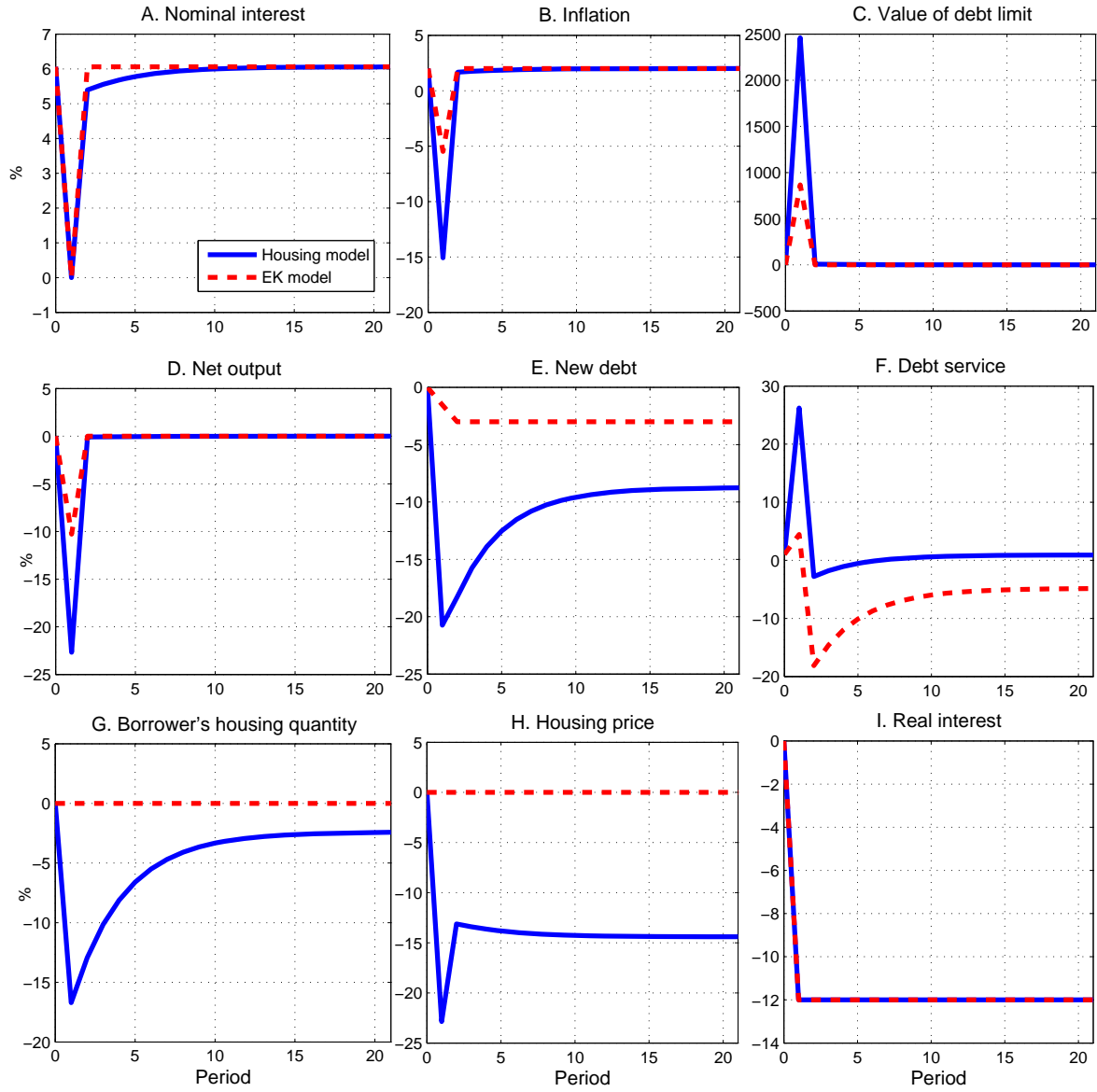
Figure 5: Impulse responses of selected economic variables under a permanent credit shock (ξ). The case with high leverage and with the ZLB. Values are percentage deviations from the initial steady state and normalized to yearly responses for the nominal interest rate (i), the inflation rate (π), the real interest rate (r), the housing price (q), and the credit shock (ξ). The EK model is the nonhousing model with exogenous debt limits.

