

**Business Statistics:
A Decision-Making Approach**
6th Edition

Chapter 7
Estimating Population Values

Fall 2006 – Fundamentals of Business Statistics

1

Confidence Intervals

Content of this chapter

- Confidence Intervals for the **Population Mean, μ**
 - when Population Standard Deviation σ is **Known**
 - when Population Standard Deviation σ is **Unknown**
- Determining the **Required Sample Size**

Fall 2006 – Fundamentals of Business Statistics

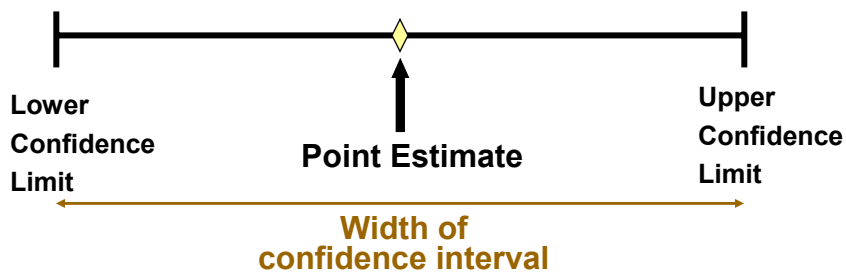
2

Confidence Interval Estimation for μ

- Suppose you are interested in estimating the average amount of money a Kent State Student (population) carries. How would you find out?

Point and Interval Estimates

- A **point estimate** is a single number,
- a **confidence interval** provides additional information about variability



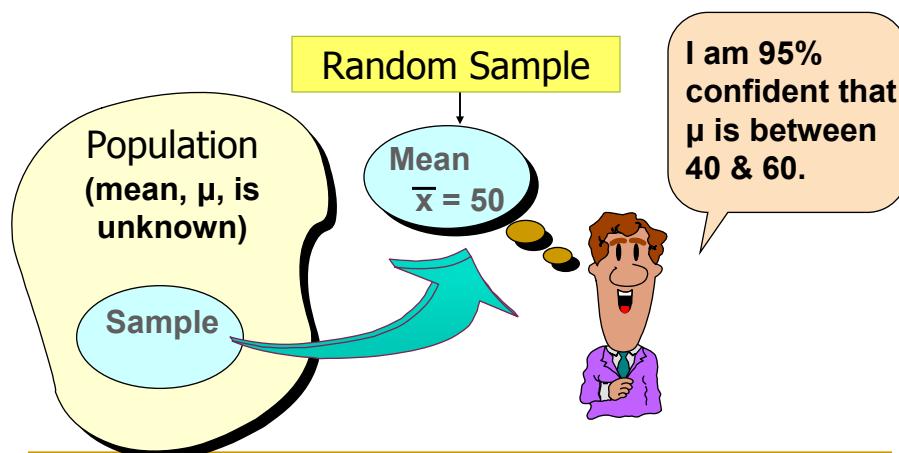
Estimation Methods

- Point Estimation
 - Provides single value
 - Based on observations from 1 sample
 - Gives no information on how close value is to the population parameter
- Interval Estimation
 - Provides range of values
 - Based on observations from 1 sample
 - Gives information about closeness to unknown population parameter
 - Stated in terms of “level of confidence.”
 - To determine exactly requires what information?

