4 Inequalities

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4.1 Graphing Inequalities

If in an equation we replace equal sign with one of the four inequality symbols, $>$, $\geq$, $<$, or $\leq$, we get an inequality. Inequalities containing $>$ or $<$ are strict. Equality is not permitted in a strict inequality. Thus $5 > 5$ or $3 < 3$ are false statements. On the other hand, $5 \geq 5$ and $3 \leq 3$ are true statements because equality is included in $\geq$ and $\leq$, the inequalities which are not strict. In other words, $x \geq 2$ means that $x > 2$ or $x = 2$ and $x \leq 2$ means that $x < 2$ or $x = 2$.

Inequalities can be represented graphically. When solving inequalities, it is usually required to graph the solution on the real-number line. Figures 1–4 show the graphs of four types of inequalities which are solved for the unknown variable $x$.

![Figure 1. The graph of $x \geq 2$.](image-url)
In each figure, the solution is indicated by the line raised above the real-number line. This is more practical than making the relevant portion of the real-number line thicker. As a reminder, blank circles show that the point (2 in all examples) is not part of the graph and the solution (i.e. \( x \neq 2 \)), whereas filled-in circles indicate that the point is part of the graph and the solution (\( x \) may equal 2). Of course, values greater than the given number are to the right of the number on the real-number line, and those that are less are to the left of it.

Sometimes, when an inequality is solved, the answer may look like \( 2 > x \). A typical mistake happens when this is graphed like in Figure 2. The symbol for “greater” is observed and misinterpreted as the graph going to the right from 2. In order to avoid such mistakes, it is useful to be able to read inequalities not only from left to right, but also from right to left. It can help here to realize that the greater quantities are always on the opening side of the inequality symbol and the smaller quantities are on the other side. Thus,

\[
\begin{align*}
greater \text{ quantity} &> \ smaller \text{ quantity}, \\
greater \text{ quantity} &\geq \ smaller \text{ quantity}, \\
smaller \text{ quantity} &< \ greater \text{ quantity}, \\
smaller \text{ quantity} &\leq \ greater \text{ quantity}.
\end{align*}
\]
4.1 Graphing Inequalities

This is why $2 > x$ means that $x$ is less than 2 and the inequality should be graphed as in Figure 4. Keep in mind that what we graph is the values of $x$.

There may be more then one inequality to graph at the same time. Sometimes, when solving inequalities, the solution is expressed in terms of two inequality symbols. This can happen when the inequality is compound, i.e. when the problem consists of a chain of two inequalities, like in the following example:

$$5 \leq 3x - 4 < 7.$$  

The meaning of this compound inequality is that $5 \leq 3x - 4$ and $3x - 4 < 7$ at the same time. Thus, $3 \leq x$ and at the same time $x < \frac{11}{3}$, so that we can write the solution as a compound inequality itself: $3 \leq x < \frac{11}{3}$. This is graphed below.

![Figure 5. The graph of $3 \leq x < \frac{11}{3}$.](image)

The above problem can be generalized as

$$\ell < ax + b < r,$$

where $\ell$, $a$, $b$, and $r$ are some constants and either or both inequality symbols $<$ may be replaced with $\leq$. Note that the compound inequality makes no sense (that is, it has no solution) if $\ell > r$, or even if $\ell = r$ and at least one of the two inequalities is strict. Also, when a compound inequality is written (either as a problem or the solution), both inequalities have to open the same way. Therefore, the inequality

$$4 \geq x > -7$$

is another example of how a compound inequality may look like. This inequality, by the way, means the same as $-7 < x \leq 4$.

**Example 1.** Determine which of the following compound inequalities are written incorrectly and which have no solution.
(a) \( 0 < 7x + 1 < \sqrt{2} \)
(b) \(-2 > x \leq 5\)
(c) \( \frac{4}{13} \geq 3 - x > 4 \)
(d) \( 3 \leq 2x - 1 \geq 8 \)
(e) \( \pi < x < 3 \)

Solution. The only compound inequality which is written correctly and has a solution is (a). Inequalities (b) and (d) are incorrect because the two inequality symbols in each of them open in opposite directions. This is not the case in (c) and (e), but those inequalities have no solution. Inequality (c) implies that \( \frac{4}{13} > 4 \) and (e) that \( \pi < 3 \); therefore, both are meaningless.