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5. Results: A multi-period deleveraging model

In this section, instead of using the two-period forced deleveraging assumption, I allow the variables to adjust gradually toward the steady state after the occurrence of a permanent credit shock. The transition from the old steady state to the new steady state can be much longer than two periods. The permanent credit shock causes the debt-to-value ratio to decrease by 12% permanently.

As we know from the previous section, in the housing model with high leverage, a negative shock of around 5% per year is able to make the nominal interest rate reach the ZLB. However, I decide to choose a negative shock of 12% to ensure that the ZLB binds in the EK model too. Thus, we are able to see the different responses between the two models in a period with a binding ZLB.

The impulse response functions are presented in Figure 5. The solid blue lines present the results from the housing model with high leverage and with the ZLB, while the dashed red lines show the results from the EK model. Panel I of Figure 5 shows that the debt-to-value ratio declines 12% in period 1, then stays there permanently.

It is interesting to note that although we allow for multi-period adjustment, the forced deleveraging phenomenon still exists under a one-time permanent credit shock. From Figure 5, in the housing model, the borrowers' new debt drops and overshoots the new steady state value at time the shock occurs. The nominal interest rate hits the ZLB only one period. The economy experiences a deep recession in the period when the shock occurs. Afterward, the debt increases and converges to the new steady state value, which is lower than the initial value. The new steady state values for the borrowers' housing quantity and the housing price are also lower than the initial steady state values.

Compared to the two-period model where all the economic variables returns to the new steady state in the second period, in this multi-period case it takes about 15 quarters for the economy to converges to the new steady state. However, the dynamic responses of some variables, including the nominal interest rate, inflation, net output, and the value of debt limit, are strikingly similar to those from the two period model. Specifically, these variables returns to the new steady state almost immediately after the occurrence of the permanent credit shock.

6. Sensitivity analysis

The benchmark housing adjustment cost parameter is set at 0.1, corresponding to 5% transaction fees of buying or selling a house of the size at the steady state. These fees are less than 8% used in Duca et al. (2011). In this section, I would like to see how a change in the housing adjustment cost parameter would affect the results. To this end, I solve the model with housing at different values of the housing price adjustment cost parameter. The results are reported in Figure 6.

The solid blue lines show the results from the benchmark model where the housing adjustment cost parameter is set at 0.1; The dot-dashed red lines show the results when the parameters is set at 0.20, corresponding to 10% transaction fees of buying or selling a house of the size at the steady state; The dashed blue lines presents the results with the parameter of 0.02, or 1% transaction fees.

