Student Name________________________________

Good approach: **Lean basic fact and rules** & **Write well** & **Connect current topic to the previous ones.**

Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31...

1. The number is divisible by 2 when the last digit is even number. Example: 10 10 / 2 = 5
2. The number is divisible by 3 when **the sum of digits is divisible by 3.**

   Is 11006 is divisible by 3? Why? No. $1+1+0+0+6=8$ 8 is not ÷ by 3

   Is 87 divisible by 3? **The sum of digits** is $8+7=15$ so 87 is divisible by 3.

Long divide: 87 by 3

29
3 ⟌ 87
−6
27
−27
0 $87 = 3 * 29$

Simplify: $\frac{87}{27} = \frac{3*29}{3\cdot 9} = \frac{29}{9}$

Is 1521 divisible by 3? If yes **long divide.**

$1+5+2+1=9$ yes

507
3 ⟌ 1521
−15
02
−0
21
−21
0

Is 1251 divisible by 3? If yes **long divide.**

$1+2+5+1=9$ yes

417
3 ⟌ 1251
−12
05
−3
21
−21
0

3. The number is divisible by 5 when the last digit is 0 or 5. Write a 3-digit number divisible by 5. 125

   25
   5 ⟌ 125
   −10
   25
   −25
   0

4. To check if the number is divisible by other prime numbers we must long divide.

Multiplication Table

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Multiplication game (do it **under 100s**): http://besttimestable.com/#game

Add 45 and 272

```
   45
+ 272
  317
```

Add 0.45 and 27.2

```
  0.45
+ 27.20
  27.65
```

Align decimal points.

Multiply 45 and 272

```
    45
 x 272
    90
   315
+  90
 12240
```

Multiply 0.45 and 27.2

```
   0.45
 x 27.2
     90
    315
+   90
 12.240
```

Add decimal places.
In this case: 2+1= 3

The divisibility by 2, 3, 5... is used to simplify fractions.

Simplify $\frac{9\div3}{15\div3}=\frac{3}{5}$

Simplify $\frac{20\div10}{30\div10}=\frac{2}{3}$ or $\frac{20\div5}{30\div5}=\frac{4\div2}{6\div2}=\frac{2}{3}$

Simplify $\frac{8\div8}{24\div8}=\frac{1}{3}$ or $\frac{8\div2}{24\div2}=\frac{4\div4}{12\div4}=\frac{1}{3}$

Simplify $\frac{24\div4}{28\div4}=\frac{6}{7}$ or $\frac{24\div2}{28\div2}=\frac{12\div2}{14\div2}=\frac{6}{7}$

Multiplying fractions. Example: $\frac{4}{15}\left(\frac{5}{2}\right)=\frac{4*5}{15*2}$ **simplify across the fraction bar before multiplying!**

Simplify $\frac{\cancel{9}^{3}}{5}\left(\frac{1}{\cancel{3}_{1}}\right)=\frac{3}{5}$

Simplify $\frac{\cancel{4}^{2}}{\cancel{15}_{3}}\left(\frac{\cancel{5}^{1}}{\cancel{2}_{1}}\right)=\frac{2}{3}$

Simplify $\frac{\cancel{9}^{3}}{\cancel{5}}\left(\frac{\cancel{4}^{2}}{15}\right)\left(\frac{1}{\cancel{3}_{1}}\right)\left(\frac{\cancel{5}}{\cancel{2}_{1}}\right)=\frac{6\div3}{15\div3}=\frac{2}{5}$

Always check if more can be simplified.

Simplify $\cancel{6}^{3}\left(\frac{1}{\cancel{2}_{1}}\right)=3$

We can view $6\left(\frac{1}{2}\right)$ as $\frac{\cancel{6}^{3}}{1}\left(\frac{1}{\cancel{2}}\right)=3$

Divide fractions: multiply first by the reciprocal of second: $\frac{2}{5}\div\frac{2}{3}=\frac{2}{5}\bullet\frac{3}{2}$

$\frac{\frac{2}{5}}{\frac{2}{3}}=\frac{2}{5}\div\frac{2}{3}=\frac{\cancel{2}}{5}\cdot\frac{3}{\cancel{2}}=\frac{3}{5}$

$\frac{\frac{3}{20}}{\frac{1}{30}}=\frac{3}{20}\div\frac{1}{30}=\frac{3}{20}\cdot\frac{30}{1}=\frac{9}{2}$

$\frac{2}{\frac{2}{5}}=2\div\frac{2}{5}=\frac{\cancel{2}}{1}\cdot\frac{5}{\cancel{2}}=\frac{5}{1}=5$

$\frac{\frac{3}{20}}{30}=\frac{3}{20}\div30=\frac{3}{20}\div\frac{30}{1}=\frac{\cancel{3}^{1}}{20}\cdot\frac{1}{\cancel{30}_{10}}=\frac{1}{200}$

- To proceed well, at this point you must know: **multiplication table**
  long division and multiplication
  prime numbers and divisibility by 2, 3, and 5
  simplifying, multiplying, and dividing fractions
  remember that $2=\frac{2}{1}=\frac{4}{2}=\frac{6}{3}\ldots$

Prime factorization (represent the number as a product of prime numbers).