When solving some more complicated inequalities, we may get solutions that consist of two inequalities which cannot be written together as a compound inequality. Such a solution may look like

\[ x < 1 \quad \text{or} \quad x \geq 2. \]

In this case we have to graph the inequalities separately but on the same real-number line.

EXERCISES 4.1

1. Graph each inequality.
   (a) \( 3 \leq x \)
   (b) \( \pi > x \)
   (c) \( x < -\frac{2}{5} \)
   (d) \( x \geq \sqrt{2} \)
   (e) \( -5 > x \)

2. Graph each compound inequality.
   (a) \( 3 > x \geq -1 \)
   (b) \( \pi < x < 7 \)
4.2 Interval Notation

(c) \(0 \geq x \geq -4\)
(d) \(\sqrt{5} < x \leq 5\)

3. Determine which of the following compound inequalities are written incorrectly and which are meaningless.

(a) \(-9 \geq 2 + 4x < 1\)
(b) \(5 \geq x \geq -5\)
(c) \(1 > 6x + 4 \geq 2\)
(d) \(-3 < 2 - 5x \leq -8\)
(e) \(e \geq x \leq 3\)

4. Graph.

(a) \(x > 7\) or \(x \leq -2\)
(b) \(x \leq 0\) or \(x \geq 5\)
(c) \(x < 9\) or \(x \geq 27\)

4.2 Interval Notation

Intervals are also used to represent solutions of inequalities. When solving inequalities, it is also usual to write the solution in interval notation. Let us consider compound inequalities first. The interval representing the graph in Figure 5 is \([3, \frac{11}{3})\). This is just a shorter notation for the following set:

\[
\left[3, \frac{11}{3}\right) = \left\{x \mid 3 \leq x < \frac{11}{3}\right\}.
\]

The interval \([3, \frac{11}{3})\) therefore denotes all the points on the real-number line which are between 3 and \(\frac{11}{3}\), including 3 but excluding \(\frac{11}{3}\). The points 3 and \(\frac{11}{3}\) are the endpoints of this interval. In general, any two real numbers \(a\) and \(b\) such that \(a < b\) can be used as the endpoints of an interval. Then we can have the following four types of intervals depending on whether an endpoint belongs to the interval or not:

\[(a, b) = \{x \mid a < x < b\},\quad [a, b) = \{x \mid a \leq x < b\},\]
\[(a, b] = \{x \mid a < x \leq b\},\quad [a, b] = \{x \mid a \leq x \leq b\}.\]
The interval \((a, b)\) is called \textit{open} and \([a, b]\) is \textit{closed}. Because of the meaning of interval notation and compound inequalities, the left endpoint of any interval should be less than its right endpoint (technically, we could have \([2, 2]\), but this interval only consists of one element, number \(2\)). It is therefore meaningless to write intervals like \((0, -2)\), \([-6, -7]\), \((3, 1]\), or \([4, 4)\) – they are all empty sets.

\textbf{Symbols \(\infty\) and \(-\infty\) may have to be used for endpoints.} Intervals corresponding to the inequalities in Figures 1–4 in Section 4.1 require the use of two special symbols, \(\infty\) (infinity) and \(-\infty\) (negative infinity). We can say that any real number \(x\) is less than infinity and greater than negative infinity:

\begin{align*}
(1) \quad -\infty < x < \infty & \text{ for any real number } x.
\end{align*}

