The Wage and Productivity Gap After a Recession

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Abstract

This paper contributes to the understanding of recessions by examining the divide between the cost to workers due to a recession and the benefit to remaining firms. This is done through the development of a dynamic coordination friction model for the labor market. An unexpected increase in the rate at which firms shut down leads to a redistribution of income with wages paid to new hires dropping 8.6% leading to a drop of 3.0% in aggregate wages while aggregate productivity experiences slight increases. This is also found in the Bureau of Labor Statistics where a one percent increase in the unemployment rate from four quarters past increases the gap between productivity and total compensation by 1.23%. The model suggests that lower wages offered during the recession are the cause of this divergence as opposed to the resorting of workers.[1]

[1]I would like to thank Fabrizio Perri, Tim Kehoe, Andy Glover and Lockwood Reynolds for their helpful comments. I would also like to thank the participants at the Northeast Ohio Economics Workshop in 2012, the Annual Conference of the Canadian Economics Association in 2012, the Annual Conference of the Southern Association in 2015, the Kent State Seminar Series and the Ohio University Seminar Series. An earlier version of this paper was titled "Income Inequality from Labor Mismatch."

This paper establishes a better understanding of the various effects of a recession by examining how the cost of a recession varies across different types of agents. A growing body of research point to vastly different experiences based on income, age and education. This paper approaches the topic in a similar vein but with a focus on the divide between firms and workers.

The motivation for this paper partially lays in the differences of wages and productivity at the aggregate level. This has received a bit of media attention as economists have debated the meaning of this finding in the data.[2] I show that the gap between total hourly compensation and productivity has been increasing over time with a sharp increase after 2001. However, changes in the unemployment rate lead this gap. A one percent increase in the unemployment rate from four quarters past increases the gap by 1.23%.

The divide between the firms and workers is examined through the lens of a dynamic coordination friction model of the labor market. Firms differ in productivity and set wages to maximize their profits. Workers, who also differ in their productivity, choose where to apply. Because there is uncertainty in being hired, some highly productive workers will apply to firms with lower productivity to ensure work. This behavior is optimal to maximize lifetime wages and optimal for maximizing output in the economy. Firms choose to either hire the best applicant or stay idle for a period before searching next period conditional on the position continuing to exist in the next period. Both workers and firms have the opportunity to break the match with the worker either quitting the job or the firm choosing to layoff the worker.

A recession in the model is an unexpected one month increase in the exogenous job destruction rate which slowly returns to a steady state rate. This results in wages paid to new hires dropping 8.6% with a return to trend 57 months after the recession ends[3] which is right in line with the findings of Oreopoulos et al. (2012). This decrease in

[2] See Sussman (2015) and Bivens and Mishel (2015).

[3] See the figure 1 for more details.

wages for new hires impacts the aggregate wages leading to a 3.0% drop in aggregate wages. Productivity experiences small increases of 0.3% in aggregate and 2.2% for new hires.

The model suggests that the decrease in wages come from wage offers as opposed to a mismatching of of workers. There can be two sources for wages decreasing in the world with a recession. Wages can drop from workers going into lower level jobs which is frequently referred to as a mismatch of workers. The other source that can cause wages to drop could be the wage offers themselves and the model suggests that this later option is the culprit with wage offers dropping 14.39%. I rerun the model with a recession but hold wages to be constant; in that case, wages for new hires increase by 1.1%.

The dynamic model in this paper has its roots in the three stage game developed in Shimer (2005). However, the static nature of the original model made it incompatible with empirical analysis. This dynamic model presents a contribution to the literature by calibrating the idea embedded in the original framework. The results from the dynamic model also differ from the original static environment. There is much less mismatch of productivity types since firms now have an outside option of waiting a period to hire in a dynamic setting.

Shimer (2007) goes on to develop a dynamic mismatch model that works differently than the work here. The focus of the two papers also differ. That paper focuses on the unemployment duration and the Beveridge curve whereas this paper is focused on the pay to workers versus the profit for firms after a recession. The friction in Shimer (2007) is due to workers lacking access to specific markets whereas the friction here is one of coordination similar to Burdett et al. (2001). Furthermore, layoffs in my model are both exogenous and endogenous based on the availability of outside workers.

I expand the original theory by introducing recessions and separations where the later are both of the exogenous and endogenous nature. A recession causes workers

with higher abilities to apply to more low level jobs resulting in high ability workers with lower wages and firms with an increased productivity. Because firms have this increase in an exogenous shut down rate, they also choose to offer lower wages than normal. Both of these effects decrease wages and the better productivity result in three avenues to increase profits.

These results present a deviation from the standard models. The typical search model has wages determined by exogenous bargaining where the weights may fluctuate while the model in this paper generates wages endogenously.[4] In the standard growth model, if productivity is held constant, a decrease in labor increases wages. If a recession is modeled with a drop in total factor productivity (TFP), productivity and wages drop. This paper ignores the typical drop in TFP and instead focuses on the changing characteristics of the labor market along with the wages offered.[5]

The model is presented in section 1 along with the efficient allocation that would result from a social planner in section 1.1. This allocation is decentralized in sections 1.2 through 1.4 before a general definition of equilibrium for this model is presented in section 1.5. Solving the model is discussed along with recessions for the model and the model's calibration. Before concluding, the paper examines phenomenon like this in the BLS data to support the model's results.

1 Model

The model consists of different types of workers that vary in their productivity but are otherwise identical. Workers apply for one job at firms that also vary in their productivity but are also otherwise identical. This creates the critical decision in the model: where to send the application? Workers enter and exit exogenously every

[4]When solving for wages in the model, I note a step taken which is similar to a bargaining weight but this will be kept constant throughout the recession.

[5]The most recent recessions have also had small drops in labor productivity relative to the drops in GDP.

period while jobs exogenously close every period as well. Vacancies also come about exogenously every period.