From Figure 6, the higher the housing adjustment cost, the more likely the nominal interest rate hits the ZLB. More importantly, the impact of the housing adjustment cost on output, inflation, and the housing price is strikingly amplified when the ZLB binds. Specifically, when the ZLB binds, all of these variables fall substantially more with an increase in the housing adjustment cost parameter, as seen in Panels D, B, and H of Figure 6.
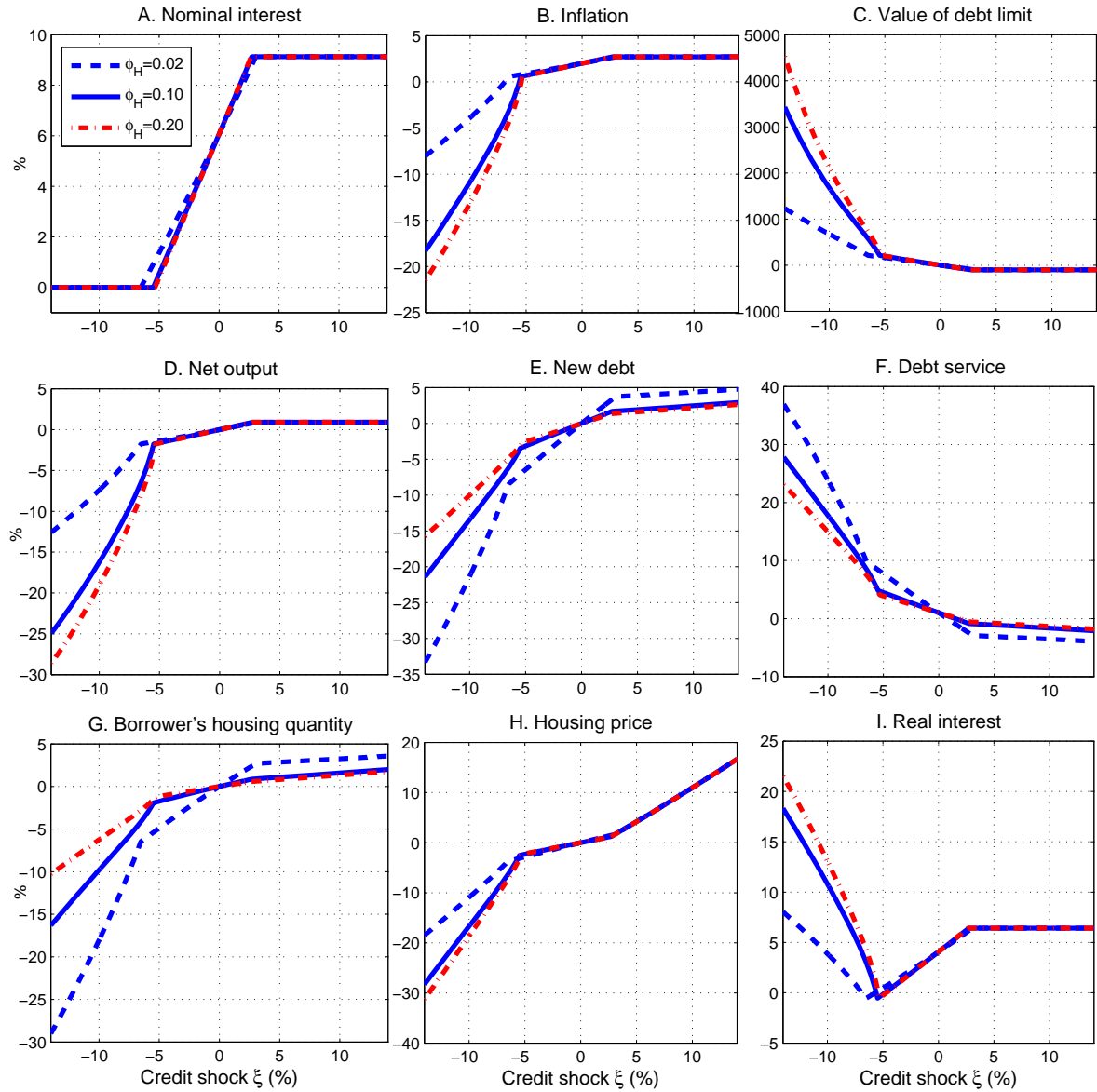
Figure 6: Responses of selected economic variables in period 1 under a permanent credit shock (ξ). The case with housing, high leverage, and the ZLB. Values are percentage deviations from the initial steady state and normalized to yearly responses for the nominal interest rate (i), the inflation rate (π), the real interest rate (r), the housing price (q), and the credit shock (ξ).

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The intuition behind these results is that the more costly the housing adjustment, the less important the durable housing good is as a cushion to mitigate the impact of an adverse shock to the borrowing constraint. More specifically, the higher the housing adjustment cost, the less equity is left from selling a house. We can see this point in Panel C of Figure 6 where the value of debt limit is higher with a higher housing adjustment cost parameter given a shock driving the economy to the ZLB.

7. Conclusion

I have already shown that incorporating high leverage and endogenizing the debt limit can generate a powerful mechanism that transmits a credit shock to a deep recession. With these features, the ZLB is more likely to bind in the housing model than in the EK model. When it binds, a great recession emerges, mostly due to the reinforcement between the increase in the real debt burden faced by credit-constrained households and the fall of the endogenous debt limit due to deleveraging.

The results from the housing model with high leverage and the ZLB challenge the widely held belief that a credit shock is not able to generate a drop in the housing price and that adding the housing durable goods in the standard deleveraging model would help the economy mitigate the negative impact of a particularly adverse credit shock.

