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Quantum amplifier: Measurement with entangled spins

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It is experimentally demonstrated that entangled quantum states can be used to amplify perturbations and to increase changes in observable values. The physical system is seven nuclear spins of single-labeled ^{13}C-benzene in a liquid crystalline matrix. An entangled state of six proton spins was used to monitor interaction with the ^{13}C spin. © *2004 American Institute of Physics.*
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With nonlinear classical dynamics, two states of a system, which are close at one moment of time, can exponentially diverge at later time. Such evolution may amplify small perturbations and result in the so-called "butterfly effect."[1] This property of nonlinear dynamics can be beneficial in measuring small signals. On the other hand, quantum dynamics is governed by linear equations of motion. Unitary evolution operators conserve distances between states: the states that are close at one moment of time remain close at all times. This means that the classical mechanisms of amplifying weak signals cannot be implemented in quantum systems. However, quantum mechanics offers another alternative of converting a small perturbation into a big change in observable values. It is based on using special entangled states of a composite quantum system. For such states, perturbation acting on a small part of a system changes a state of the whole system in a coherent way and produces changes of "macroscopic" observables. The experimental demonstration described below was made with nuclear magnetic resonance (NMR) technique on a system of seven dipolar-coupled nuclear spins of single-labeled ^{13}C-benzene molecules in a liquid–crystalline matrix. The maximally entangled quantum state of six proton spins was used to amplify the effect of interaction with the ^{13}C nuclear spin.

The experimental scheme is shown in Fig. 1. Two evolution periods with the effective Hamiltonians of opposite signs constitute a time-reversal sequence. The ^{13}C spin is decoupled all the time except for a short perturbation period between the two evolution periods. The double-quantum effective Hamiltonian $H_{av} = -\frac{1}{2}\Sigma_{i<j}D_{ij}(S_i^+ S_j^+ + S_i^- S_j^-)$, where $S^{\pm} = S_x \pm iS_y$ and D_{ij}'s are the dipolar coupling constants, is created by 20 cycles of the eight-pulse sequence.[2] This effective Hamiltonian generates multiple-quantum coherences of even orders. The highest-order multiple-quantum (HOMQ) coherence, six-quantum ($6Q$) in this experiment, was filtered by a combination of phase cycling and 180° pulse.[3] The first evolution period of 3.1 ms duration with $6Q$ filtering converts the initial thermal equilibrium state of proton spins into the state with the deviation density matrix $i(|u\rangle\langle d| - |d\rangle\langle u|)$, where $|u\rangle$ is the state with all spins up and $|d\rangle$ is the state with all spins down. This matrix with two

elements is, essentially, the off-diagonal part of the maximally entangled "cat" state $(|u\rangle + |d\rangle)(\langle u| + \langle d|)$. The second, time-reversed, evolution period converts this state into the diagonal state with only two nonzero elements $|u\rangle\langle u| - |d\rangle\langle d|$. Further details of this state preparation are described elsewhere.[4] The final states are verified by recording spectra using a small-angle reading pulse.

The experiment has been performed with a Varian Unity/Inova 500 MHz NMR spectrometer. The sample contained 5% of single-labeled ^{13}C-benzene (Aldrich) dissolved in liquid–crystalline solvent MLC-6815 (EMD Chemical). In this system, fast molecular motions average out all intermolecular spin–spin interactions. Intramolecular dipole–dipole interactions are not averaged to zero due to orientational order induced by a liquid–crystalline matrix. Therefore, the system is a good example of an ensemble of noninteracting spin clusters, where each benzene molecule contains seven spins, one ^{13}C and six protons, coupled by residual dipole–dipole interactions.