Factor 9.	Factor 15.	Factor 20.	Factor 30.
3 3	3 5	4 5 2 2	6 5 2 3
$9 = 3 \times 3$	$15 = 3 \times 5$	$20 = 2 \times 2 \times 5$	$30 = 2 \times 3 \times 5$

Greatest common factor (GCF) of two (or more) numbers is a product of all common prime factors.

GCF(9, 15) = 3

$9 = 3 \times 3$
$15 = 3 \times 5$

GCF(20, 30) = 2*5 = 10

$20 = 2 \times 2 \times 5$
$30 = 2 \times 3 \times 5$
$2 \times 5 = 10$

Add fractions : we must first bring fractions to same units by using **Least Common Multiple (LCM)**

Find LCM for 20 and 30.

Write increments of the largest number and verify if smaller number(s) are divisible by it:

Numbers: 20

30, 60

The smallest number divisible by both 20 and 30 is 60.

$\mathbf{LCM}(\mathbf{20}, \mathbf{30}) = 60$

Find LCM for 3 and 5.

Numbers: 3

5, 10, 15

The smallest number divisible by both 3 and 5 is 15.

$\mathbf{LCM}(\mathbf{3}, \mathbf{5}) = 15$

Rewrite $\frac{1}{30}$ with the denominator of 60.

$$\frac{1 \times 2}{30 \times 2} = \frac{2}{60}$$

Rewrite $\frac{2}{5}$ with the denominator of 15.

$$\frac{2 \times 3}{5 \times 3} = \frac{6}{15}$$

Rewrite $\frac{3}{20}$ with the denominator of 60.

$$\frac{3 \times 3}{20 \times 3} = \frac{9}{60}$$

Rewrite $\frac{2}{3}$ with the denominator of 15.

$$\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

Simplify $\frac{1 \times 2}{30 \times 2} + \frac{3 \times 3}{20 \times 3} = \frac{2}{60} + \frac{9}{60} = \frac{11}{60}$

Simplify $\frac{2 \times 5}{3 \times 5} + \frac{2 \times 3}{5 \times 3} = \frac{10}{15} + \frac{6}{15} = \frac{16}{15}$

Simplify $\frac{2}{5} + \frac{4}{5} = \frac{6}{5}$

Simplify (write an improper fraction) $3 + \frac{2}{5} =$

$$\frac{3 \times 5}{1 \times 5} + \frac{2}{5} = \frac{15}{5} + \frac{2}{5} = \frac{17}{5}$$

- To proceed well, at this point you must know **everything so far AND:** prime factorization
  how to find least common multiplier
  how to add fractions

Number line and opposite numbers.

Opposite number of $\mathbf{1}$ is $-\mathbf{1}$ (add minus in front)

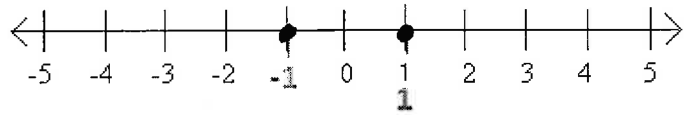
Opposite number of $-\mathbf{1}$ is $\mathbf{1}$

BUT

Opposite number of $-\mathbf{1}$ is also $-(-\mathbf{1})$

so we can write: $-(-\mathbf{1}) = \mathbf{1}$

opposite of 1 is -1



opposite of -1 is 1 so $-(-1)=1$

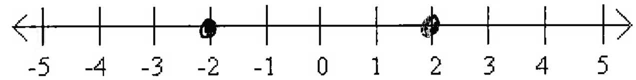
When we write 1 instead of $-(-1)$ we make math writing simpler.

Plot 2 and opposite of 2. Opposite is -2

Plot opposite of -2.