The timing works as follows. At the beginning of every period new firms and new workers enter the world and join the existing job vacancies along with the existing unemployed workers. Firms announce whether they have vacancies and the wage offer; workers then choose to apply to one job. Firms will choose to either hire a worker for the opening or decline all applicants (if any) and stay vacant for the time period. Production takes place and at the end of the period, some jobs end exogenously at rate ρ. Firms choose whether they want to terminate any jobs and search for a new worker before workers choose whether or not to quit. Finally, some workers exogenously exit the labor force at rate δ. The employed workers that exit the labor force create a job vacancy for next period. Workers that do not exit the labor force but were separated by quitting, lay off or exogenously at rate ρ enter the next period as unemployed workers.

1.1 Social Planner Problem

Consider a Social Planner's problem to maximize output for the economy. The planner cannot distinguish between individual workers and individual vacancies and therefore will simply create a rule for specific types of workers (m) to apply to certain types of firms (n). Hence, the planner is maximizing the value of each type of firm searching for workers ($\Gamma_{n,t}$). The value functions are time dependent based on the measure of available workers ($\mu_{m,t} \forall m$) and vacancies for all types of firms ($v_{n,t} \forall n$). This is formally expressed as follows:

$$\max \sum_{n=1}^{N} v_{n,t} \Gamma_{n,t}$$

subject to the resource constraint for the economy $\forall m$:

$$\sum_{n=1}^{N} v_{n,t} q_{m,n,t} \leq \mu_{m,t} \tag{1}$$

where

$$\Gamma_{n,t} = \sum_{m=1}^{M} P_{m,n,t} \Phi_{m,n,t} + (1 - \sum_{m=1}^{M} P_{m,n,t})(0 + (1-\rho) E[\Gamma_{n,t+1}]) \tag{2}$$

The probability that a type n firm matches a particular type m worker is denoted $P_{m,n,t}$ and $\Phi_{m,n,t}$ is the value of employing a specific type of worker. Therefore, the value function is formally stating that each firm is maximizing the expected production for the time period. With probability (1-$\sum$P), the firm does not get an match for the job and goes into next period searching for employment conditional on surviving (1-ρ). Once a match is made it may be broken by the firm shutting down exogenously or by the worker exogenously leaving the job with probability (δ). The value of a match is thus given as:

$$\Phi_{m,n,t} = y_{m,n} + \delta(1-\rho) E[\Gamma_{n,t+1}] + (1-\rho)(1-\delta) E[\Phi_{m,n,t+1}]$$

1.1.1 Probability of Matching

Firms can end each period with several applications. These applications form a queue ($q_{m,n}$) for each job. Since the planner is looking to match the most productive workers, it is beneficial to reintroduce notation from Shimer (2005) that identifies workers that are at least as productive as the applicant m:

$$Q_{m,n} = \sum_{m'=m}^{M} q_{m',n}$$

When several type n firms have an opening and type m workers apply for the job, one type n firm may get several applications while another type n firm may receive

zero applications. These applications (or queues) from type m applicants when firm n has an opening are Poisson random variables. The probability that firm n gets zero m type applications with the implication on at least one application:

Zero m Applications: $\left(\frac{1}{0!}\right) q_{m,n}^{0} e^{-q_{m,n}} = e^{-q_{m,n}} \Rightarrow$ At least one m application: $(1 - e^{-q_{m,n}})$

Therefore the probability that the planner successfully matches the job opening for firm n with a worker is the probability that a worker actually applies and that for each worker type m applying, nobody better ($Q_{m+1,n}$) applies:

$$P_{m,n,t} = \sum_{m=1}^{M} e^{-Q_{m+1,n}} (1 - e^{-q_{m,n}}) \tag{3}$$

1.2 Decentralizing the Planner's Problem

In the decentralized version of the model, firms wish to maximize their profits while workers are looking to maximize their lifetime wages. To maximize profits, firms attract the best workers through wage offers for all types of workers and choose whether to keep workers employed. Workers choose where to send an application after viewing these offers. Firms will choose to either hire the best worker for maximizing profit or to decline all applicants and stay vacant for the time period. Separations also occur in the decentralized version of the model where firms or workers have the opportunity to layoff or quit respectively.

1.3 Workers

Workers maximize wages. The workers vary in their productivity which is denoted $m \in \{1, ...M\}$. Every time period, workers of all types enter the labor market. New entrants together with unmatched workers, workers recently laid off and workers who had quit make up a pool of unemployed workers with measures of type m available

workers denoted $\mu_{m,t}$. Workers apply to one job. Therefore, a type m unemployed worker faces the following problem of where to send the application:

$$U_{m,t} = \max_{n} P^{w}_{m,n,t} W_{m,n,w,t} + (1 - P^{w}_{m,n,t})(b + (1-\delta)E[U_{m,t+1}]) \tag{4}$$

$$W_{m,n,t,\tau} = w_{m,n,\tau} + \rho(1-\delta)E[U_{m,t+1}] + (1-\rho)(1-\delta)E[W_{m,n,t+1,\tau}] \tag{5}$$

$P^{w}_{m,n,t}$ is the probability of employment for a specific type m worker being hired by the specific type n firm which receives the application. If unsuccessful with a match, the worker receives the unemployment benefit (b), which does not vary by type, before entering the the unemployment market next period with probability $(1-\delta)$. The value for working with a firm of type n in time t is denoted $W_{m,n,t,\tau}$. The year the worker was hired is indicated by τ while the wage for the worker does not change over time. With probability $(1-\delta)$, the worker survives to see next period where the worker may become unemployed if the firm shuts down, which occurs exogenously at rate ρ.

