Single-Energy Partial-Wave-Analyses
-Determining The Structure of Baryon Resonances -

Brian Hunt
Advisor: D. Mark Manley
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Outline

- Introduction
- Partial-wave-analyses (PWA)
- Fitting procedure
- Preliminary results
- Summary
- Future Goals
- Acknowledgments
Photoproduction Reactions

- My work focuses on analyzing the reactions
  \[ \gamma p \rightarrow \eta p \quad \gamma n \rightarrow \eta n \quad \gamma p \rightarrow K^+ \Lambda \]
- Photoproduction reactions depend on 4 complex helicity amplitudes
- Refitting reactions
  \[ \pi N \rightarrow \eta N \quad \pi N \rightarrow K \Lambda \]
Why study photoproduction reactions?

- Search for resonances predicted by quark models and lattice QCD
  - A resonance can be thought of as an excited state of a particle $N^*, \Delta^*$, etc.
- Pion beams have been primary tool to study resonances
  - What about resonances that don’t couple strongly to the $\pi N$ channel?
- The reactions in this work are pure isospin $\frac{1}{2}$ states – Only couple to $N^*$ resonances
Helicity Amplitudes

- Helicity amplitudes are expanded into electric and magnetic multipoles, e.g.,

\[ H_N = \sqrt{\frac{1}{2}} \cos \left( \frac{\theta}{2} \right) \sum_{l=0}^{\infty} [(l + 2) E_{l+} + lM_{l+} + lE_{(l+1)-} - (l + 2) M_{(l+1)-}] \left( P'_l - P'_{l+1} \right) \]

- All observables are bilinear combinations of the helicity amplitudes in the form \( \sum H_i H^*_j \)
  - 16 possible combinations leading to 16 observables
    - Eight are linearly independent
    - Causes a global phase ambiguity
Observables

\[ \frac{d\sigma}{d\Omega}(\theta) = \frac{q}{2k} \left[ |H_N(\theta)|^2 + |H_D(\theta)|^2 + |H_{SA}(\theta)|^2 + |H_{SP}(\theta)|^2 \right] \]

\[ \Sigma(\theta) \sigma(\theta) = \frac{q}{k} \text{Re} \left[ H_{SP}(\theta)H_{SA}^*(\theta) - H_N(\theta)H_D^*(\theta) \right] \]

\[ T(\theta) \sigma(\theta) = \frac{q}{k} \text{Im} \left[ H_{SP}(\theta)H_N^*(\theta) + H_D(\theta)H_{SA}^*(\theta) \right] \]

\[ P(\theta) \sigma(\theta) = -\frac{q}{k} \text{Im} \left[ H_{SP}(\theta)H_D^*(\theta) + H_N(\theta)H_{SA}^*(\theta) \right] \]

\[ G(\theta) \sigma(\theta) = -\frac{q}{k} \text{Im} \left[ H_{SP}(\theta)H_{SA}^*(\theta) + H_N(\theta)H_D^*(\theta) \right] \]

\[ H(\theta) \sigma(\theta) = -\frac{q}{k} \text{Im} \left[ H_{SP}(\theta)H_{D}^*(\theta) + H_{SA}(\theta)H_{N}^*(\theta) \right] \]

\[ E(\theta) \sigma(\theta) = \frac{q}{2k} \left[ |H_N(\theta)|^2 + |H_{SA}(\theta)|^2 - |H_D(\theta)|^2 - |H_{SP}(\theta)|^2 \right] \]

\[ F(\theta) \sigma(\theta) = \frac{q}{k} \text{Re} \left[ H_{SA}(\theta)H_D^*(\theta) + H_{SP}(\theta)H_N^*(\theta) \right] \]

\[ O_x(\theta) \sigma(\theta) = -\frac{q}{k} \text{Im} \left[ H_{SA}(\theta)H_D^*(\theta) + H_{SP}(\theta)H_N^*(\theta) \right] \]

\[ O_z(\theta) \sigma(\theta) = -\frac{q}{k} \text{Im} \left[ H_{SA}(\theta)H_{SP}^*(\theta) + H_N(\theta)H_D^*(\theta) \right] \]

\[ C_x(\theta) \sigma(\theta) = -\frac{q}{k} \text{Re} \left[ H_{SA}(\theta)H_N^*(\theta) + H_{SP}(\theta)H_D^*(\theta) \right] \]

\[ C_z(\theta) \sigma(\theta) = \frac{q}{2k} \left[ |H_{SA}(\theta)|^2 + |H_D(\theta)|^2 - |H_N(\theta)|^2 - |H_{SP}(\theta)|^2 \right] \]

\[ T_x(\theta) \sigma(\theta) = \frac{q}{k} \text{Re} \left[ H_{SP}(\theta)H_{SA}^*(\theta) + H_N(\theta)H_D^*(\theta) \right] \]