Fall 2006 – Fundamentals of Business Statistics

5

Estimation Process



Fall 2006 – Fundamentals of Business Statistics

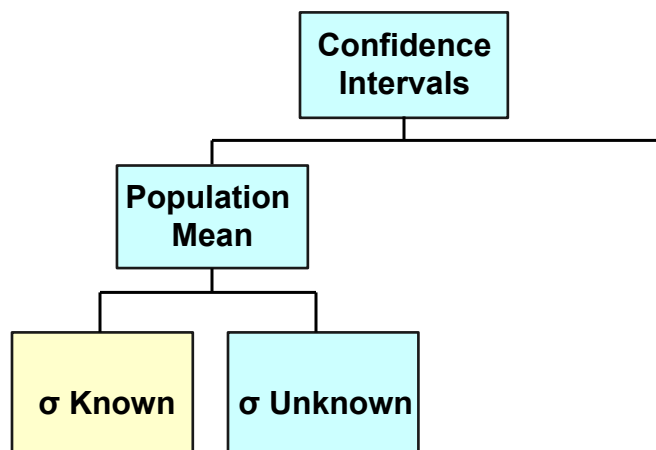
6

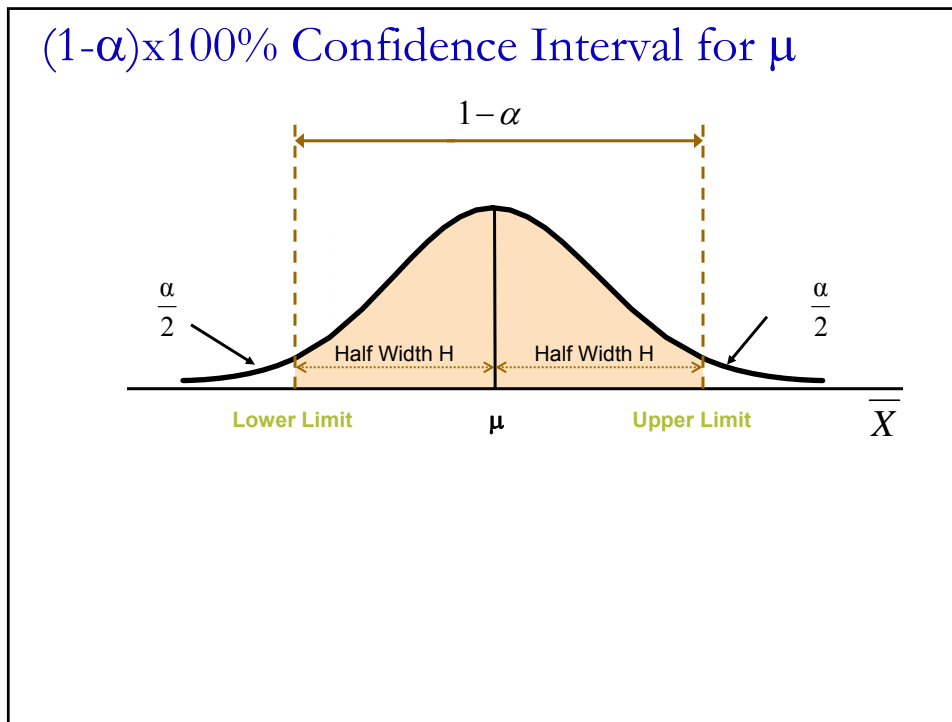
General Formula

- The general formula for all confidence intervals is:

$$\text{Point Estimate} \pm (\text{Critical Value})(\text{Standard Error})$$

Confidence Intervals





CI Derivation Continued

1. Parameter = Statistic \pm Error (Half Width)

$$\mu = \bar{X} \pm H$$

$$H = \bar{X} - \mu \text{ or } \bar{X} + \mu$$

$$Z = \frac{\bar{X} - \mu}{\sigma_{\bar{X}}} = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{H}{\sigma / \sqrt{n}}$$

$$H = Z \times \sigma / \sqrt{n}$$

$$\mu = \bar{X} \pm Z \times \sigma / \sqrt{n}$$

Confidence Interval for μ (σ Known)

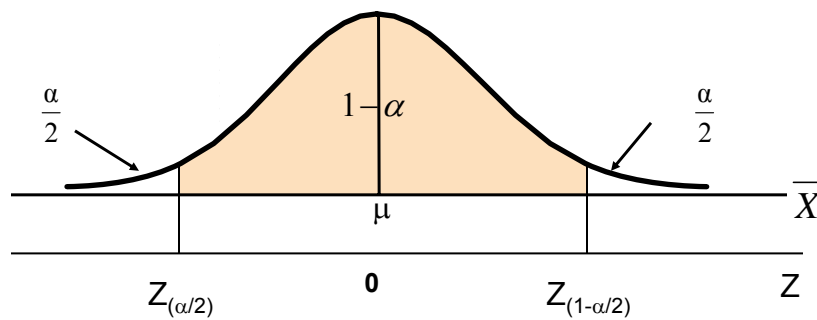
■ Assumptions

- Population standard deviation σ is known
- Population is normally distributed
- If population is not normal, use large sample