Therefore, \(-\infty\) and \(\infty\) can be used as interval endpoints (we can call them \textit{infinite endpoints}) even though they are not real numbers, only some convenient symbols. Since no real number can equal \(\infty\) or \(-\infty\), only parentheses can be used next to \(\infty\) or \(-\infty\) in interval notation. Intervals like \([2, \infty]\) or \([-\infty, 10)\) are written incorrectly. The above statement (1) means that \(\mathbb{R} = (-\infty, \infty)\). When writing the interval for \(x \geq 2\), we can graph the inequality first and then realize that \(x\) takes values from \(x = 2\) “to infinity”. Then the corresponding interval is \([2, \infty)\). Alternatively, we can conclude that because of (1) the inequality \(2 \leq x\) automatically means \(2 \leq x < \infty\) and the interval for this is \([2, \infty)\).

\textbf{Example 1.} Determine which of the following intervals are written incorrectly and why.

\begin{itemize}
  \item[(a)] \([-3, \infty]\)
  \item[(b)] \((\infty, 3]\)
  \item[(c)] \((0, \frac{1}{2})\)
  \item[(d)] \([\sqrt{2}, \infty)\)
  \item[(e)] \((4, -1)\)
\end{itemize}

\textit{Solution.} Interval notation is used correctly in (c) and (d). It is incorrect in (a) because a bracket cannot be written by \(\infty\), whereas in (b) and (e), the
4.2 Interval Notation

left endpoint is not less than the right one.

Example 2. Convert each interval into set notation.

(a) \((-\sqrt{11}, 7]\) 
(b) [25, \(\infty\)) 
(c) \((-9, -1)\)

Solution.

(a) \((-\sqrt{11}, 7]\) = \(\{x \mid -\sqrt{11} < x \leq 7\}\)
(b) [25, \(\infty\)) = \(\{x \mid 25 \leq x\}\) or \(\{x \mid x \geq 25\}\). Note that it is not necessary to write \(x < \infty\) since this is always satisfied because of (1).
(c) \((-9, -1)\) = \(\{x \mid -9 < x < -1\}\)

Intervals are sets, not statements. We should realize that a compound inequality like \(-3 \leq x < 5\) is a statement about the variable \(x\), which is not the same as \([-3, 5)\), since this indicates a set of numbers. The statement corresponding to \(3 \leq x < 5\) is \(x \in [-3, 5)\). We say that these two statements are equivalent, i.e. they mean one and the same thing.

Example 3. Write a statement equivalent to the given one.

(a) \(3 \geq x > -4\)
(b) \(x \in [7, \infty)\)
(c) \(x < 6\)
(d) \(x \in [0, 2\pi)\)

Solution.

(a) \(x \in (-4, 3]\)
(b) \(x \geq 7\)
(c) \(x \in (-\infty, 6)\)
(d) \(0 \leq x < 2\pi\)
Intervals corresponding to compound inequalities can be represented using intersection of intervals with one infinite endpoint. As we have seen, a compound inequality like the above-mentioned \(-3 \leq x < 5\) means the following:

(2) \(-3 \leq x \) and \(x < 5\).

Of course, \(-3 \leq x < 5\) means the same as \(x \in [-3, 5)\), but each of the inequalities in (2) also has its corresponding interval, \([-3, \infty)\) and \((-\infty, 5)\) respectively. Note that \([-3, 5)\) is nothing else but the intersection of \([-3, \infty)\) and \((-\infty, 5)\),

\([-3, \infty) \cap (-\infty, 5) = [-3, 5)\).

We have already mentioned inequalities that cannot be combined into a compound inequality. An earlier example is repeated below,

(3) \(x < 1\) or \(x \geq 2\).

The interval notation for (3) makes use of the union of the two intervals corresponding to \(x < 1\) and \(x \geq 2\),

\(x \in (-\infty, 1) \cup [2, \infty)\).

Notice the importance of the words “and” and “or” in (2) and (3), indicating how to combine the two inequalities.