\[ T_z(\theta) \sigma(\theta) = \frac{q}{k} \text{Re} \left[ H_{SP}(\theta)H_N^*(\theta) - H_{SA}(\theta)H_D^*(\theta) \right] \]

\[ L_x(\theta) \sigma(\theta) = \frac{q}{k} \text{Re} \left[ H_{SA}(\theta)H_N(\theta) - H_{SP}(\theta)H_D(\theta) \right] \]

\[ L_z(\theta) \sigma(\theta) = \frac{q}{2k} \left[ |H_{SP}(\theta)|^2 + |H_{SA}(\theta)|^2 - |H_N(\theta)|^2 - |H_D(\theta)|^2 \right] \]
Nucleon Resonance

Basic Reactions

s-channel process

\[
\begin{align*}
\gamma & \rightarrow N^* \\
N^* & \rightarrow p + \eta
\end{align*}
\]

*Also u-channel processes

\[
\begin{align*}
\gamma & \rightarrow \rho^0, \omega \\
\rho^0, \omega & \rightarrow p + p
\end{align*}
\]
Resonances as 3-D H.O. Excitations

$N^p = 1^-$

$1 \otimes \frac{1}{2} = \frac{1}{2} \oplus \frac{3}{2}$

$1 \otimes \frac{3}{2} = \frac{1}{2} \oplus \frac{3}{2} \oplus \frac{5}{2}$

$N^p = 2^+$

$0 \otimes \frac{1}{2} = \frac{1}{2}$

$0 \otimes \frac{3}{2} = \frac{3}{2}$

$2 \otimes \frac{1}{2} = \frac{3}{2} \oplus \frac{5}{2}$

$2 \otimes \frac{3}{2} = \frac{1}{2} \oplus \frac{3}{2} \oplus \frac{5}{2} \oplus \frac{7}{2}$

$\hbar \omega \bigg\{ \begin{array}{c} 2S \ 1D \\ \hline \\ 1P \\ \hline \\ 1S \end{array}$
Known Resonances (N=1,2 Bands)

- D13(1520) ****
- S11(1535) ****
- S11(1650) ****
- D15(1675) ****
- D13(1700) ***
- P11(1440) ****
- F15(1680) ****
- P11(1710) ***
- P13(1720) ****
- P13(1900) ***
- F15(1860) **
- F15(1860) **
- F17(1990) **
KSU Fitting Procedure

- Start by analyzing one channel $\pi N \rightarrow \eta N$
  - Fit observables in small energy bins $f(\theta)$
  - Generate single-energy amplitudes $T_{\pi N, \eta N}^{IJ}$
- Fit single-energy amplitudes with energy-dependent parametrization
  - Energy-dependent code requires unitarity and analyticity
  - Must fit all reactions with a consistent set of parameters
- Iterate process until a good energy-dependent fit of the observables is found
Energy-Dependent Solution

- Add in all two-body channels
  \[ \pi N \rightarrow \pi \Delta \]
  \[ \pi N \rightarrow \omega N \]
  \[ \pi N \rightarrow \gamma N \]
  \[ \pi N \rightarrow \rho N \]
  \[ \pi N \rightarrow \eta N \]
  \[ \pi N \rightarrow K \Lambda \]

- Missing Resonances
  - Even after adding these channels, some predicted resonances are not seen

- Look for resonances that couple weakly to \( \pi N \)
Energy-Dependent Fit

- Single-energy points are fitted rather than observables
  - Include all possible reactions
- Model dependent
  - Resonances are added by hand
  - Background added as needed
  - Dummy channels to saturate unitary bound
- Each partial wave analyzed separately
  - Lose interference effects between partial waves in approach
Difficulties of a Single-Energy PWA

- Inconsistent data
- Can’t directly use unitary and analytic constraints
- Global Phase Ambiguity
  - Global phase for a single reaction is determined from the energy-dependent fit
- Lose interference effects when moving to energy-dependent fits
- Insufficient data to determine amplitudes uniquely
  - Creates a noisy bin-to-bin solution
    - Handle this by truncating higher-order multipoles and small amplitudes
    - Penalty terms are added to constrain fits
  - May miss resonances
Importance of Complete Data Sets

\[ \cos \theta = -0.4 \]

\[ \cos \theta = -0.2 \]

\[ \gamma p \rightarrow \eta p \]

\[ \cos \theta = 0.4 \]