There are several directions in which to extend the paper. It would be interesting to extend the model by investigating different monetary policy regimes instead of using a simple Taylor rule. By doing so, we might answer how monetary policy would be conducted optimally in this framework. In addition, it is important to see what kind of fiscal policy would be best in cases where the ZLB binds.[6]

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[6]Investigating how large the government expenditure multiplier is in the framework of endoge-

8. Acknowledgements

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nous debt limit and the ZLB, as described in this paper, is included in my future research agenda.

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Appendix A. Equilibrium equations

The system of equations governing the equilibrium of the model is as follows:

$$C_{bt} + D_{bt-1}\left(\frac{1+i_{t-1}}{1+\pi_t}\right) + q_t\left(H_{bt} - H_{bt-1}\right)$$

$$+\frac{\phi_H}{2}\left(\frac{H_{bt}}{H_{bt-1}} - 1\right)^2 q_t H_{bt} - w_t N_{bt} - D_{bt} = 0 \tag{A.1}$$

$$\frac{\eta_b N_{bt}^{\phi}}{C_{bt}^{-\gamma}} - w_t = 0 \tag{A.2}$$

$$C_{bt}^{-\gamma_b} - \phi_{bt}\left(1+i_t\right)E_t\left[\frac{1}{1+\pi_{t+1}}\right] - \beta_b\left(1+i_t\right)E_t\left[\frac{C_{bt+1}^{-\gamma_b}}{1+\pi_{t+1}}\right] = 0 \tag{A.3}$$

$$j_b H_{bt}^{-\psi} + \xi_t\phi_{bt}E_t\left[q_{t+1}\right] + \beta_b E_t\left[C_{bt+1}^{-\gamma}q_{t+1}\left(1+\phi_H\left(\frac{H_{bt+1}}{H_{bt}} - 1\right)\left(\frac{H_{bt+1}}{H_{bt}}\right)^2\right)\right]$$

$$-C_{bt}^{-\gamma}q_t\left(1+\frac{\phi_H}{2}\left(\frac{H_{bt}}{H_{bt-1}} - 1\right)^2 + \phi_H\left(\frac{H_{bt}}{H_{bt-1}} - 1\right)\frac{H_{bt}}{H_{bt-1}}\right) = 0 \tag{A.4}$$

$$\max\left\{-\xi_t H_{bt}E_t\left[q_{t+1}\right] + D_{bt}\left(1+i_t\right)E_t\left(\frac{1}{1+\pi_{t+1}}\right), 0 - \phi_{bt}\right\} = 0 \tag{A.5}$$

$$\chi_b C_{bt} + \chi_b\frac{\phi_H}{2}\left(\frac{H_{bt}}{H_{bt-1}} - 1\right)^2 q_t H_{bt} + \chi_s C_{st} + \chi_s\frac{\phi_H}{2}\left(\frac{H_{st}}{H_{st-1}} - 1\right)^2 q_t H_{st}$$

$$-Y_t\left(1 - \frac{\phi_P}{2}\left(\pi_t - \theta\pi^*\right)^2\right) = 0 \tag{A.6}$$

$$\frac{\eta_s N_{st}^{\phi}}{C_{st}^{-\gamma}} - w_t = 0 \tag{A.7}$$

$$C_{st}^{-\gamma} - \beta_s \left(1 + i_t\right) E_t \left[\frac{C_{st+1}^{-\gamma}}{1 + \pi_{t+1}}\right] = 0 \tag{A.8}$$

$$j_s \left(\frac{\overline{H} - \chi_b H_{bt}}{\chi_s}\right)^{-\psi} + \beta_s E_t \left[C_{st+1}^{-\gamma} q_{t+1} \left(1 + \phi_H \left(\frac{H_{st+1}}{H_{st}} - 1\right)\left(\frac{H_{st+1}}{H_{st}}\right)^2\right)\right]$$
$$-C_{st}^{-\gamma} q_t \left(1 + \frac{\phi_H}{2}\left(\frac{H_{st}}{H_{st-1}} - 1\right)^2 + \phi_H \left(\frac{H_{st}}{H_{st-1}} - 1\right)\frac{H_{st}}{H_{st-1}}\right) = 0 \tag{A.9}$$