**Example 4.** Write the given inequalities using either intersection or union of the corresponding intervals.

(a) \(x > 7\) or \(x \leq -2\)

(b) \(x \leq 0\) and \(x \geq -5\)

(c) \(x > 6\) and \(x < 10\)

(d) \(x < 9\) or \(x \geq 27\)

**Solution.**

(a) \(x \in (7, \infty) \cup (-\infty, -2]\)
4.2 Interval Notation

(b) \( x \in (-\infty, 0] \cap [-5, \infty) \)
(c) \( x \in (6, \infty) \cap (-\infty, 10) \)
(d) \( x \in (-\infty, 9) \cup [27, \infty) \)

Example 5. Find the interval representing each intersection.

(a) \((-\infty, 8) \cap (-4, \infty)\)
(b) \((-\infty, 7) \cap [6, \infty)\)
(c) \([-3, \infty) \cap (-\infty, 0]\)

Solution.

(a) \((-4, 8]\)
(b) \([6, 7)\)
(c) \([-3, 0]\)

EXERCISES 4.2

1. Determine which of the following intervals are written incorrectly and why.

(a) \((-7, -3]\)
(b) \((\infty, 3)\)
(c) \([0, \infty]\)
(d) \((-\infty, -\sqrt{3})\)
(e) \((4, 0]\)
(f) \([-3, -5]\)

2. Convert each interval into set notation.

(a) \([\pi, \infty)\)
(b) \((5, 105]\)
(c) \((-\infty, 8)\)
3. Write a statement equivalent to the given one.
   (a) \(5 \geq x\)
   (b) \(x \in (-\infty, \sqrt{7})\)
   (c) \(-6 \leq x < 11\)
   (d) \(x \in [9, 10]\)

4. Write the given inequalities using either intersection or union of the corresponding intervals.
   (a) \(x < 7\) and \(x \geq -2\)
   (b) \(x \leq 0\) or \(x \geq 5\)
   (c) \(x \geq 1\) and \(x \leq 6\)
   (d) \(x < 0\) or \(x > 10\)

5. Find the interval representing each intersection.
   (a) \((-\infty, 0) \cap (-4, \infty)\)
   (b) \((7, \infty) \cap (-\infty, 8]\)
   (c) \((-\infty, 3] \cap [-3, \infty)\)

4.3 Solving Inequalities

Like equations, inequalities are divided into linear and non-linear based on the same principles.

Example 1. Determine for each inequality whether it is linear or non-linear.
   (a) \(\frac{2x - 7}{3} + 1 \geq 2x - 6\)
   (b) \(x^2 < 3x + 4\)
   (c) \(\frac{4x + 3}{2x - 1} > 5 - \frac{8}{x}\)
   (d) \(\sqrt{x} \leq 4\)
   (e) \(5x + 6 > \sqrt{3} - 2x\)
4.3 Solving Inequalities

Solution. Inequalities (a) and (e) are linear. Inequality (b) is non-linear since the variable is squared. This is a polynomial inequality. Inequality (c) is non-linear because of the variable in the denominator. This is a rational inequality. Inequality (d) is non-linear because its variable is under a radical.

From now on, we shall only consider polynomial and rational inequalities. Polynomial inequalities include the linear ones, which are polynomial inequalities of degree 1. Linear inequalities are solved using the same steps as for linear equations. The only thing that is different and should be paid attention to is that the inequality symbol should be switched when the inequality is multiplied or divided by a negative number. For instance,

(1) \[-2x > 5 \iff x < -\frac{5}{2},\]

or

(2) \[-\frac{x}{4} \leq -3 \iff x \geq 12.\]

The reason for this can be illustrated by considering any two numbers, say, 2 and 5. Whereas 2 < 5, when we multiply both numbers by −1 and get −2 and −5, then −2 > −5. Forgetting to change the inequality symbol in such situations is a typical mistake when solving inequalities. Of course, if an inequality is multiplied or divided by a positive number, the inequality symbol remains unchanged.

Notice in (1) and (2) that when an inequality is transformed during the solution process, in each step we get a new inequality which is not equal but equivalent to the previous one. Recall from Chapter 3 that this also is the case with equations.