The ^{1}H thermal equilibrium spectrum of ^{13}C-benzene in MLC-6815 without ^{13}C decoupling is shown in Fig. 2(a). The proton spectrum with ^{13}C decoupling is shown in Fig. 2(b). The peaks are somewhat broader in the ^{13}C-decoupled spectrum due to nonperfect decoupling and RF heating of the sample. We have found that CW decoupling is not very efficient. The TPPM (two-pulse phase-modulation) decoupling sequence[5] with the phase excursion angle 54° was found to give better results, and it was used in the experiment. The RF power level was a compromise between the decoupling quality and the RF heating, which creates temperature gradients
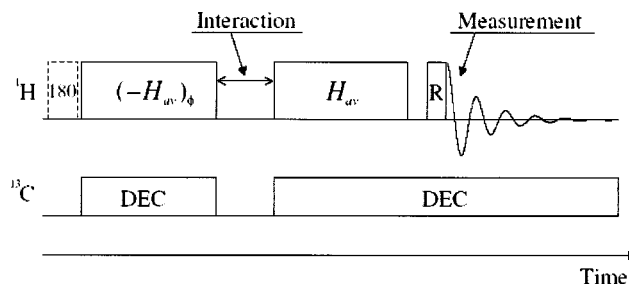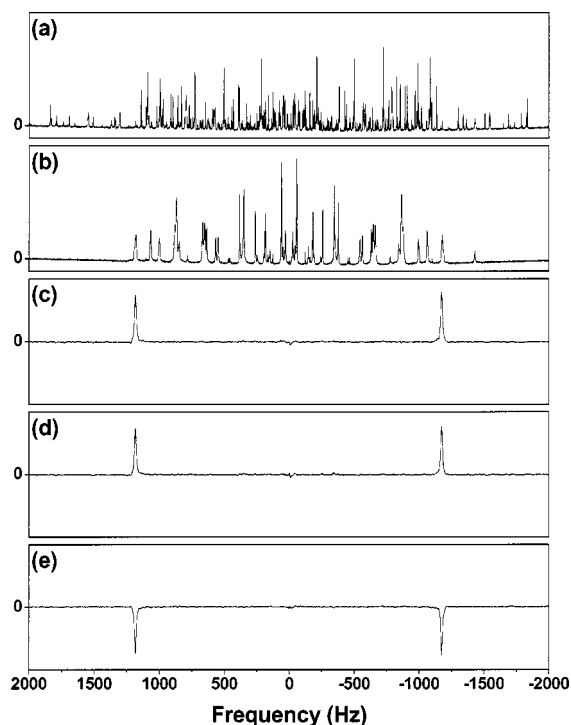

FIG. 1. Schematic of the experiment.

FIG. 2. (a) ^{1}H thermal equilibrium spectrum of ^{13}C-benzene in MLC-6815; (b) ^{1}H spectrum with ^{13}C decoupling; (c) linear-response spectrum after the time-reversal sequence with six-quantum filtering (^{13}C spin decoupled); (d) spectrum with the delay of 382.5 μs (perturbation period) in between the two evolution periods (^{13}C spin decoupled); (e) spectrum with the decoupling turned off during the perturbation period.

and broadens the peaks. The rightmost peak in Fig. 2(b) is due to impurity.

A linear-response spectrum corresponding to the state $|u\rangle\langle u| - |d\rangle\langle d|$ is presented in Fig. 2(c). Due to the high symmetry of a benzene molecule, there is only one allowed single-quantum transition from each of the states: all spins up and all spins down. The two peaks in the spectrum 2(c) are at the frequencies of these transitions. When 382.5 μs delay was introduced between the two evolution periods, and the ^{13}C spin was decoupled during this perturbation period, there were no noticeable changes in the spectrum [Fig. 2(d)]. However, if the decoupling was off, the spectrum reversed, indicating that the magnetization of proton spins changed its sign [Fig. 2(e)].

The mechanism of this magnetization reversal is the following. The $6Q$ deviation density matrix ρ_{6Q} after the first evolution period can be written, without a concern for a phase factor, as

$$\rho_{6Q} = S_1^+ S_2^+ S_3^+ S_4^+ S_5^+ S_6^+ + S_1^- S_2^- S_3^- S_4^- S_5^- S_6^- , \qquad (1)$$

where S_j^+ (S_j^-) are the raising (lowering) operators for the j-th proton spin. The rotation of any one of the spins by the angle φ around z-axis adds only a phase factor to each of the two product terms of the density matrix (1) according to the relation $\exp(-i\varphi S_j^z) S_j^{\pm} \exp(i\varphi S_j^z) = S_j^{\pm} \exp(\mp i\varphi)$. Therefore, the result does not depend on whether all the spins are rotated by the angle $\varphi/6$ or only one spin is rotated by the angle φ. As an example, when one of the spins is rotated by π the whole density matrix (1) changes its sign. As a result, the deviation density matrix after the second evolution period
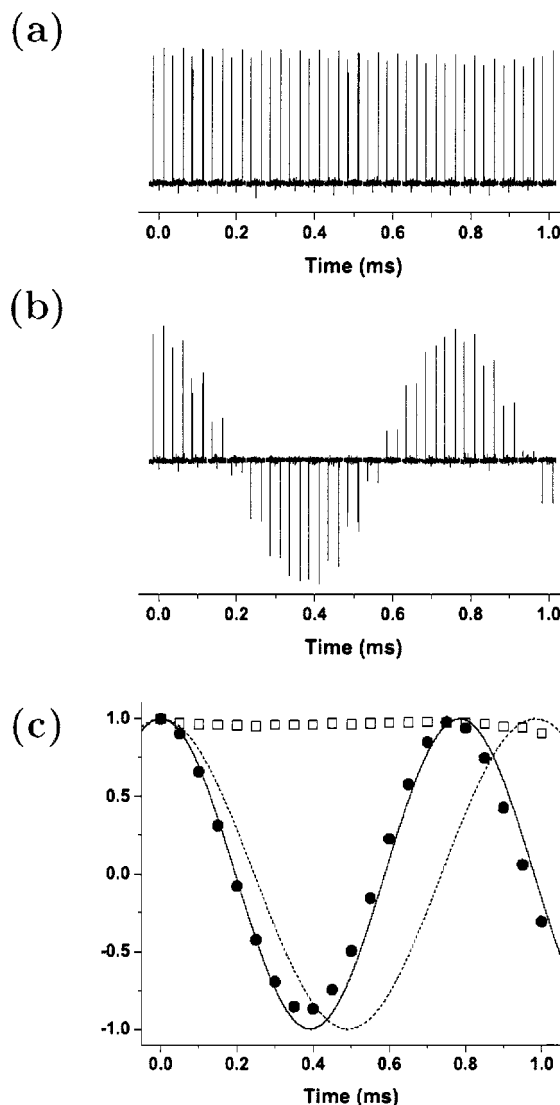

