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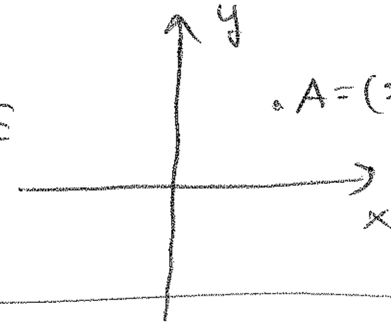
INFORMAL LECTURE NOTES
MTH 234.

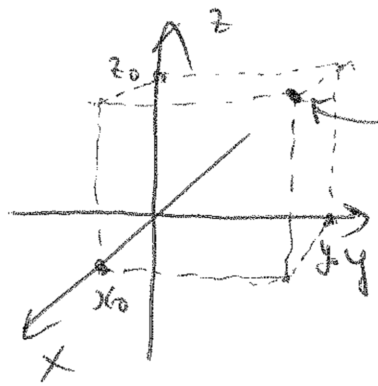
LECTURE 1. $\boxed{\text{Section } 1[redacted].1}$

2D - COORDINATE SYSTEM

Example: $y = 2x+1$ IS A LINE



3D - COORDINATE SYSTEM.



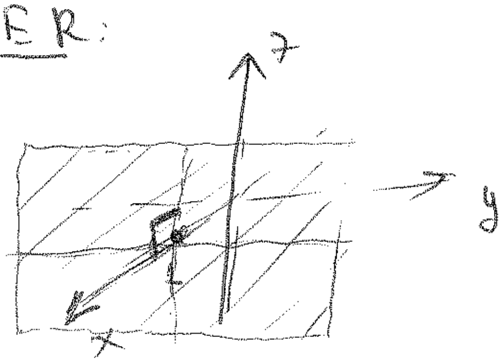
$A = (x_0, y_0, z_0)$

POINTS HAVE 3 COORDINATES

Q: DESCRIBE SET OF POINTS IN 3D (x, y, z)

such that $x = 1$.

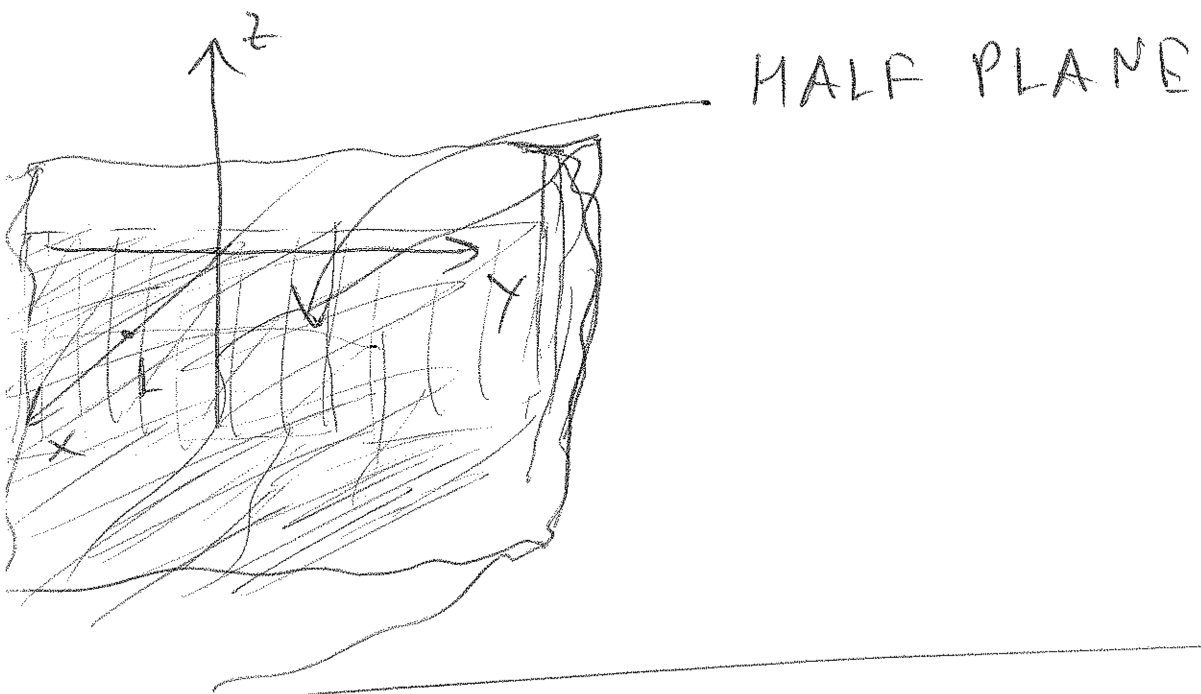
ANSWER:



IT IS A PLANE PERPENDICULAR TO THE X-AXIS AT POINT $x=1$.