Workers can choose whether to accept the wage offered to them in unemployment and employment using notation from above. Unemployed workers choose whether to accept the best wage offered to them when searching or they can choose the unemployment benefits before entering unemployment next period which is summarized in the following problem:

$$\max\{[W_{m,n,t,\tau}], b + (1-\delta)E[U_{m,t+1}]\} \tag{6}$$

Employed workers choose whether to keep their job or quit as summarized in the similar problem:

$$\max\{E[W_{m,n,t+1,\tau}], E[U_{m,t+1}]\} \tag{7}$$

1.3.1 Probability of Employment

The probability of a worker gaining employment is similar to the probability that the social planner makes a match. One difference comes from workers applying for one opening. The second difference comes from the fact that several type m workers may apply for the same job. Therefore, the probability that a specific m worker is hired by firm n is the probability of a match together with the proportion of m type workers that apply for the job:

$$\frac{e^{-Q_{m+1,n}}(1-e^{-q_{m,n}})}{q_{m,n}}$$

1.4 Firms

Firms vary in their productivity denoted $n \in \{1, ...N\}$. Every time period a measure of these firms $\nu_{n,t}$ have vacancies and are hiring. Firms are maximizing profits by choosing to hire the best applicant or by choosing to stay vacant. Hence, the problem for a firm with a vacancy is twofold: set wages to attract the optimal queue of workers then choose whether to hire the best applicant. This is expressed formally with the following equations.

$$V_{n,t} = \max_{w_{m,n,t}} \sum_{m=1}^{M} \left\{ \hat{P}_{m,n,t} \Pi_{m,n,t} \right\} + (1 - \sum_{m=1}^{M} \hat{P}_{m,n,t})(0 + (1-\rho)E[V_{n,t+1}]) \tag{8}$$

$$\Pi_{m,n,t,\tau} = y_{m,n} - w_{m,n,\tau} + \delta(1-\rho)E[V_{n,t+1}] + (1-\delta)(1-\rho)E[\Pi_{m,n,t+1,\tau}] \tag{9}$$

$$\max\{[\Pi_{m,n,t}], 0 + (1-\rho)E[V_{n,t+1}]\} \tag{10}$$

For time period t, $\hat{P}_{m,n,t}$ is the probability that the type n firm gets an application from an unemployed type m worker.[6] $\Pi_{m,n,t}$ is the value of hiring a worker of type m which is simply the profits received from that worker going forward. This is conditional on the job not ending at rate ρ while the worker does not exit the labor force at rate δ. Recall that the year the worker was hired is indicated by τ and that wages do not vary over time. The production function ($y_{m,n}$) for a type m worker at a type n firm does not vary with time as well. The best applicant type is indicated as $\tilde{m}$.

Firms choose whether to layoff their worker based on the following:

$$\max\{E[\Pi_{m,n,t+1,\tau}], E[V_{n,t+1}]\} \tag{11}$$

1.5 Equilibrium

Given the unemployment benefit b and output combinations $\{y_{m,n}\}_{n=1}^{N}$, the equilibrium for the model is made up of measures of workers of every type m ($\{\mu_{m,t}\}_{t=1}^{T}$), measures of firms of every type n ($\{\nu_{n,t}\}_{t=1}^{T}$), wage offers ($\left\{[w_{m,n,t}]_{n=1}^{N}\right\}_{t=1}^{T}$ $\forall m$), acceptance thresholds that satisfy 6, quitting thresholds that satisfy 7, hiring thresholds that satisfy 10, layoff thresholds that satisfy 11 along with application strategies for workers ($\left\{[q_{m,n,t}]_{n=1}^{N}\right\}_{t=1}^{T}$ $\forall m$) that satisfy the vacancy value functions ($\left\{[V_{n,t}]_{n=1}^{N}\right\}_{t=1}^{T}$) and the unemployed value functions ($\left\{[U_{m,t}]_{n=1}^{N}\right\}_{t=1}^{T}$) subject to the resource constraints ($\sum_{n=1}^{N} v_{n,t} q_{m,n,t} \leq \mu_{m,t}$ $\forall m$ $\forall t$) for the economy.

1.6 Recessions in the Model

Recessions in the model will come from an unexpected increase in ρ that gradually returns to the ρ in steady state. Formally, the exogenous job destruction rate follows:

[6] The probability that firm n gets a match with a m type worker is exactly the same as the probability facing the social planner in making a match. See Equation 3.

$$\rho_t = \theta\rho_{t-1} + (1-\theta)\rho_{SteadyState}$$

1.7 Solving the Model

The first order conditions with respect to $q_{m,n,t}$ of the Planner's Problem 1.1 together with the resource constraint 1 are sufficient for determining optimal queues for the economy. Firms set wages to obtain these queues as an average[7] of the maximum wage offered through the hiring thresholds from 10 and the minimum wage that workers are willing to take accept summarized in 6.

1.7.1 Maximum Wages Offered to Workers

The maximum wage offer comes from the firm being indifferent to the profit today and going a period without operating before searching again. This comes when the two options in 10 are equal which results in the maximum wage ($w_{m,n,t,\chi}$) a firm of type n would offer a worker of type m. Specifically:

$$\begin{aligned}
(1-\rho)EV_{n,t+1} &= \Pi_{m,n,t,\chi} \\
(1-\rho)EV_{n,t+1} &= y_{m,n} - w_{m,n,t,\chi} + \delta(1-\rho)EV_{n,t+1} + (1-\delta)(1-\rho)\big[y_{m,n} - w_{m,n,t,\chi} + \delta(1-\rho)EV_{n,t+2} + \\
&\quad + (1-\delta)(1-\rho)[y_{m,n} - w_{m,n,t,\chi} + \delta(1-\rho)EV_{n,t+3} + ...]\big] \\
(1-\rho)EV_{n,t+1} &= \tfrac{y_{m,n} - w_{m,n,t,\chi}}{1-(1-\delta)(1-\rho)} + \delta(1-\rho)\Big[EV_{n,t+1} + (1-\delta)(1-\rho)EV_{n,t+2} + \big((1-\delta)(1-\rho)\big)^2 EV_{n,t+3} + ...\Big] \\
w_{m,n,t,\chi} &= y_{m,n} + (1-\rho)\Big(1-(1-\delta)(1-\rho)\Big)\Big(\delta\left[EV_{n,t+1} + (1-\delta)(1-\rho)EV_{n,t+2} + ...\right] - EV_{n,t+1}\Big)
\end{aligned}$$