Example 2. Determine for each pair of inequalities whether they are equivalent or not.

(a) \(-6 \geq 2x\) and \(-3 \leq x\)

(b) \(-4x > 1\) and \(x < -\frac{1}{4}\)

(c) \(-\frac{2}{3}x > 5\) and \(x > -\frac{15}{2}\)
(d) $5x \leq -10$ and $x \leq -2$

(e) $-3x \geq 6$ and $x < -2$

(f) $-\frac{x}{8} > -1$ and $x < 8$

**Solution.** Pairs of equivalent inequalities are given in (b), (d), and (f). In (a), $-6 \geq 2x$ is divided by 2, which is a positive number, so this inequality is equivalent to $-3 \geq x$. In (c) and (e), the second inequality symbol should be $<$ and $\leq$ respectively.

Non-linear inequalities have to be solved using a sign chart. Although some other algebraic approaches are also possible, this is the most practical one. Before the sign chart can be constructed, the non-linear inequality has to be transformed appropriately: there should be 0 on one side and the other side should be factored completely; if the inequality is rational, its non-zero side should be written as a single fraction whose numerator and denominator should both be factored completely. This is illustrated by the following polynomial inequality:

(3)  
\[
x^2 > x \\
x^2 - x > 0 \\
x(x - 1) > 0
\]

Here is an example of the preparatory steps for a rational inequality:

(4)  
\[
x + \frac{6}{x} \leq 5 \\
x - 5 + \frac{6}{x} \leq 0 \\
\frac{(x - 5)x}{x} + \frac{6}{x} \leq 0 \\
x^2 - 5x + 6 \leq 0 \\
\frac{(x - 2)(x - 3)}{x} \leq 0
\]

**Example 3.** Determine for each inequality whether it is prepared for the sign chart or not.

(a) $x(2x - 1)(x + 3) \leq 1$

(b) $\frac{x}{(x - 2)(x + 4)} < 0$
4.3 Solving Inequalities

(c) \( 1 - \frac{x(x + 5)}{x - 3} > 0 \)

(d) \( 0 \geq (2x + 7)(x^2 + 1) \)

(e) \( x^4 \geq x^3 \)

Solution. Inequalities (a) and (e) do not have 0 on one side and the left side of (c) is not in the form of a fraction. These inequalities are not prepared for the sign chart. Inequalities (b) and (d) are in the right form.

It is not always possible to transform an inequality so that a sign chart can be made. When this is the case, graphing methods are the only remaining ones. They are not discussed here because, generally speaking, they give approximate solutions.

The sign chart is made for the expression on the non-zero side of the inequality, not for the inequality itself. This means that the same sign chart should be used when solving (3) or any of the following inequalities:

\[ x^2 \geq x, \quad x^2 < x, \quad \text{or} \quad x^2 \leq x. \]

Like (3), the three inequalities above can be transformed so that they have the expression \( x(x - 1) \) on the left side. The sign chart is then constructed for this expression. It tells us on what intervals the expression is positive, negative, or equal to 0. If we are solving (3), for instance, then the intervals where \( x(x - 1) \) is positive form the solution. This happens to be \((−∞, 0) \cup (1, ∞)\). On the other hand, suppose we have to solve \( x^2 \leq x \), i.e. \( x(x - 1) \leq 0 \). Then the intervals where \( x(x - 1) \) is negative or equal to 0 represent the solution. In this case, the solution is \([0, 1]\). This illustrates the general situation with polynomial inequalities of order greater than 1. On the non-zero side of the inequality, we get a completely factored polynomial. For different values of the real variable, the polynomial can be positive, negative, or equal to 0. The sign chart shows where each of these three possibilities takes place on the real-number line. We then just select the intervals that correspond to the inequality we are solving. The numbers used as endpoints of these intervals are those that make the polynomial equal to 0. Such a number is included in the solution (i.e. the interval is closed at that point) if the inequality involves \( \leq \) or \( \geq \). If the inequality is strict (that is, with \( > \) or \( < \)), the number is not part of the solution (the interval is open at that point).