Q: ~~$x \geq 1$~~ Describe $x \geq 1$.

ANSWER: IT IS A HALF PLANE



REMEMBER: ~~Som~~ SOMETHING $= 0$

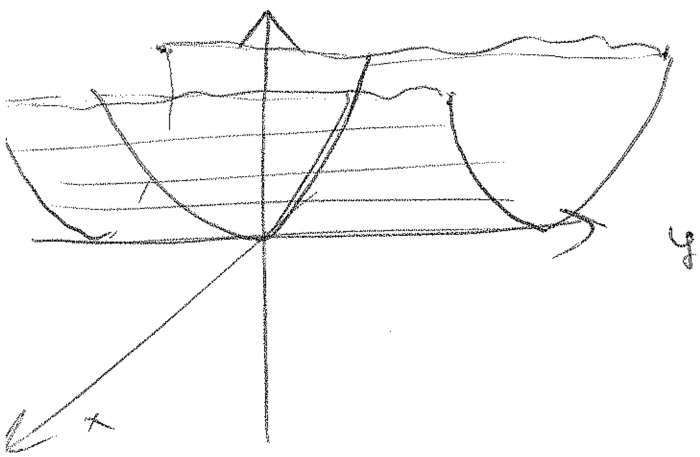
Gives Boundary (Surface).
And SOMETHING ≥ 0 is one side of this boundary and SOMETNING ≤ 0 Is another side of this boundar

Ex. Describe

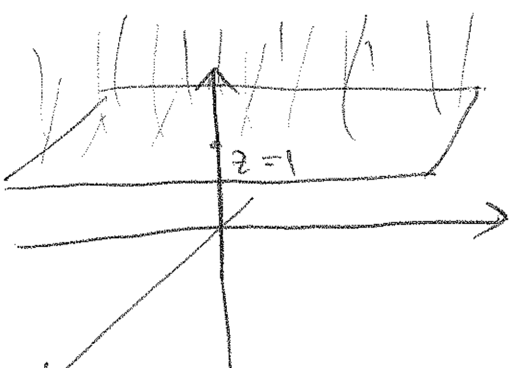
$$\begin{cases} y = x^2 \\ z \geq 1 \end{cases}$$

Answer:

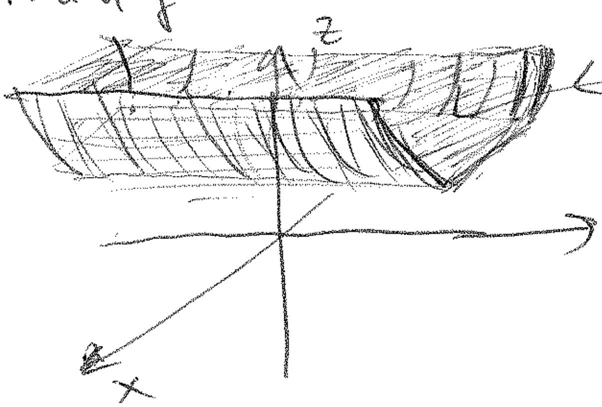
First we draw $y = x^2$ in 3D.



AFTER That WE DRAW $z \geq 1$



Finally we take intersection:

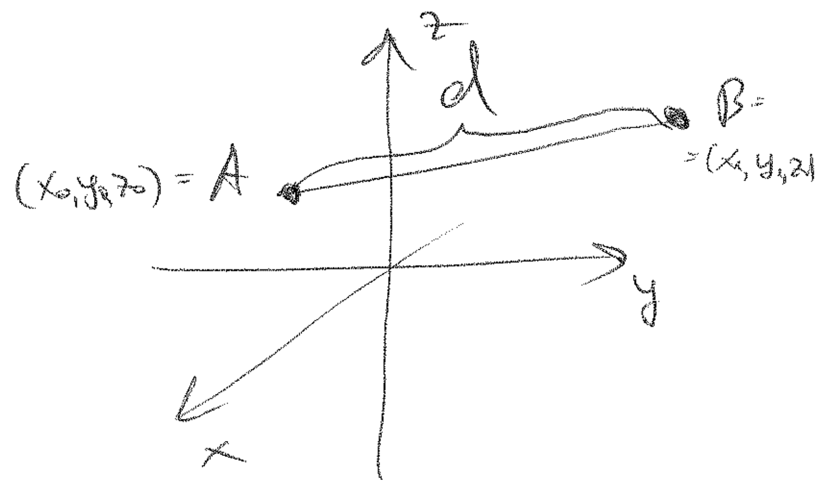


$$\begin{cases} y \geq x^2 \\ z \geq 1 \end{cases}$$

DISTANCE BETWEEN POINTS

Let $A = (x_0, y_0, z_0)$

$B = (x_1, y_1, z_1)$



What is the value of $d = ?$

$$d = \sqrt{(x_1 - x_0)^2 + (y_1 - y_0)^2 + (z_1 - z_0)^2}$$

PROOF: The Pythagorean theorem.