1.7.2 Minimum Wage Acceptable to Workers

Similar to the maximum wage offer acceptable for firms, the minimum wage offer that workers would take comes from the worker being indifferent between taking the job

[7] This is similar to keeping the bargaining weight for each type of agent constant at 50%.

and taking unemployment for one period before entering unemployment again next period. This comes from the two options in the acceptance threshold in 6 being equal through a minimum wage ($w_{m,t,\mu}$):

$$W_{m,t,\mu} = b + (1-\delta)EU_{m,t+1}$$

$$w_{m,t,\mu} + \rho(1-\delta)EU_{m,t+1} + (1-\delta)(1-\rho)\Big[w_{m,t,\mu} + \rho(1-\delta)EU_{m,t+2} + (1-\delta)(1-\rho)[w_{m,t,\mu} + ...]\Big] = b + ...$$

$$\frac{w_{m,t,\mu}}{1-(1-\delta)(1-\rho)} + \rho(1-\delta)\Big[EU_{m,t+1} + (1-\delta)(1-\rho)EU_{m,t+2} + \Big((1-\delta)(1-\rho)\Big)^2 EU_{m,t+3} + ...\Big] = b + ...$$

$$w_{m,t,\mu} = \Big(1-(1-\delta)(1-\rho)\Big)\Big(b + (1-\delta)EU_{m,t+1} - \rho(1-\delta)\Big[EU_{m,t+1} + (1-\delta)(1-\rho)EU_{m,t+2} + ...\Big]\Big)$$

During a recession created by an unexpected increase in ρ that gradually returns to the ρ in steady state, $w_{m,n,t,\chi}$ must be less than the righthand side of its expression and $w_{m,t,\mu}$ must be greater than its righthand side. When using $w_{m,n,t,\tau}$ as the average of $w_{m,n,t,\chi}$ and $w_{m,n,t,\mu}$ as approximated above, the wage offered will not violate the righthand sides of $w_{m,n,t,\mu}$ nor $w_{m,n,t,\chi}$.

1.7.3 Computing the Solution

After the optimal $q_{m,n,t}$ are determined for all m type matches with firms of type n, these determine the MxN ($w_{m,n,t,\chi}$) variables along with M ($w_{m,t,\mu}$) variables for the MxN $w_{m,n,\tau}$ variables. These MxN $w_{m,n,\tau}$ variables determine the M unknown $U_{m,t}$ from 4 which I use to find $\{EU_{m,t+1}\}_t^T$ and the N unknown $V_{n,t}$ from 8 which I use to find $\{EV_{n,t+1}\}_t^T$.[8]

[8] All future values $\{E[U_{m,t+1}]\}_t^T$, $\{E[V_{n,t+1}]\}_t^T$ and $\{E[\Gamma_{n,t+1}]\}_t^T$ are found with $E[X_{t+1}] = \theta X_t + (1-\theta)X_{SteadyState}$ and thus $E[X_{t+n}] = \theta X_{t+n-1} + (1-\theta)X_{SteadyState}$ where θ is set such that the difference of $E[\Gamma_{n,t+1}]$ and $\Gamma_{n,t+1}$ is never more than 0.5% in absolute value. This results in average errors in absolute terms for $E[V_{n,t+1}]$ and $E[U_{m,t+1}]$ that are less than 1%.

2 Calibration

Consider the model at the monthly frequency. The model runs for 300 periods with 100,000 initial workers running for a 20 period initialization period before the labor force increases by the rate at which workers leave (δ) to roughly create a steady state. The exogenous rate for which workers leave the labor force is set to $\delta = 1/480$ for an expectation of 40 years in the labor force. The initial distribution of the workers is set using labor force data for those with less than a high school diploma ($m = 1$), those with a high school diploma but no college experience($m = 2$), those with some college experience or an associate's degree($m = 3$) and those with at least a bachelor's degree($m = 4$) based on the averages of from 2001 to 2007.

The distribution of jobs is based on the BLS Occupational Employment Statistics. I take all occupations at the two digit level and sort them by mean annual wages and look for the three biggest breaks beyond the breaks for legal occupations and supervisors to create my four types of jobs.[9] Below, you will see that the model does a decent job creating distributions like that in the data without major misses.

Table 1: Worker and Job Distributions

	Data	Model
$\mu_{1,ss}$	18.0%	12.9%
$\mu_{2,ss}$	32.8%	33.9%
$\mu_{3,ss}$	27.0%	28.9%
$\mu_{4,ss}$	22.2%	24.4%
$v_{1,ss}$	18.6%	21.4%
$v_{2,ss}$	40.0%	37.5%
$v_{3,ss}$	19.2%	23.1%
$v_{4,ss}$	22.1%	18.1%

Following Shimer (2005), I consider output with the case of no comparative advantage with a slight adjustment which will help in calibrating the model. Formally, output follows $y_{m,n} = (n+x_n)*(m+x_m)$. There are four types of workers ($m = 1, 2, 3$

[9] I agree that these breaks seem arbitrary but the results are not sensitive to the distribution of job types.

or 4) with four types of firms ($n = 1, 2, 3$ or 4).