■ Confidence interval estimate

$$\bar{X} \pm Z_{(.5-\alpha/2)} \frac{\sigma}{\sqrt{n}}$$

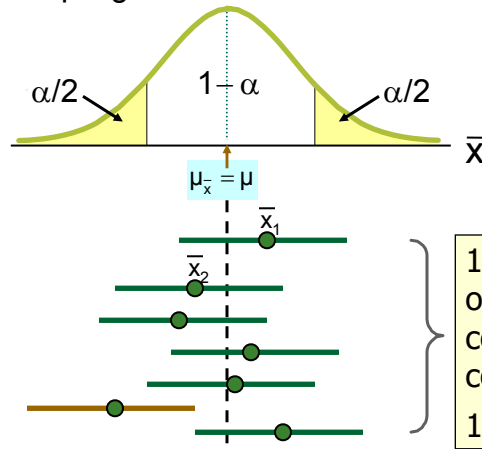
(1- α)x100% CI



Conf. Level	(1- α)	α	(.5- $\alpha/2$)	$Z_{(.5-\alpha/2)}$
90	0.90	0.10	0.450	
95				
99				

Interpretation

Sampling Distribution of the Mean



100(1-α)% of intervals constructed contain μ; 100α% do not.

Confidence Intervals

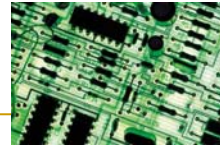
Factors Affecting Half Width

$$H = z_{(.5-\alpha/2)} \frac{\sigma}{\sqrt{n}}$$

- Data variation, σ : H as σ ↓
- Sample size, n : H as n ↑
- Level of confidence, $1 - \alpha$: H if $1 - \alpha$ ↓

Example

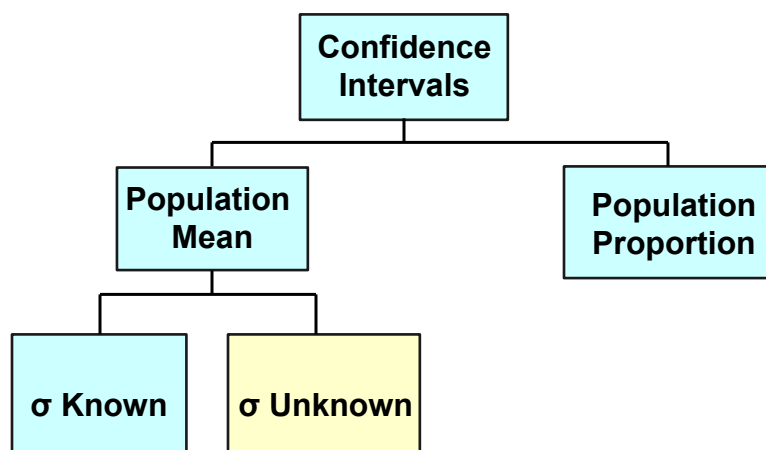
- A sample of 11 circuits from a large normal population has a mean resistance of 2.20 ohms. We know from past testing that the population standard deviation is .35 ohms.
- Determine a 95% confidence interval for the true mean resistance of the population.



Fall 2006 – Fundamentals of Business Statistics

15

Confidence Intervals



Fall 2006 – Fundamentals of Business Statistics

16

Confidence Interval for μ (σ Unknown)

- If the population standard deviation σ is unknown, we can substitute the sample standard deviation, s
- This introduces extra uncertainty, since s is variable from sample to sample
- So we use the t distribution instead of the standard normal distribution

Fall 2006 – Fundamentals of Business Statistics

17

Confidence Interval for μ (σ Unknown)