FIG. 3. Spectra at different durations of the perturbation period (a), (b) and integrated intensities (c); decoupling is on for (a) and squares in (c), and off during the perturbation period for (b) and circles in (c). Solid and dashed lines in (c) are the theoretical curves with the interaction strengths of 2549 and 2035 Hz, respectively.

changes its sign and all observables, including the protons' magnetization, also change their signs. In our experiment such rotation was performed by interaction with the ^{13}C spin. The interaction between ^{13}C and the nearest proton (2035 Hz), which includes the dipole–dipole interaction (1877 Hz) and J-coupling (158 Hz), amounts to 80% of the ^{13}C—protons interaction (2549 Hz). During the 382.5 μs perturbation period the spin of the proton nearest to ^{13}C is rotated by almost π, while the phases acquired by other proton spins remain small. However, the entangled state (1) is not "spoiled" by these inhomogeneous rotations, as if all the spins performed a coherent rotation. An interesting implementation of a coherent rotation of all spins in an entangled state is a high-precision spectroscopy, which can reach the Heisenberg limit.[6]

The arrays of spectra, similar to those in Figs. 2(d) and 2(e), for varying duration of the perturbation period are displayed in Figs. 3(a) and 3(b), respectively. The array 3(a) is for the ^{13}C spin decoupled during the perturbation period and

the array 3(b) is for the case when decoupling is off during this period. Integrated intensities of the spectra are shown in Fig. 3(c). The solid line is the theoretical curve, and the dashed line is the theoretical curve when only the interaction between ^{13}C and the nearest proton is taken into account during the perturbation period.

In NMR experiments the initial thermal equilibrium spin state is a mixed one. Some filtering, temporal or spatial averaging, is required to get a deviation density matrix corresponding to the HOMQ coherence or a pseudopure state. This averaging reduces the total signal intensity. The loss of a signal rapidly increases with system's size.[7] For our system, 11% of the protons' equilibrium magnetization was converted into magnetization of the state $|u\rangle\langle u| - |d\rangle\langle d|$ with the spectrum in Fig. 2(c). As a result, the maximum change of the protons' magnetization, due to interaction with the ^{13}C spin, was about 20% of the protons' equilibrium magnetization. However, even with the loss due to filtering, this change is larger than the theoretical maximum of 1/7 for the indirect ^{13}C detection,[8] when the initial polarization of six proton spins is redistributed in the seven-spin system. If the experiment could be performed with a pure initial state, e.g., the ground state for the proton spins, then 100% of the initial protons magnetization would be reversed.

It should be noted that the experiment presented here was not designed to measure the state of the ^{13}C spin. Only its presence and the strength of interaction between ^{13}C and protons [Fig. 3(c)] have been measured.

The term "quantum butterfly effect" has been used[9] to describe sensitivity of quantum evolution to perturbation of the Hamiltonian during the evolution period, starting with the same initial state. In a classical version, the effect originates from perturbation of the initial condition under identical equations of motion. Similar to the classical case, the Hamiltonian during the evolution period is fixed in our experiment.

The scheme of the time-reversal experiment in Fig. 1 follows a scenario of the story[10] where a small perturbation performed in the past produces dramatic changes in the present. The scenario can be implemented both with a classical dynamics and also with a quantum evolution, as demonstrated here. In contrast to a classical version, in a quantum experiment, big change of the state occurs "instantly" during the perturbation period. However, this change cannot be observed directly. A long evolution period is needed for correlations to propagate through the entire system and to convert a change of the state into a change of observable values.

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