The calibration of the x_m and x_n parameters are important because they indicate the comparative productivity of each worker type and the comparative productivity of each job type. The parameters for x_n and x_m are estimated to match the wage ratios for the different types of workers in the model to the different types of workers in the BLS data. Specifically, the x parameters minimize the square differences of the ratio of median usual weekly earnings for workers with the various levels of education noted above. I compare the ratio of wages for type $m = 4$ workers to type $m = 3$ workers with its counterpart in the data while being sure that wages are increasing in productivity. I continue to do this for every combination until finding $x_n = 3$ and $x_m = 41$.[10]

Table 2: Wage Ratios

	Data	Model
4v3	1.55	1.39
4v2	1.79	1.65
4v1	2.50	2.18
3v2	1.15	1.19
3v1	1.61	1.57
2v1	1.40	1.32

The last key parameter is the exogenous rate of job loss ρ. I set this parameter to target the unemployment rate in times outside the recession since this parameter drives this unemployment rate in model. (Endogenous separations only occur during recessions in the model.) The recession as an increase in the rate of exogenous job loss increases to target an increase in the unemployment rate like the one seen during the 2008 recession. Values of $\rho = 0.0325$ generates an average unemployment rate of 5.78% which is in line with historical averages outside of recessions in the US. However, $\rho = 0.0325$ indicates jobs lasting 2.56 years on average which misses the 4.0

[10]Since this paper will make the argument that mismatched workers are not the cause of a decrease in wages, I make x_n as small as possible. The results are very similar (within 2% points) when I instead use $x_n = x_m$.

year median tenure from 2004 and 2006 as reported in BLS (2014).

The recession of a one period increase to $\rho = 0.0449$ which gradually returns to the steady state generates a recessionary unemployment rate which peaks at 10.66% for a month before dropping to 8.75% in the next month. The unemployment returns to less than 6% 28 months after the recession. Some of this is driven by the exogenous rate of job destruction with a persistence parameter ($\theta = 0.95$) which brings the exogenous shut down rate to the steady state after 54 months. This persistence parameter is determined computationally from the planner's problem as noted above.

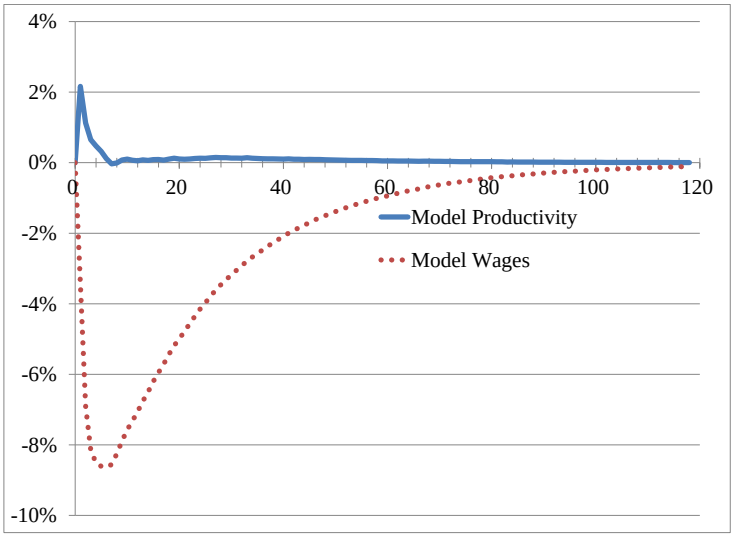
The final exogenous parameters involve the unemployment benefits and the vacancy rate of the model. The unemployment benefits (b) are equal to 38% of wages similar to US unemployment benefits. I set this as a constant fraction of the average wages offered to the lowest skilled workers in the model. I keep the job vacancy rate equal to one meaning that there is always enough jobs for everybody.

2.1 Results

Figure 1 demonstrates the loses to workers compared to firms for new hires during and after the recession. The y-axis is the change in the line's value compared to a world with no recession. Newly hired workers wages are 8.6% lower seven months into the recession compared to their wages without a recession taking place. This does not return to within 1% of the non-recessionary world's wages until 60 months after the start of the recession. Firms on the other hand experience a small bump in productivity which increases by 2.2% one month into the recession due to a re-sorting of workers but it quickly returns to within 1% of the non-recessionary world's productivity.

The aggregate economy per labor unit for the model demonstrates a similar fact to that above but should not take away from the overall total loss to output. Specifically, Figure 2 that wages for the entire economy drop 3.0% while productivity experiences

Figure 1: New Hires in a Recession



small gains of 0.3%.[11] Figure 3 demonstrate that both total wages and total output drop during the recession due to unemployment. However, this figure also points out the bigger picture: workers experience a bigger loss as opposed to firms. Total wages in the economy drop by 17.4% compared to output which drops by 14.1% with a much faster recovery.

Wage offers drop 14.39%. Therefore, there is actually a sorting of workers that leads to more efficient output as noted with the small increase of productivity. I further demonstrate this by rerunning the environment with a recession but holding wages to be the same as the wages in the non-recessionary world. In that case, wages for new hires increase by 1.1% while the increase in output is still the same at 2.2%.

[11] This picture is similar to the data found in the BLS and will be explored in the next section.