EQUATION OF A SPHERE OF RADIUS R

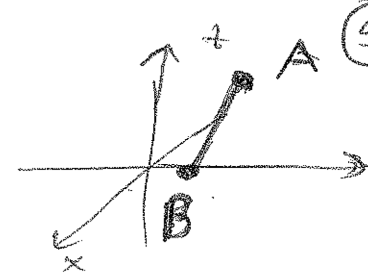
$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = R^2$$

where (x_0, y_0, z_0) is a center of a sphere and R is a radius.

~~Th~~ GOOD TO KNOW:

This equation tells you that distance between the points (x, y, z) and (x_0, y_0, z_0) is constant $= R$.

Example:



$A = (1,2,3)$, $\quad B = (0,1,0)$

$$\text{Distance} = \sqrt{1^2 + (2-1)^2 + 3^2} =$$

$$= \sqrt{1+1+9} = \sqrt{11}$$

Example:

Describe the set of points (x,y,z) such that $x^2+2x+y^2+z^2-2z = 10$

Solution:

$$(a+b)^2 = a^2 + 2ab + b^2$$

$$0 = (x^2+2x) + y^2 + (z^2-2z) =$$

$$= (x^2+2x+1) - 1 + y^2 + (z^2-2z+1) - 1 =$$

$$= (x+1)^2 + y^2 + (z-1)^2 - 2$$

Hence:

$$12 = (x+1)^2 + y^2 + (z-1)^2$$

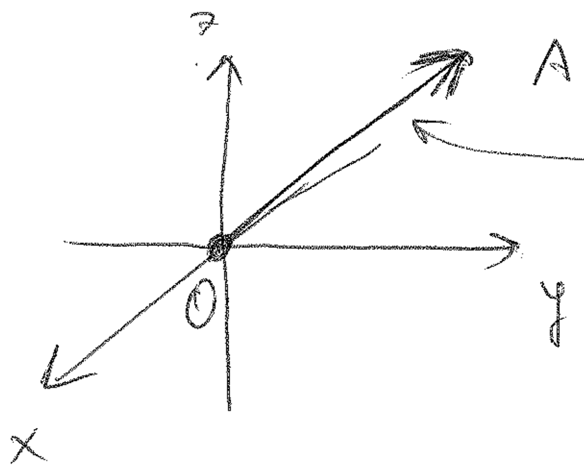
radius $= \sqrt{12}$

center $= (-1, 0, 1)$

Section 1[redacted].2

IN 3D WE HAVE A POINT

$A = (x_0, y_0, z_0)$. WHAT IS VECTOR?



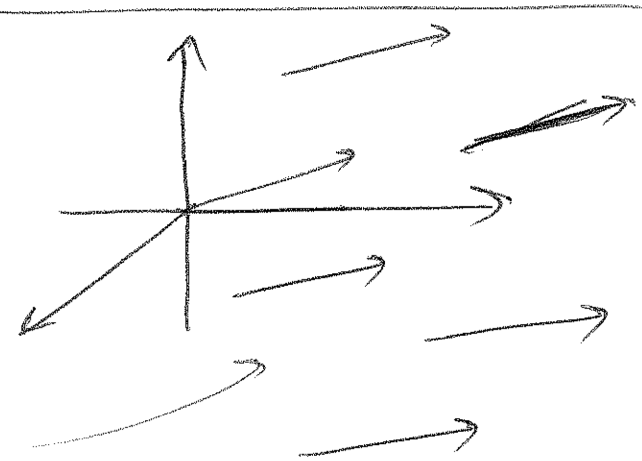
vector $\overrightarrow{OA} = \langle x_0, y_0, z_0 \rangle$

different brackets

WHEN WE SAY VECTOR $\overrightarrow{OA}$ or $\vec{v}$

we care about <u>length</u> of OA and

<u>direction</u>.



These are one and the same vectors because they have the same length and direction.