$$\left(1 - \varepsilon + \varepsilon\frac{w_t}{A_t} - \phi_P\left(\pi_t - \theta\pi^*\right)\left(1 + \pi_t\right)\right) Y_t$$
$$+\phi_P \beta_s C_{st}^\gamma E_t \left[\frac{\left(\pi_{t+1} - \theta\pi^*\right)\left(1 + \pi_{t+1}\right) Y_{t+1}}{C_{st+1}^\gamma}\right] = 0 \tag{A.10}$$

$$\chi_b N_{bt} + \chi_s N_{st} - Y_t = 0 \tag{A.11}$$

$$\max\left\{(1 + i)\left(\frac{Y_t}{Y^*}\right)^{\phi y}\left(\frac{1 + \pi_t}{1 + \pi^*}\right)^{\phi_\pi} - (1 + i_t), 0 - i_t\right\} = 0 \tag{A.12}$$

Appendix B. Initial steady state and parameter calibration

At the initial steady state, the inflation is the same as the target inflation. The equilibrium labor supply is calibrated to be 1/3 for both the borrower and the saver. Thus, the target output is the same as the labor supply. The total housing quantity is normalized to be 1.

$$\pi = \pi^* = 0.005$$

$$N_b = N_s = 1/3$$

$$Y^* = Y = \chi_b N_b + \chi_s N_s = 1/3$$

$$H = 1$$

$$\text{Eq.}A.10 : w = \frac{\varepsilon - 1}{\varepsilon} + \frac{\phi_P \left(1 - \theta\right) \pi^* \left(1 + \pi^*\right) \left(1 - \beta_s\right)}{\varepsilon}$$

$$\text{Eq.}A.8 : \frac{1 + i}{1 + \pi^*} = 1 + r = R = \frac{1}{\beta_s}$$

First, I calibrate ξ to match the total debt to income ratio of 1.32, the total housing asset to income ratio of about 2.45, as in the flow of funds table, and the saver's housing to borrower's housing ratio of 1:

$$\text{Eq.}A.5 : \xi = \frac{\left(\frac{\chi_b D_b}{Y}\right) R}{\chi_b \left(\frac{qH}{Y}\right) \left(\frac{H_b}{H}\right)}$$

$$\frac{H_b}{H} = \frac{1}{\chi_b + \chi_s \left(\frac{H_s}{H_b}\right)}$$

I then compute the other steady state values:

$$H_b = H_s = H = 1$$

$$q = \left(\frac{qH}{Y}\right) \frac{Y}{H}$$

$$\text{Eq.}A.5 : D_b = \frac{\xi H_b q}{R}$$

$$\text{Eq.}A.1 : C_b = w N_b - r D_b$$

$$\text{Eq.}A.6, \text{Eq.}A.1 : C_s = \frac{1}{\chi_s} Y \left(1 - \frac{\phi_P}{2} \left(\pi - \theta \pi^*\right)^2\right) - \frac{\chi_b}{\chi_s} C_s$$

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Finally, I calibrate the free parameters j_b, j_s, η_b, η_s:

$$Eq.A.4 : j_b = \frac{C_b^{-\gamma} q \left(1 - \xi \left(\beta_s - \beta_b\right) - \beta_b\right)}{H_b^{-\psi}}$$

$$Eq.A.9 : j_s = \frac{C_s^{-\gamma} q \left(1 - \beta_s\right)}{\left(\frac{\overline{H} - \chi_b H_b}{\chi_s}\right)^{-\psi}}$$

$$Eq.A.2 : \eta_b = \frac{w C_b^{-\gamma}}{N_b^{\phi}}$$

$$Eq.A.7 : \eta_s = \frac{w C_s^{-\gamma}}{N_s^{\phi}}$$

In sum, I calibrate the 5 free parameters $\xi, j_b, j_s, \eta_b, \eta_s$ to match with the 5 stylized facts: (i) the total debt to income ratio of 1.32 ; (ii) the total housing asset to income ratio of about 2.45; (iii) the saver's housing to borrower's housing ratio of 1; (iv) the borrower's steady state labor of 1/3; and (v) the saver's steady state labor of 1/3. The parameters are uniquely determined by these facts.