Figure 2: Overall Change in the Economy's Hourly Income

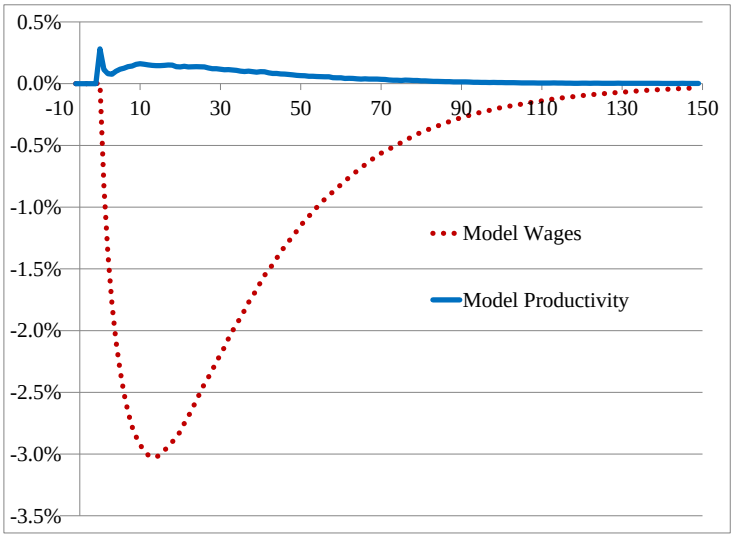
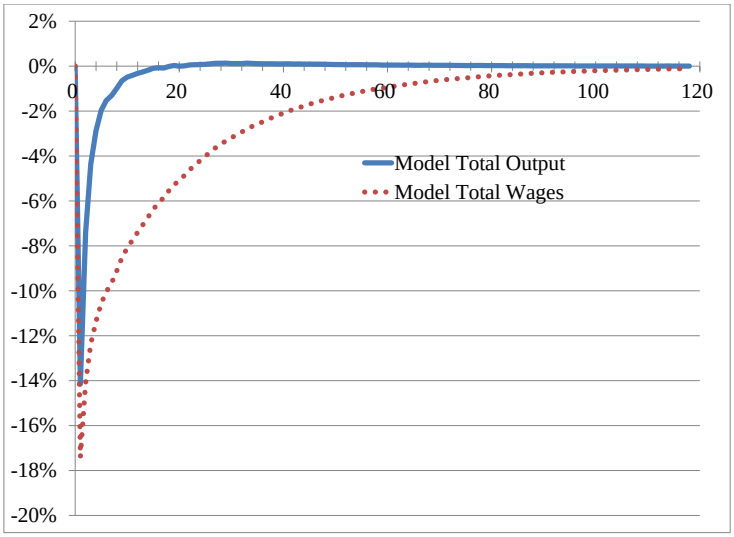


Figure 3: Overall Change in the Economy's Total Income



3 Testing the Model

I compare hourly compensation for the nonfarm business sector and real nonfarm business sector output per hour from the Bureau of Labor Statistics. Real business sector output is measured as the country's GDP excluding output from government spending, nonprofit institutions and private households (including owner-occupied housing).[12] Hourly compensation includes compensation from wage, salaries, supplements such as payments in kind, social insurance, private pension, profit sharing plans, group health and life insurance plans and private workers' compensation. This also includes commission, tips, bonuses, stock options and employer contributions to benefits such as a 401(k).

Following critiques such as Feldstein (2008) made on comparing these two data sets, I start with nominal hourly compensation and make it real using the same price deflator applied to the nonfarm business sector output per hour. Specifically, I take the nominal hourly compensation from the BLS and make the data real with the implicit price deflator for nonfarm business sector output.[13] I then make take both time series indexed to 1962 resulting in Figure 4.

Figure 5 provides an examination of the gap between productivity and compensation while Figure 6 indicates how this gap moves along with the unemployment rate.[14] After 2001, the gap really jumps as can be seen in Figure 7 and studied in the recent literature regarding a changing labor share.[15]

Figure 6 and figure 7 point to unemployment leading the gap. I examine this closer with by testing up to eight lags on unemployment and find that the unemployment from four quarters past is the optimal lag based on the likelihood-ratio tests, the final

[12] The BLS labels this variable Series ID PRS85006093.

[13] These variables come from the BLS with nominal hourly compensation being Series ID PRS85006103 and the implicit price deflator being Series ID PRS85006143.

[14] The unemployment rate here is the seasonally adjusted unemployment rate for those 16 years of age and older. (BLS Series ID LNS14000000Q.

[15] See Karabarbounis and Neiman (2014) and Glover and Short (2016) for example.