Rational inequalities are somewhat more complicated than the polynomial
ones. In a rational inequality, the non-zero side is expressed as an algebraic fraction with completely factored numerator and denominator. However, there are four, not three, possibilities for any algebraic fraction: it may be positive, negative, equal to 0, or undefined (for the values of the variable that make the denominator equal to 0). The sign chart then shows where these four possibilities occur. The numbers that make the algebraic fraction undefined can never be included in the solution, i.e. the solution intervals are always open at such endpoints.

Example 4. Each given number is an endpoint of a solution interval of the given inequality. Should it be included in the solution or not?

(a) $x = -3$ in $(2x - 1)(x + 3) < 0$

(b) $x = 2$ in $\frac{x}{(x-2)(x+4)} \geq 0$

(c) $x = 0$ in $\frac{x(x + 5)}{x - 3} \leq 0$

(d) $x = 1$ in $0 \geq (2x + 7)(x - 1)$

Solution. The answer is ‘yes’ in (c) and (d). In (a), $x = -3$ cannot be included since the inequality is strict. In (b), $x = 2$ makes the fraction undefined.

Is it possible to avoid using a sign chart? Yes, but this may be more complicated. Consider inequalities (3) and (4) again. Can we divide (3) by $x$ and simplify it to $x > 1$? As we have mentioned above, the solution of this inequality is not just $x > 1$ but also $x < 0$. What is then wrong with the suggested simple approach? The point is that $x$ stands for any real number, so it may be negative. Therefore, $x^2 > x$ is equivalent to $x > 1$ if and only if $x > 0$. If $x < 0$, $x^2 > x$ is equivalent to $x < 1$ since we divided the original inequality by a negative number! These two cases should now be discussed: 1) $x > 1$ and $x > 0$ should hold true at the same time – this gives $x > 1$, and 2) $x < 0$ and $x < 1$ should be true at the same time, resulting in $x < 0$. All these details are covered by the sign-chart approach and we do not have to explore all the logical possibilities which make the preceding discussion dangerously complicated.

Similarly, fractions with variable denominators cannot, in general, be cleared from inequalities. If we want to multiply (4) by $x$, we have to keep in mind
that $x$ may be negative and then we have to switch the inequality symbol. This makes (4) equivalent to

\[
\begin{cases}
  x^2 + 6 \leq 5x & \text{if } x > 0, \\
  x^2 + 6 \geq 5x & \text{if } x < 0.
\end{cases}
\]

It is much more complicated to solve (5) than to use a sign chart.

A frequent mistake is to take all the preparatory steps for a sign chart, but to “solve” the nonlinear inequality without it. In problem (3), this would go as follows:

\[
x^2 > x \\
x^2 - x > 0 \\
x(x - 1) > 0 \\
x > 0 \text{ and } x - 1 > 0 \quad \text{WRONG!} \\
x > 0 \text{ and } x > 1 \quad \text{WRONG!} \\
x > 1 \quad \text{WRONG!}
\]

We can see again that this gives another incomplete “solution” since the part $x < 0$ is missing. This happened because not all logical possibilities were analyzed. It holds true that $x(x - 1) > 0$ not only when $x > 0$ and $x - 1 > 0$, but also when $x < 0$ and $x - 1 < 0$. Since it is easier to construct a sign chart than to discuss all such logical possibilities, the sign-chart approach is strongly recommended.

**Example 5.** Determine for each stated equivalence whether it is true or not.

(a) \((2x - 1)(x + 3) \leq 0 \iff 2x - 1 \leq 0 \text{ and } x + 3 \leq 0\)

(b) \(\frac{2x - 3}{x + 5} \geq 1 \iff 2x - 3 \geq x + 5\)

(c) \(x^3 \geq x^2 \iff x \geq 1\)

(d) \(\frac{2x + 3}{x^2 + 1} < 0 \iff 2x + 3 < 0\).