- Assumptions *(continued)*
 - Population standard deviation is unknown
 - Population is normally distributed
 - If population is not normal, use large sample
- Use Student's t Distribution
- Confidence Interval Estimate

$$\bar{X} \pm t_{(1-\alpha/2)}^{(n-1)} \frac{s}{\sqrt{n}}$$

Fall 2006 – Fundamentals of Business Statistics

18

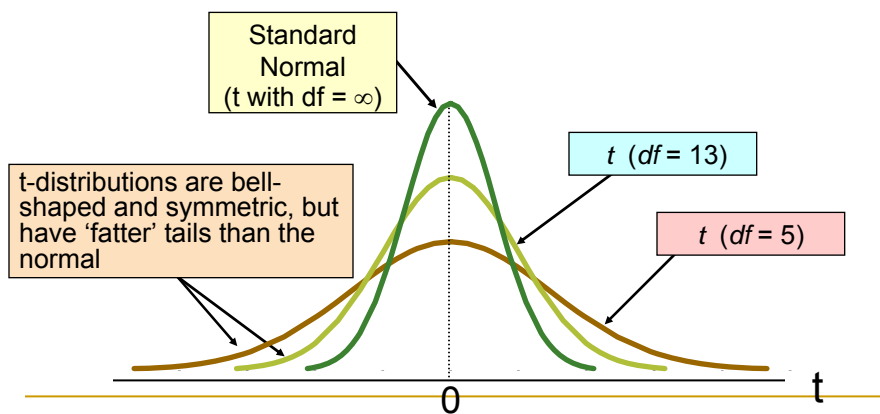
Student's t Distribution

- The t is a family of distributions
- The t value depends on **degrees of freedom (d.f.)**
 - Number of observations that are free to vary after sample mean has been calculated

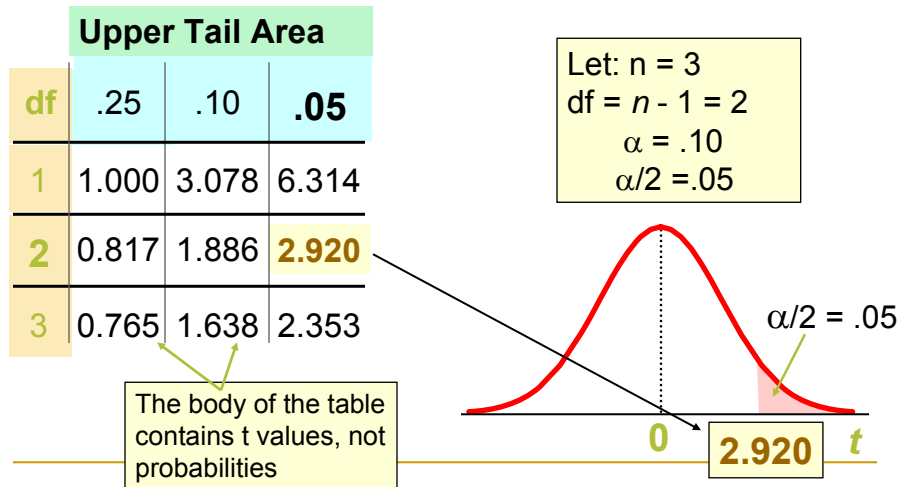
$$d.f. = n - 1$$

Student's t Distribution

Note: $t \rightarrow z$ as n increases



Student's t Table



Fall 2006 – Fundamentals of Business Statistics

21

t distribution values

With comparison to the z value

Confidence Level	t (10 d.f.)	t (20 d.f.)	t (30 d.f.)	z
.80	1.372	1.325	1.310	1.28
.90	1.812	1.725	1.697	1.64
.95	2.228	2.086	2.042	1.96
.99	3.169	2.845	2.750	2.58

Note: $t \rightarrow z$ as n increases

Fall 2006 – Fundamentals of Business Statistics

22

Example

A random sample of $n = 25$ has $\bar{x} = 50$ and $s = 8$. Form a 95% confidence interval for μ

Approximation for Large Samples

- Since t approaches z as the sample size increases, an approximation is sometimes used when $n \geq 30$:

Correct
formula

$$\bar{X} \pm t_{(1-\alpha/2)}^{(n-1)} \frac{s}{\sqrt{n}}$$

Approximation
for large n

$$\bar{X} \pm z_{(0.5-\alpha/2)} \frac{s}{\sqrt{n}}$$

Determining Sample Size

- The required sample size can be found to reach a desired half width (H) and level of confidence (1 - α)