Figure 4: US Data

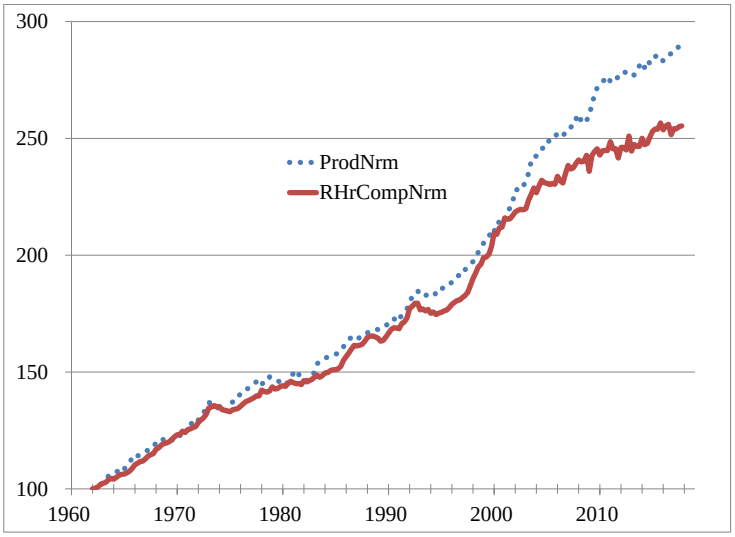


Figure 5: US Gap

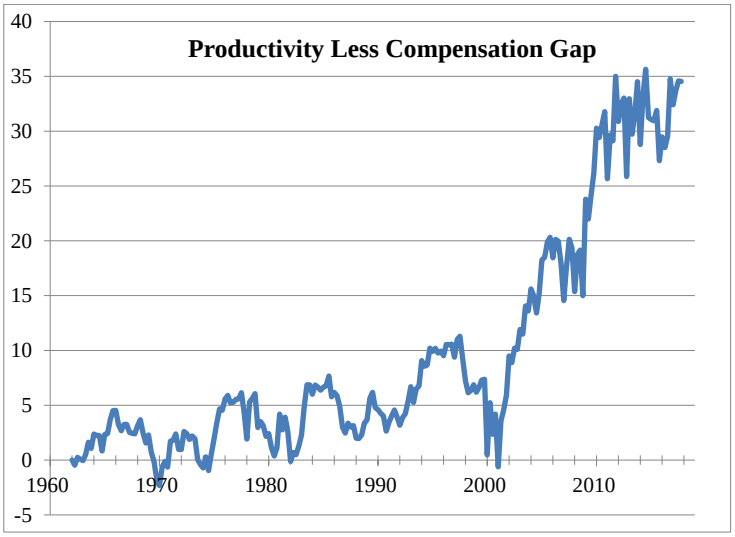


Figure 6: US Gap with Unemployment

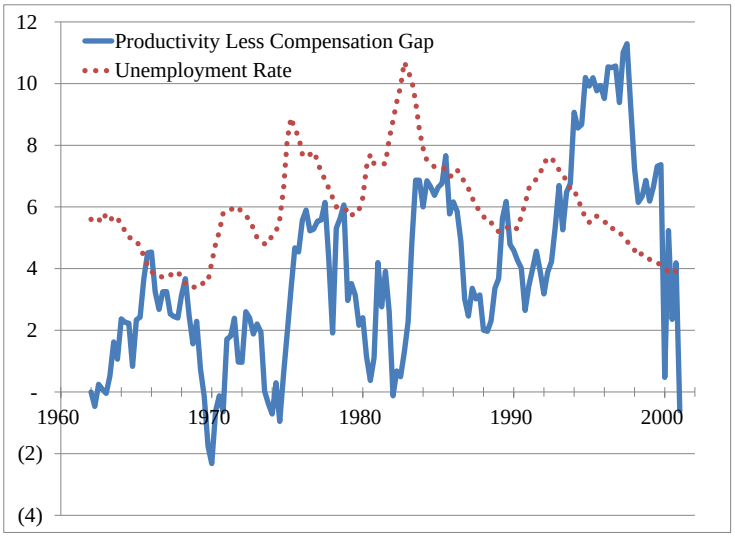
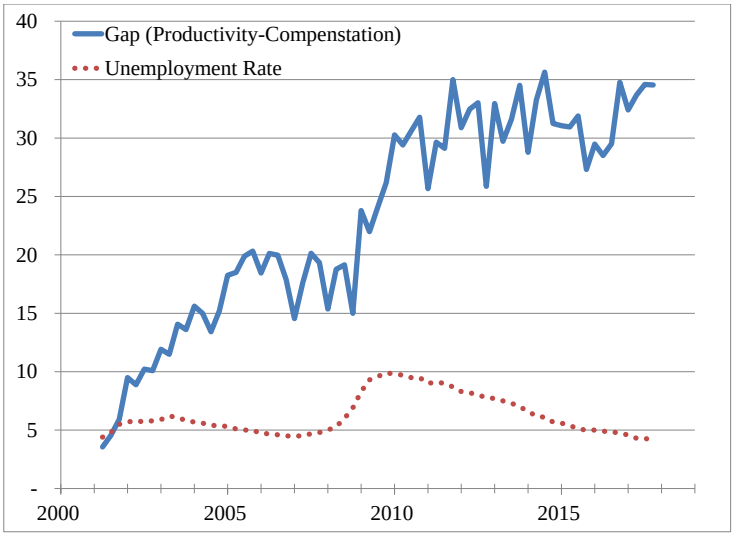


Figure 7: US Gap with Unemployment Since 2001



prediction error and Akaike's information criterion.[16] The following specification also provides the best fit in terms of adjusted R squared for the entire sample:

$$Gap_t = \alpha + \beta U_{t-4} + \gamma t + \epsilon_t \tag{12}$$

Here Gap_t is the difference between real productivity and real compensation at the quarterly level while U_{t-4} provides the unemployment rate from 4 quarters past. The general increasing trend for the gap is captured with γ. The results of this estimation are found in table 3.

Table 3: The Estimation of Equation 12 on Quarterly US Data

		Coefficient (Std. Err.)	
Variable			
Lagged Unemployment	1.227***	0.431***	1.468***
	(0.22)	(0.12)	(0.26)
Time Trend	0.454***	0.154***	1.492***
	(0.02)	(0.02)	(0.12)
Constant	-902.9***	-303.1***	-2983.7***
	(47.68)	(33.09)	(241.92)
Time	$\forall t$	$t < 2001Q2$	$t \geq 2001Q2$
Adj. R^2	0.692	0.425	0.920
N	208	153	55

Significance levels: $*$: 10% $**$: 5% $***$: 1%

Note: The dependent variable for this estimation is the difference between productivity and compensation at the quarterly level while "Lagged Unemployment" is the unemployment rate from 4 quarters past.

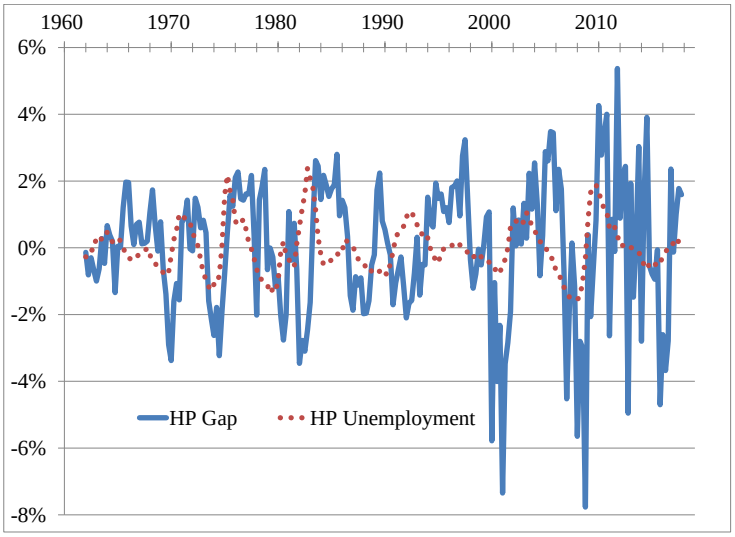
The main coefficient of interest, β, suggests a one percentage increase of last year's unemployment rate raises the gap between productivity and compensation 1.23%.[17] The value is not as strong although still significant at the 1% level when I consider data before the second quarter of 2001. For this time period, a one percentage increase of last year's unemployment rate raises the gap between productivity and

[16] Hannan and Quinn information criterion finds the optimal lag to be three quarters and Schwarz's Bayesian information criterion finds the optimal lag to be two quarters.

