HOW (NOT) TO WRITE MATHEMATICS

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Being able to express oneself precisely is a skill which is needed in all aspects of life. Math classes are perfect for practicing and improving this important skill because of the precise notation that is used in mathematics. In mathematics, we say what we mean and we mean what we say. Moreover, knowing how to write mathematics correctly is helpful for a better understanding of mathematics itself. Elementary math problems require just a few steps and we usually do them in one go, without looking back. Some of the steps can be even done mentally and there is very little to write down. However, as we start encountering more and more complicated math problems, the need arises for solution methods which require going back and forth between the steps. If we do not write down the previous steps in a clear way, we won't be able to understand them and use them to continue solving the problem.

Mathematics should be written like English, only, in mathematics we have math sentences and math rules which correspond to spelling and grammatical rules. There are even rules that govern the style and organization of an essay. Of course, one also has to know what to write about. Math writing mistakes can therefore be characterized like those in English. Two math problems and their solutions are given below to illustrate all this. The correct way of writing mathematics is compared to the typical mistakes.

1. A precalculus problem. Solve the equation for x,

$$x^2 - x = 1$$

Solution. [Correct steps are numbered, mistakes are bulleted.]

1. Subtract 1 from both sides to get $x^2 - x - 1 = 0$.

- *Inappropriate passage*
  Left-hand side factored, $x(x - 1) = 1$, and the "solution(s)" found somehow from there.

- *Incomplete sentence (predicate missing)*
  Step 1 done, but only $x^2 - x - 1$ written.

2. Try to factor $x^2 - x - 1$ as $(x - m)(x - n)$, where the integers m and n should satisfy $m \cdot n = -1$ and $m + n = -1$. Conclude that this is not possible and that the quadratic formula needs to be used.

- *Inappropriate passage*
  The trinomial is factored somehow and the "solution(s)" are found.

- *Wrong conclusion*
  Upon realizing that $x^2 - x - 1$ cannot be factored, it is concluded that the equation has no solution.

3. Use the quadratic formula with $a = 1$, $b = -1$, $c = -1$:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{1 \pm \sqrt{(-1)^2 - 4(-1)}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

- *Spelling mistake*
  Fraction bar too short, like $-b$ is not in the numerator:

$$x = \begin{array}{c} 1 \pm\sqrt{5} \\ \hline 2 \end{array}$$

- *Spelling mistake*
  Parentheses omitted when calculating $b^2 = (-1)^2$, resulting in the wrong solution:

$$x = \frac{1 \pm \sqrt{-1^2 - 4(-1)}}{2} = \frac{1 \pm \sqrt{3}}{2}$$

- *Inappropriate passage*
  Something else miscalculated.

2. A calculus problem. Find all critical numbers of $f(x) = \sqrt[3]{x^3 - 3x}$. Then apply to each of them, if possible, the Second-Derivative Test. If this is not possible to do, say so.

Solution. [Correct steps are numbered, mistakes are bulleted.]

1. Discuss the domain of the function and prepare f for differentiation:

$$f(x) = (x^3 - 3x)^{1/3}, \quad \text{domain} = \text{all real numbers}$$

2. Differentiate f:

$$f'(x) = \frac{1}{3}(x^3 - 3x)^{-2/3}(3x^2 - 3)$$

- *Incomplete sentence (subject missing)*
  '$f'(x) =$' not written:

$$\frac{1}{3}(x^3 - 3x)^{-2/3}(3x^2 - 3)$$

- *Spelling mistake*
  Parentheses omitted:

$$f'(x) = \frac{1}{3}(x^3 - 3x)^{-2/3} \cdot 3x^2 - 3$$

3. Simplify f':

$$f'(x) = \frac{3x^2 - 3}{3(x^3 - 3x)^{2/3}} = \frac{x^2 - 1}{(x^3 - 3x)^{2/3}}$$

- *Phrase too complicated*
  Fraction not reduced:

$$f'(x) = \frac{3x^2 - 3}{3(x^3 - 3x)^{2/3}}$$

4. Find critical numbers of the first kind:

$$x^2 - 1 = 0, \quad x = \pm 1$$

5. Find critical numbers of the second kind:

$$(x^3 - 3x)^{2/3} = 0, \quad x^3 - 3x = 0$$

$$x(x^2 - 3) = 0, \quad x = 0, \quad x = \pm\sqrt{3}$$

- *Inappropriate passage*
  The wrong technique is used to solve the equation $x^3 - 3x = 0$ and the solution $x = 0$ is missed:

$$x^3 = 3x, \quad \frac{x^3}{x} = \frac{3x}{x}, \quad x^2 = 3, \quad x = \pm\sqrt{3}$$

6. List all critical numbers: $x = 0, \pm 1, \pm\sqrt{3}$

- *No conclusion*
  Critical numbers not identified and thus part of the question left unanswered.

- *Irrelevant passage*
  Step 6 followed by the sign chart for $f'(x)$.

- *Missed theme*
  Critical numbers discussed using the First-Derivative Test and the work ended.

7. Find f'':

$$f''(x) = \frac{2x(x^3 - 3x)^{2/3} - (x^2 - 1)\frac{2}{3}(x^3 - 3x)^{-1/3}(3x^2 - 3)}{(x^3 - 3x)^{4/3}}$$

- *Spelling mistake, incomplete sentence*
  Step 7 in the form

$$2x(x^3 - 3x)^{2/3} - (x^2 - 1)\frac{2}{3}(x^3 - 3x)^{-1/3} \cdot 3x^2 - 3$$

- *Unnecessary passage*
  Step 7 followed by the sign chart for $f''(x)$.

8. Conclude:

   $f''(1) > 0$, therefore local minimum at $(1, -\sqrt[3]{2})$

   $f''(-1) < 0$, therefore local maximum at $(-1, \sqrt[3]{2})$

- *No conclusion*
  Only $f''(1) > 0$ and $f''(-1) < 0$ written.

9. State that the test does not apply to critical numbers 0 and $\pm\sqrt{3}$.

- *Incomplete conclusion*
  Step 9 omitted.

- *Inappropriate passage*
  Critical numbers 0 and $\pm\sqrt{3}$ substituted for x in $f''(x)$.

How would an English essay look like with all the mistakes listed above: incomplete sentences, over-complicated phrases, spelling mistakes, inappropriate or irrelevant/unnecessary passages, missed theme, incomplete conclusion, or no conclusion at all? Well, let's see – here's, not an essay, but a short passage from J.R.R. Tolkien's *The Return of The King*:

The door was open, the passage dark save for the glimmer of the torch and the red glare from outside filtering through the window-slit. But here the stair stopped and climbed no further. Sam crept into the passage. On either side there was a low door; both were closed and locked. There was no sound at all.

And here's how this passage might look like with all those mistakes:

Was open, the passage dark save for the glimmer of the torch and the red glare from out side filtering thru the window-slit. I don't like to write. But hear the stair stopped and climbed no further and did not go up any longer. Crept into the passage. On either side there was a low door; both were doors. Stir slowly until it starts boiling. Their was no at all.

As you study math, also learn how to write it. You can only benefit from this!