The photographs shown on these two pages represent the polarizing-microscopy textures of nematic films. No external fields, no temperature gradients — just thin films of a standard nematic liquid crystal 5CB. However, the textures reveal many striking features.

Firstly, they show the existence of different periodic domain patterns. The ability to form periodic spatial structures either because of specific molecular interactions (cholesterics, smectics, blue phases) or under the influence of electric or magnetic fields is one of the most important and well-known properties of liquid crystals. In the present case, however, one deals with a translationally symmetric phase with no external field.

Secondly, the textures contain defect points (boojums) with enormously high topological charges, or strengths. They manifest themselves as nuclei with numerous black brushes. General rules of polarizing microscopy tell us that the director should undergo a \( \frac{2}{3} \) rotation between each pair of successive brushes [1]. The presence of 12 brushes indicates that defect contains as many as \( \frac{12}{2} \) turns! It is quite amazing, because the greater the number of turns the greater the elastic energy. Thus only defects with the lowest possible number of turns, such as disclinations with total \( \pi \)-rotation or boojums with \( 2\pi \)-rotation, are supposed to exist.

Thirdly, the pairs of boojums turn out to be connected by strings that appear in textures as four or three parallel black brushes. All deformations induced by the defect pair are stretched out into this string of constant width, across which the director field undergoes rotation through an angle \( 2\pi \). It is noteworthy that the constant thickness of the string means that the attractive force between boojums does not depend on the separation, as in the case of quarks [2]. The strings can appear also as isolated loops or form sophisticated cellular patterns.

Despite the absence of external fields or temperature gradients, these strange structures are not present without a reason. The cause lies in the very nature of the film preparation that is different from conventional methods of putting a liquid crystal between two rigid plates. The samples used for microphotography are nematic films with a free surface placed onto an isotropic liquid substrate, such as polyethylene glycol, glycerin, or water. The thinnest films (\( \leq 0.5 \) \( \mu \)m) show up different periodic textures [3]. The films of intermediate thickness (1-20 \( \mu \)m) manifest strings, defects with large strengths as well as nucleation of defect pairs [2,4]. Finally, the textures of thick films are similar to those of conventional cells: periodical structures, high-strength defects, and strings do not appear.

There are two crucial features of the liquid crystalline films that make their properties unique. First, the polar tilt angles of the director at two surfaces need not necessarily coincide since the two ambient media are different. Thus the film is hybrid aligned. Secondly, because of the isotropic nature of the ambient media, the molecular interactions do not fix the azimuthal orientation. Thus the boundary conditions degenerate in the film plane and the director can rotate on the film surface without anchoring energy losses.

With non-zero vertical curvature and zero azimuthal anchoring, how can the system gain energy? Surprisingly, the total energy of distortions can be reduced if additional deformations arise in the film plane. To demonstrate this, let us imagine surface \( \Sigma \) that is perpendicular to \( n \), Fig 1. Deformations of \( \Sigma \) are characterized by principal radii of curvature \( R_1 \) and \( R_2 \), and \( R_3 \) are algebraic lengths, whose signs depend on the orientation chosen on the normal to \( \Sigma \). They define the mean curvature \( (1/R_1 + 1/R_2) \) and the Gaussian curvature \( 1/R_1 R_2 \) of \( \Sigma \). The mean and Gaussian curvatures can be expressed in terms of derivatives on \( n \) [1]:

\[
1/R_1 + 1/R_2 = -\text{div} n \quad \text{and} \quad 1/R_1 R_2 = \frac{1}{4} \text{div}(\text{div} n + n \times \text{rot} n).
\]
sponding contributions to the elastic energy density are splay
\( \frac{1}{2}K_{s}\Delta \text{div} n \) and saddle-splay \( K_{ss}\Delta \text{div}(n \times n + n \times \text{rot} n) \) terms respectively.

The difference in the polar anchoring fixes the non-zero value of
one of the principal radii of curvature, say \( R_1 \). The hybrid-aligned
film that is uniform in the horizontal plane \((a,b)\) is characterised
by one radius of curvature; as a result, the splay term is equal to
\( K_{s}/2R_1^2 \) and the saddle-splay term is zero. In the states with
horizontal deformations \((c,d)\) both radii are finite. The splay
contribution \( \frac{1}{2}K_{s}(1/R_1 + 1/R_2)^2 \) decreases when \( R_1 \) and \( R_2 \) are of
opposite signs (so-called splay cancelling mechanism [5]).
Moreover, with finite \( R_1 \) the saddle-splay term becomes non zero
and reduces the total energy when \( K_{ss}/R_1 R_2 \) is negative. It does
not mean that the Gaussian curvature of the energetically pre-
ferred states should always be negative: the saddle-splay con-
stant \( K_{ss} \) can be either positive or negative, in contrast to standard
elastic moduli such as \( K_{ss} \). Thus the positive \( K_{ss} \) will favour
the deformation with \( 1/R_1 R_2 < 0 \), \( K_{ss} > 0 \) will favour \( 1/R_1 R_2 > 0 \). Despite
the evident importance and long history of the saddle-splay
problem in nematics, up to now there are only a few tentative
experimental estimations of \( K_{ss} \).[6,7].

Qualitative consideration suggests that the deformed states can
be energetically preferable to the expected "uniform" hybrid
aligned film because of non-zero Gaussian curvature and zero
azimuthal anchoring. Furthermore, these states can provide a
number of independent methods for determination of the saddle-
splay elastic constant \( K_{ss} \). For example, \( K_{ss} \) defines
the thickness threshold for periodic domain patterns [3] and domain behaviour
under the action of external field [8] or thickness changes [9] can
allow the measurements of \( K_{ss} \). Another approach is based on
the peculiarities of strings which can be viewed as two \( \pi \)-substrings with opposite signs of Gaussian curvature [7].

Liquid crystalline films placed on an isotropic substrate reveal
many other interesting properties, connected for example, with
their polar structure and corresponding flexoelectric or
thermochemical effects. We believe that the beauty and some-
times unexpected behaviour of these definitely "soft-matter"
objects deserves detailed study.

Fig 1. Director field \((a,c)\) and corresponding surface \( \Sigma \) \((b,d)\) for
uniform and deformed states of a hybrid aligned film.

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