**Solution.** Only (d) is true. This illustrates a special case when a fraction can be cleared from a rational inequality. In this example, the denominator $x^2 + 1$ is always positive and both sides of the inequality can be multiplied by $x^2 + 1$ without the need to discuss any additional possibilities. Inequality (c) is almost true, but the solution $x = 0$ is still missing. When both sides of $x^3 \geq x^2$ are divided by $x^2$, $x$ cannot equal 0, so it has been verified separately
that \( x = 0 \) is a solution. The equivalence in (b) is true if \( x + 5 > 0 \), but there is another possibility: \( \frac{2x - 3}{x + 5} \) if \( 2x - 3 \leq x + 5 \) and \( x + 5 < 0 \). Finally, if we could compare incorrect statements, (a) would be even worse than (b) or (c). If \( 2x - 1 \leq 0 \) and \( x + 3 \leq 0 \), then \( (2x - 1)(x + 3) \geq 0 \), which is not the original inequality. Note that none of these “equivalences” result when the sign chart is used.

\[ \Box \]

**Inequalities with the symbol \( \neq \) are solved like equations.** For instance, when solving the inequality \( x^2 \neq x \), we can solve the equation \( x^2 = x \). Its solutions are \( x = 0 \) and \( x = 1 \). Then the solution of the inequality contains all real numbers \( x \neq 0,1 \). In interval notation, this solution is \((-\infty, 0) \cup (0,1) \cup (1,\infty)\).

**EXERCISES 4.3**

1. Determine for each inequality whether it is linear or non-linear.
   
   (a) \( 3x - 7(x + 1) > \sqrt{2x + 4} \)
   
   (b) \( \frac{1}{2}x - 9 \leq 5x + \frac{3}{4} \)
   
   (c) \( x^3 - 2x + 6 < 0 \)
   
   (d) \( 5 - 2(x + 6) > 3(x - \sqrt{7}) \)
   
   (e) \( \frac{2x - 7}{3x} - 8 \geq 0 \)

2. Determine for each pair of inequalities whether they are equivalent or not.
   
   (a) \( 4x < -12 \) and \( x < -3 \)
   
   (b) \( -\frac{1}{2}x \geq 1 \) and \( x \leq -2 \)
   
   (c) \( -\frac{x}{5} \geq 2 \) and \( x > -10 \)
   
   (d) \( -6x > -9 \) and \( x < \frac{3}{2} \)
   
   (e) \( \frac{2x}{7} \geq 3 \) and \( x \leq \frac{21}{2} \)
   
   (f) \( \frac{3}{4} \leq -x \) and \( -\frac{3}{4} > x \)

3. Determine for each inequality whether it is prepared for the sign chart or not.
4.3 Solving Inequalities

(a) $(2x - 5)(3 - x) \geq 0$

(b) $\frac{(3x + 1)(x - 1)}{x(x + 1)} < 1$

(c) $\frac{x^2 - 5}{x} \leq 0$

(d) $x^2 > 2x - 1$

(e) $0 < 4 + (x + 3)(x - 2)$

4. Each given number is an endpoint of a solution interval of the given inequality. Should it be included in the solution or not?

(a) $x = 1$ in $(x - 1)(8x + 3) \geq 0$

(b) $x = -2$ in $\frac{x + 2}{(x - 7)(x - 2)} \leq 0$

(c) $x = 3$ in $\frac{x(x + 5)}{x - 3} \leq 0$

(d) $x = -\frac{7}{2}$ in $0 < (2x + 7)(x - 1)$

5. Determine for each stated equivalence whether it is true or not.

(a) $(x - 4)(x + 2) \geq 0 \iff x - 4 \geq 0$ and $x + 2 \geq 0$

(b) $\frac{3x - 1}{x + 2} > 0 \iff 3x - 1 > 0$ and $x + 2 > 0$

(c) $x^2 < x^4 \iff 1 < x^2$

(d) $(7x - 3)(x^2 + 3) < 0 \iff 7x - 3 < 0$. 