\[ \cos \theta = 0.6 \]
**Current World Database**

<table>
<thead>
<tr>
<th>Observable</th>
<th>$\gamma p \rightarrow \eta p$</th>
<th>$\gamma p \rightarrow K^+ \Lambda$</th>
<th>$\gamma n \rightarrow \eta n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>DSG</td>
<td>4890</td>
<td>4545</td>
<td>879</td>
</tr>
<tr>
<td>T</td>
<td>358</td>
<td>458</td>
<td>96</td>
</tr>
<tr>
<td>$\Sigma$</td>
<td>295</td>
<td>438</td>
<td>88</td>
</tr>
<tr>
<td>P</td>
<td>7</td>
<td>1810</td>
<td>0</td>
</tr>
<tr>
<td>F</td>
<td>265</td>
<td>92</td>
<td>96</td>
</tr>
<tr>
<td>E</td>
<td>210</td>
<td>72</td>
<td>140</td>
</tr>
<tr>
<td>Ox/Oz</td>
<td>0</td>
<td>363/363</td>
<td>0</td>
</tr>
<tr>
<td>Cx/Cz</td>
<td>0</td>
<td>133/133</td>
<td>0</td>
</tr>
<tr>
<td>Other Data</td>
<td>296 (3 observables)</td>
<td>289</td>
<td></td>
</tr>
<tr>
<td>Bins With 8 independent obs</td>
<td>0</td>
<td>10</td>
<td>0</td>
</tr>
</tbody>
</table>

*Some data are preliminary*
\[ \gamma p \rightarrow \eta p \]

\[ W = 1580 \text{ MeV} \]

\[ \frac{d\sigma}{d\Omega} \]

\[ \cos \Theta \]

\[ W = 1580 \text{ MeV} \]

\[ \sum \]
$\gamma p \rightarrow \eta p$

\[
\frac{d\sigma}{d\Omega}
\]

$W = 1670$ MeV

RSU Fit

Data of Fit: 2016-03-31 00:05:41 PM
\[ \gamma p \rightarrow K^+ \Lambda \]
\[ \gamma p \rightarrow K^+ \Lambda \]

\[ W = 1830 \text{ MeV} \]
$\gamma p \rightarrow K^+ \Lambda$

$W = 1910 \text{ MeV}$
\[
\frac{d\sigma}{d\Omega} \quad \pi^- p \rightarrow \eta n \quad P
\]
\[ \frac{d\sigma}{d\Omega} \quad \pi^- p \rightarrow K^0 \Lambda \quad P \]
Integrated Cross Section

\[ \gamma p \rightarrow \eta p \]

\[ \sigma_i (\text{MeV}) \]

\[ W (\text{MeV}) \]

\[ \text{Date of Fit: 2016-03-31} \]
Integrated Cross Section

$\gamma p \rightarrow K^+ \Lambda$

- ERBE 1969
- BOCKHORST 1994
- BRADFORD 2006

Legend:
- SX1
- SX1+PX1
- SX1+PX1+PX3E
- SX1+PX1+PX3E+PX3M
- SX1+PX1+PX3+DX3
- SX1+PX1+PX3+DX3+DX5
- SGT

Date of Fit: 2016-03-31

$\sigma_{int}$ (mb) vs. $W$ (MeV)
Integrated Cross Section
Integrated Cross Section

\[ \gamma n \rightarrow \eta n \]

Werthmuller 2014

\( \sigma_\gamma (\mu b) \)

Date of Fit: 2016-03-31
Resonances and Argand Diagrams

Assume \( T_{ij} = \frac{\sqrt{\Gamma_i \Gamma_j}}{M - W - i \frac{1}{2}} \) near the resonance

\[
T_r^2 + \left( T_i - \frac{1}{2} \sqrt{x_i x_j} \right)^2 = \left( \frac{1}{2} \sqrt{x_i x_j} \right)^2 \text{ where } x_i \equiv \frac{\Gamma_i}{\Gamma}
\]

Reading an Argand Diagram
### S11

<table>
<thead>
<tr>
<th>S11(1535) and S11(1650)</th>
<th>KSU</th>
<th>BnGa</th>
<th>SAID</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (MeV) S11(1535)</td>
<td>1530</td>
<td>1519 ± 5</td>
<td>1547 ± 0.7</td>
<td>1525-1545</td>
</tr>
<tr>
<td>Width (MeV) S11(1535)</td>
<td>157</td>
<td>128 ± 14</td>
<td>188.4 ± 3.8</td>
<td>125-175</td>
</tr>
<tr>
<td>Mass (MeV) S11(1650)</td>
<td>1665</td>
<td>1651 ± 6</td>
<td>1634.7 ± 1.1</td>
<td>1645-1670</td>
</tr>
<tr>
<td>Width (MeV) S11(1650)</td>
<td>151</td>
<td>104 ± 14</td>
<td>115.4 ± 2.8</td>
<td>110-170</td>
</tr>
</tbody>
</table>
### S11