Properties

1) IF $\vec{v} = \langle v_1, v_2, v_3 \rangle$

Then $\boxed{|\vec{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2}}$

length amplitude magnitude

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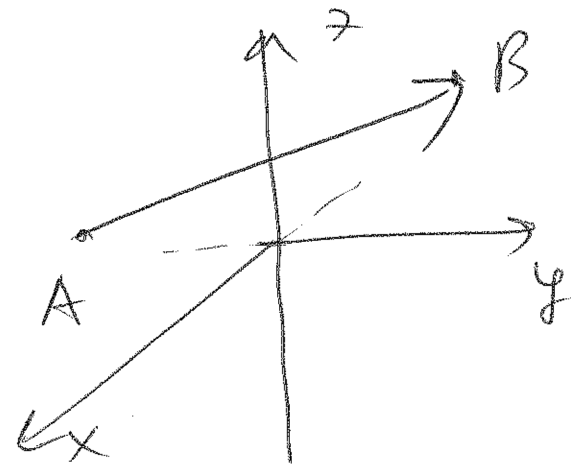
2) IF $A = (x_0, y_0, z_0)$

$B = (x_1, y_1, z_1)$



$\overrightarrow{AB} = \langle x_1 - x_0, y_1 - y_0, z_1 - z_0 \rangle$

$= \overrightarrow{OB} - \overrightarrow{OA}$. Therefore $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

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3) $\overrightarrow{AB} = -\overrightarrow{BA}$

<u>Example.</u> $A = (0, 1, 2)$

$B = (3, 4, 5)$

$\overrightarrow{AB} = ?$

Answer: $\overrightarrow{AB} = \cancel{\langle 3,4,5 \rangle} \cdot \langle 3-0, 4-1, 5-2 \rangle =$

$= \langle 3, 3, 3 \rangle$

4) IF $\vec{v} = \langle v_1, v_2, v_3 \rangle$

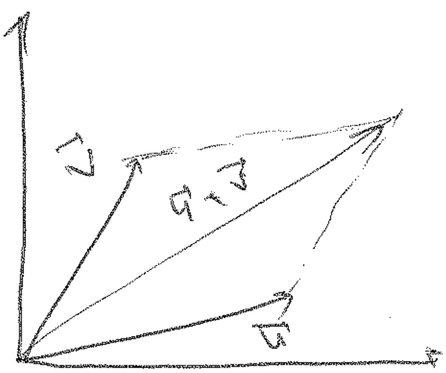
$\vec{u} = \langle u_1, u_2, u_3 \rangle$

Then

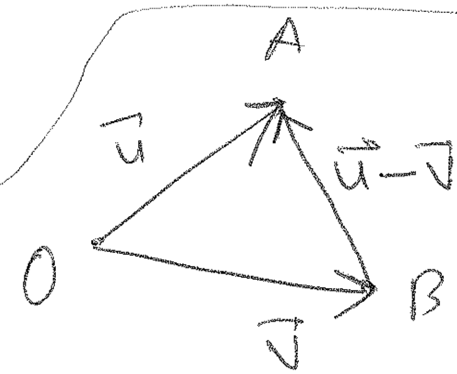
$$\vec{v} + \vec{u} = \langle v_1 + u_1, v_2 + u_2, v_3 + u_3 \rangle$$

5.) IF $\vec{v} = \langle v_1, v_2, v_3 \rangle$ and λ is a number (scalar) then

$$\lambda \cdot \vec{v} = \langle \lambda v_1, \lambda v_2, \lambda v_3 \rangle$$



$\boxed{\text{Parallelogram law}}$



IF

$\overrightarrow{OA} = \vec{u}$

$\overrightarrow{OB} = \vec{v}$

then $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -\overrightarrow{OB} + \overrightarrow{OA} =$

$= -\vec{v} + \vec{u}$

IF k - is a number then

Then

$$|k \cdot \vec{u}| = |k| \cdot |u|$$

PROOF: BY DEFINITION.

Example:

$\vec{u} = \langle 1, 2, 3 \rangle$, $\vec{v} = \langle 1, 1, 1 \rangle$

Find $\frac{1}{2}\vec{u} - 3\vec{v} = ?$

Answer:

$$\frac{1}{2}\vec{u} - 3\vec{v} = \frac{1}{2}\langle 1, 2, 3 \rangle -$$

$$- 3 \cdot \langle 1, 1, 1 \rangle = \left\langle \frac{1}{2}, 1, \frac{3}{2} \right\rangle - \langle 3, 3, 3 \rangle =$$

$$= \left\langle -\frac{5}{2}, -2, -\frac{3}{2} \right\rangle$$

~~More properties~~:

IF $\vec{v}$ is a vector

then $\dfrac{\vec{v}}{|\vec{v}|}$ - is called <u>direction</u>.

The unit vector is a vector such that its length is 1.

$|u| = 1$

<u>Proposition</u>: IF $\vec{u}$ is a vector then $\dfrac{\vec{u}}{|\vec{u}|}$ - is a unit vector

<u>PROOF</u>: INDEED.

$$\left| \frac{\vec{u}}{|\vec{u}|} \right| = \frac{1}{|\vec{u}|} \cdot |\vec{u}| = 1.$$

Example: $\vec{u} = \langle \frac{3}{5}, \frac{4}{5}, 0 \rangle$ - is a unit vector

<u>Indeed</u>:

$$|\vec{u}| = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 + 0^2} = \sqrt{\frac{9+16}{25}} = 1.$$