[17] I run this same specification on the model's data and find that the results are not statistically different although the model's estimated β is smaller at 0.71%. Additionally, the best fit for the model's data is with a four quarter lag in unemployment.

compensation 0.43%. The estimation for the data after the second quarter of 2001 results in a 1.47% increase in the gap from a lagged unemployment rate from four quarters past.[18] The estimations on these coefficients during these different time periods are also statistically different from one another.

Figure 8: HP Detrended Gap with Detrended Unemployment



One last examination of this result comes from the original specification detrended with the Hondrick Prescott filter using the standard smoothing for quarterly data. Figure 8 demonstrates the cyclical data with the dotted line representing the HP filtered unemployment deviation while the solid line is the deviation of the HP filtered difference between real productivity and real compensation. This demonstrates more visual evidence of the unemployment rate leading the gap between productivity and wages. Using this detrended data, I estimate the following where the variables listed

[18]The Chow test also indicates a structural break at this quarter.

are those detrended variables:

$$Gap_t^{HP} = \beta^{HP} U_{t-4}^{HP} + \epsilon_t^{HP} \tag{13}$$

The estimation for β^{HP} suggests a one percentage increase of last year's the cyclical unemployment rate raises the cyclical gap between productivity and compensation 1.26%. The estimated value is still larger after the second quarter of 2001 at 1.78% but this is now no longer statistically different from the earlier time period's estimation of 0.96%.[19] The rest of the details on the results of this estimation are found in table 4.

Table 4: The Estimation of Equation 13 on H-P Filtered Quarterly US Data

		Coefficient (Std. Err.)	
Variable			
Lagged Unemployment	1.262***	0.959***	1.782***
	(0.161)	(0.180)	(0.325)
Time Trend	none	none	none
Constant	none	none	none
Time	$\forall t$	$t < 2001Q2$	$t \geq 2001Q2$
Adj. R^2	0.225	0.152	0.346
N	208	153	55

Significance levels: $*$: 10% $**$: 5% $***$: 1%

Note: The dependent variable for this estimation is the deviation from the trend of the H-P filtered difference between productivity and compensation at the quarterly level while "Lagged Unemployment" is the deviation from trend of the unemployment rate from 4 quarters past.

Furthermore, I look into causality with Granger causality tests and find that the unemployment rate Granger causes the gap. I can reject the null hypothesis that the coefficients on the lag terms of the unemployment are equal to zero when the gap is the explanatory variable. This holds true whether I use the entire sample or either of the subsamples although I can only reject the null hypothesis at the 5% level for the time period after 2001. However, when unemployment is the explanatory variable, I cannot reject the null hypothesis at the 1% level that the coefficients on the lag terms of the gap are equal to zero except for the time period after 2001 when I cannot reject

[19] For more details showing that the result is in fact robust, please consult Barnette (2016) which has many more details of these results along with more tests for robustness.

the null hypothesis at the 5% level.

To adjust for the growth in the gap after the second quarter of 2001, consider the following specification:

$$Gap_t = \alpha_1 I_1 + \alpha_2 I_2 + \beta U_{t-4} + \gamma_1 I_1 t + \gamma_2 I_2 t + \epsilon_t$$

Here Gap_t is still the gap or the difference between productivity and compensation at the quarterly level while U_{t-4} continues to provide the unemployment rate from four quarters past. The changing increasing trend in the gap is captured with the various γ's and α's depending on whether the quarterly data is before the second quarter of 2001 or after. Specifically, the indicator function I_1 takes the value of one if the time period is before the second quarter of 2001 and the value of zero otherwise. The indicator function I_2 takes the value of zero if the time period is before the second quarter of 2001 and the value of one otherwise.

Once again, I find significant values on all the coefficients at the 1% level. This specification demonstrates the jump in the growth of this gap after 2001 but this paper does not have a theory for this increase. Instead, what concerns this theory is whether the the lagged unemployment rate also contributes significantly to this gap and the specification above suggests a one percentage increase of last year's unemployment rate raises the gap between productivity and compensation 0.64%. Furthermore, this specification is constructed for the Chow test of these two time periods. Indeed, the Chow test rejects the hypothesis of the constant γ's and α's using the second quarter of 2001.

One last examination of this result comes from the original specification detrended with the Hondrick Prescott filter using the standard smoothing for quarterly data. Figure 8 demonstrates the cyclical data with the dotted line representing the HP filtered unemployment deviation. This demonstrates evidence of the unemployment

rate leading the gap between productivity and wages.

4 Conclusion

I show that a dynamic coordination friction model for the labor market is helpful for understanding the foundations of changing inequality in the United States. The model captures the tradeoff between firms and workers in the marketplace suggesting more productive existing firms after a recession. This leads to a redistribution of income from workers to firms and I show evidence of this from the Bureau of Labor Statistics. This gap between total hourly compensation and productivity has been increasing in the economy since 2001 but I also show that an increase in the previous year's unemployment contributes to this effect. The model suggests that this increase is due lower wages offered by the firms as opposed to workers moving into lower level jobs. I leave it to future work to investigate this empirical suggestion further.

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