#### Branching Ratios

<table>
<thead>
<tr>
<th>Branching Ratios</th>
<th>KSU</th>
<th>BnGa</th>
<th>SAID</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi N$</td>
<td>41.4</td>
<td>54 ± 5</td>
<td>35.5 ± 0.2</td>
<td>35-55</td>
</tr>
<tr>
<td>$\eta N$</td>
<td>51.1</td>
<td>33 ± 5</td>
<td>N/A</td>
<td>32-52</td>
</tr>
<tr>
<td>$K^+ \Lambda$</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>$\pi N$</td>
<td>53</td>
<td>51 ± 4</td>
<td>100</td>
<td>50-70</td>
</tr>
<tr>
<td>$\eta N$</td>
<td>0.1</td>
<td>18 ± 4</td>
<td>N/A</td>
<td>5-15</td>
</tr>
<tr>
<td>$K^+ \Lambda$</td>
<td>10.5</td>
<td>10 ± 5</td>
<td>N/A</td>
<td>2.5-3.4</td>
</tr>
</tbody>
</table>
### P13(1720) and P13(1900)

<table>
<thead>
<tr>
<th></th>
<th>KSU</th>
<th>BnGa</th>
<th>SAID</th>
<th>PDG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (MeV) P13(1720)</td>
<td>1721</td>
<td>1690 ± 70 -35</td>
<td>1763.8 ± 0.7</td>
<td>1700 - 1750</td>
</tr>
<tr>
<td>Width (MeV) P13(1720)</td>
<td>151</td>
<td>420 ± 100</td>
<td>210 ± 22</td>
<td>150 ± 400</td>
</tr>
<tr>
<td>Mass (MeV) P13(1900)</td>
<td>1912</td>
<td>1905 ± 30</td>
<td>N/A</td>
<td>1900 ± 30</td>
</tr>
<tr>
<td>Width (MeV) P13(1900)</td>
<td>423</td>
<td>250 +120 -50</td>
<td>N/A</td>
<td>200 ± 50</td>
</tr>
<tr>
<td>Br. Ratio</td>
<td>KSU</td>
<td>BnGa</td>
<td>SAID</td>
<td>PDG</td>
</tr>
<tr>
<td>-----------</td>
<td>-----</td>
<td>------</td>
<td>---------</td>
<td>--------</td>
</tr>
<tr>
<td>$\pi N$</td>
<td>19</td>
<td>10 ± 5</td>
<td>9.4 ± .5</td>
<td>8-14</td>
</tr>
<tr>
<td>$\eta N$</td>
<td>1.7</td>
<td>3 ± 2</td>
<td>N/A</td>
<td>.6 – 3.5</td>
</tr>
<tr>
<td>$K^+ \Lambda$</td>
<td>8</td>
<td>N/A</td>
<td>N/A</td>
<td>4-4.8</td>
</tr>
<tr>
<td>$\pi N$</td>
<td>7</td>
<td>3 ± 2</td>
<td>N/A</td>
<td>~5</td>
</tr>
<tr>
<td>$\eta N$</td>
<td>6</td>
<td>10 ± 4</td>
<td>N/A</td>
<td>~12</td>
</tr>
<tr>
<td>$K^+ \Lambda$</td>
<td>24</td>
<td>16 ± 5</td>
<td>N/A</td>
<td>0 - 10</td>
</tr>
</tbody>
</table>
Summary

- Missing resonances problem is still an open question
- Knowing the resonance structure provides information about the quark degrees of freedom inside a hadron
- Added new S11, D13, and F15 resonances into our fits, and constrained the properties of other resonances, especially in the fourth resonance region
- We have increased maximum energy for the fits from 2100 to 2250 MeV because more data at higher energies are becoming available
Future Goals

- Improve fits of my photoproduction reactions, especially at higher energies
- Have a consistent explanation of bump in $\gamma n \rightarrow \eta n$ cross section
- Compare new fits of $\gamma N \rightarrow \pi N$ with observables
- Add in $K\Sigma$ final-state reactions (new Ph.D. project)
Acknowledgments

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- Thanks to Kent State University Physics Department for partial support of my research
Properties of S and T

Using unitarity and the relationship $S_i = I + 2iT_i$

$$T^\dagger T = \frac{T - T^\dagger}{2i}$$

Considering time-reversal symmetry

$$T_{if} = T_{fi}$$

we can show for elastic scattering

$$(T^\dagger T)_{ii} = \frac{T_{ii} - T_{ii}^*}{2i} = \text{Im}T_{ii}$$

This is an expression of the Optical Theorem
Strengths of a Single-Energy PWA

- Starting point has little model dependence
- Get a feel for the resonance structure
- Focus on small number of parameters/data
BACKUP SLIDE – INELASTICITY PLOTS

- Show 4 plots of inelasticity
BACKUP SLIDE – INELASTICITY PLOTS

- Show 4 more plots of inelasticity
BACKUP – Another Single Angle set of Plots K Lambda
LHS particle Accelerator is 22 km in circumference

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