- Required sample size, σ known:

$$n = \frac{z_{(0.5-\alpha/2)}^2 \sigma^2}{H^2} = \left(\frac{z_{(0.5-\alpha/2)} \sigma}{H} \right)^2$$

Determining Sample Size

- The required sample size can be found to reach a desired half width (H) and level of confidence (1 - α)

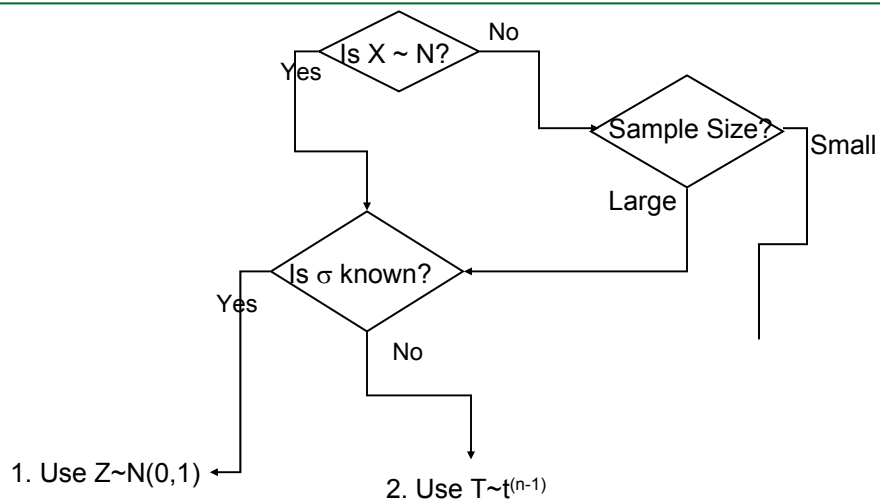
- Required sample size, σ unknown:

$$n = \frac{z_{(0.5-\alpha/2)}^2 S^2}{H^2} = \left(\frac{z_{(0.5-\alpha/2)} S}{H} \right)^2$$

Required Sample Size Example

If $\sigma = 45$, what sample size is needed to be 90% confident of being correct within ± 5 ?

Confidence Interval Estimates



Confidence Intervals $(1-\alpha)\%$

1. Standard Normal

$$\text{Two-sided: } \bar{X} \pm Z_{(0.5-\alpha/2)} \frac{\sigma}{\sqrt{n}}$$

$$\text{One-sided Upper: } \mu \leq \bar{X} + Z_{(0.5-\alpha)} \frac{\sigma}{\sqrt{n}}$$

$$\text{One-sided Lower: } \mu \geq \bar{X} - Z_{(0.5-\alpha)} \frac{\sigma}{\sqrt{n}}$$

2. T distribution

$$\text{Two-sided: } \bar{X} \pm t_{(1-\alpha/2)}^{(n-1)} \frac{s}{\sqrt{n}}$$

$$\text{One-sided Upper: } \mu \leq \bar{X} + t_{(1-\alpha)}^{(n-1)} \frac{s}{\sqrt{n}}$$

$$\text{One-sided Lower: } \mu \geq \bar{X} - t_{(1-\alpha)}^{(n-1)} \frac{s}{\sqrt{n}}$$

YDI 10.17

A beverage dispensing machine is calibrated so that the amount of beverage dispensed is approximately normally distributed with a population standard deviation of 0.15 deciliters (dL).

- Compute a 95% confidence interval for the mean amount of beverage dispensed by this machine based on a random sample of 36 drinks dispensing an average of 2.25 dL.
- Would a 90% confidence interval be wider or narrower than the interval above.
- How large of a sample would you need if you want the width of the 95% confidence interval to be 0.04?

YDI 10.18

A restaurant owner believed that customer spending was below the usual spending level. The owner takes a simple random sample of 26 receipts from the previous weeks receipts. The amount spent per customer served (in dollars) was recorded and some summary measures are provided:

$$n = 26, \bar{X} = 10.44, s^2 = 7.968$$

- Assuming that customer spending is approximately normally distributed, compute a 90% confidence interval for the mean amount of money spent per customer served.
- Interpret what the 90% confidence interval means.