Appendix C. New steady state with housing

Suppose the debt-to-value ratio changes permanently from ξ to ξ'. Given the parameters calibrated in the previous section, I am able to compute the new steady state corresponding to the new debt-to-value ratio as described below:

$$\pi' = \pi = \pi^* = 0.005$$

$$Eq.A.10 : w' = w = \frac{\varepsilon - 1}{\varepsilon} + \frac{\phi_P \left(1 - \theta\right) \pi^* \left(1 + \pi^*\right) \left(1 - \beta_s\right)}{\varepsilon}$$

$$Eq.A.8 : i' = i = R \left(1 + \pi^*\right)$$

$$\text{Eq.}A.2 : N_b' = \left(\frac{wC_b'^{-\gamma}}{\eta_b}\right)^{\frac{1}{\phi}} \tag{C.1}$$

$$\text{Eq.}A.7 : N_s' = \left(\frac{wC_s'^{-\gamma}}{\eta_s}\right)^{\frac{1}{\phi}} \tag{C.2}$$

$$\text{Eq.}A.11 : Y' = \chi_b N_b' + \chi_s N_s' \tag{C.3}$$

$$\text{Eq.}A.5 : D_b' = \frac{\xi' q' H_b'}{R} \tag{C.4}$$

$$\text{Eq.}A.3 : \phi_b' = C_b'^{-\gamma}(\beta_s - \beta_b) \tag{C.5}$$

$$\text{Eq.}A.1 : C_b' = wN_b' - rD_b' \tag{C.6}$$

$$\text{Eq.}A.6, \text{Eq.}A.1 : C_s' = \frac{1}{\chi_s}Y'\left(1 - \frac{\phi_P}{2}(\pi - \theta\pi^*)^2\right) - \frac{\chi_b}{\chi_s}C_b' \tag{C.7}$$

$$\text{Eq.}A.4 : j_b H_b'^{-\psi} = C_b'^{-\gamma}q'(1 - \xi(\beta_s - \beta_b) - \beta_b) \tag{C.8}$$

$$\text{Eq.}A.9 : \left(\frac{\overline{H} - \chi_b H_b'}{\chi_s}\right)^{-\psi} = \frac{C_s'^{-\gamma}q'(1 - \beta_s)}{j_s} \tag{C.9}$$

The algorithm is as below:

- Step 1: Initialize $X^0 = (C_b^0, H_b^0, C_s^0, q^0)$.

- Step 2: Compute $(N_{b,}' N_s', Y', D_b', \phi_b')$ using Eq.$C.1, ..., $Eq.$C.5$.

- Step 3: Compute $X^{(n)} = (C_b, H_b, C_s, q)$ that solves the system of nonlinear equations Eq.C.6, ...,Eq.C.9.

- Step 4: Compute $||X^{(n)} - X^{(n-1)}||$, go to Step 5 if the norm is less than the tolerant value. Otherwise go back to Step 2.

- Step 5: Report the solution $X = X^{(n)}$

Appendix D. Algorithm to solve multi-period models, including two-period models.

I use the shooting method to solve for the multi-period housing model. The same method can be used to solve the multi-period EK model. First, I find the initial and the new steady state. Then I determine, T, the number of periods that is long enough to make sure the economy will converge to the steady state after T periods. Then I follow the algorithm below:

- Step 1: Initialize $X_t^0 = \{C_{b,t}^0, N_{b,t}^0, D_{b,t}^0, H_{b,t}^0, C_{s,t}^0, N_{s,t}^0, i_t^0, \pi_t^0, q_t^0, w_t^0, \phi_{b,t}^0, y_t^0\}$ for $t = 1, ..., T-1$.

- Step 2: Using the initial steady state and the new steady state value for $X_0^{(n)}$ and $X_T^{(n)}$, compute $X_t^{(n)}$ that solves the system of equibrium equations Eq.A.1, ...,Eq.A.12. for $t \in (1, ..., T-1)$.

- Step 3: Compute $||X^{(n)} - X^{(n-1)}||$, go to Step 4 if the norm is less than the tolerant value. Otherwise go back to Step 2.

- Step 4: Report the solution $X = X^{(n)}$.

Note that we can use the same algorithm described above to solve for the two-period models in the paper. However, we have to impose $T = 2$.