Opposite of -2 is $-(-2)=2$

$-(-2)$

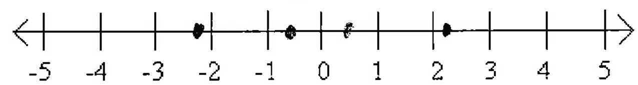


Plot -2.25 and $-\frac{1}{2}$

Plot their opposites: $-(-2.25)$ and $-(-\frac{1}{2})$

2.25 $\frac{1}{2}$

-2.25 $-\frac{1}{2}$ $-(-\frac{1}{2})$ $-(-2.25)$



Simplify: $-(-2.25) = 2.25$

$-\left(-\frac{1}{2}\right) = \frac{1}{2}$

Simplify: $0.63 \underbrace{-(-2.25)}_{+2.25} = 0.63 + 2.25$
$= 2.88$

$\frac{3}{5} \underbrace{-\left(-\frac{1}{2}\right)}_{+\frac{1}{2}} = \frac{3}{5} + \frac{1}{2} = \frac{6}{10} + \frac{5}{10} = \frac{11}{10}$

Work:

$$\begin{array}{r} 0.63 \\ +\,2.25 \\ \hline 2.88 \end{array}$$

Simplify: $-(-375) = 375$

$-(-0.043) = 0.043$

Simplify: $101.25 \underbrace{-(-375)}_{+375} = 476.25$

$1.06 \underbrace{-(-0.043)}_{+0.043} = 1.103$

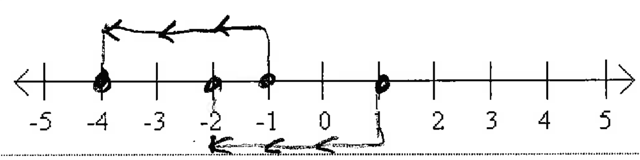
Work:

$$\begin{array}{r} 101.25 \\ +375 \\ \hline 476.25 \end{array} \qquad \begin{array}{r} 1.06 \\ +\,0.043 \\ \hline 1.103 \end{array}$$

Adding positive and negative numbers.

$-1 + (-3) = -1 - 3 = -4$

$1 + (-3) = 1 - 3 = -2$



Simplify: $-1.73 - 3.55 = -5.28$

$$\begin{array}{r} -1.73 \\ -3.55 \\ \hline -5.28 \end{array}$$

Simplify: $-1.07 + 2.25 = 1.18$

$$\begin{array}{r} 2.25 \\ -1.07 \\ \hline 1.18 \end{array}$$

Simplify $-\frac{1\times 2}{3\times 2} - \frac{3\times 3}{2\times 3} = -\frac{2}{6} - \frac{9}{6} = -\frac{11}{6}$

Simplify $-\frac{2\times 5}{3\times 5} + \frac{2\times 3}{5\times 3} = -\frac{10}{15} + \frac{6}{15} = -\frac{4}{15}$

Simplify $-\frac{2}{5} - \frac{4}{5} = -\frac{6}{5}$

Simplify $-4 + \frac{2}{5} = -3\frac{5}{5} + \frac{2}{5} = -3\frac{3}{5}$

Multiplication of positive and negative numbers $(+)(+)=(+)$

$(+)(-)=(-)(+)=(-)$

$(-)(-)=(+)$ because $-(-1)=(-1)(-1)=1$

$5(2)=10$	$-5(2)=(-1)(5)(2)=(-1)(10)=-10$	$-5(-2)=(-1)(5)(-1)(2)=(-1)(-1)(5)(2)=10$

Determine the sign of the product and compute.

$\overbrace{(+)(-)}^{(-)}(+)=(-)(+)=(-)$ $(-)(+)(+)=(-)(+)=(-)$ $(+)(+)(-)=(+)(-)=(-)$	$\overbrace{(-)(+)}^{(-)}(-)=(-)(-)=(+)$ $(+)(-)(-)=(-)(-)=(+)$ $(-)(-)(+)=(+)(+)=(+)$	$\overbrace{(-)(-)}^{(+)}(-)=(+)(-)=(-)$ $(+)(-)(-)(+)=(-)(-)(+)=(+)(+)=(+)$ $(-)(-)(+)(-)=(+)(+)(-)=(+)(-)=(-)$
$5(-2)(3)=(-10)(3)=-30$ $-5(2)(3)=(-10)(3)=-30$ $5(2)(-3)=10(-3)=-30$	$-5(2)(-3)=(-10)(-3)=30$ $5(-2)(-3)=(-10)(-3)=30$ $-5(-2)(3)=10(3)=30$	$-5(-2)(-3)=10(-3)=-30$ $5(-2)(-3)(2)=(-10)(-3)(2)=30(2)=60$ $-5(-2)(3)(-2)=(-10)(3)(-2)$ $=(-30)(-2)=-60$
$0.5(-0.2)(3)=(-0.1)(3)=-0.3$ $0.5 \times 0.2 = 0.10 = 0.1$ $\quad 0.1 \times 3 = 0.3$	$\underbrace{-0.5(0.2)}_{-0.1}(-3)=(-0.1)(-3)=0.3$	$\underbrace{-0.5(-0.2)}_{0.1}(-3)=0.1(-3)=-0.3$
$\frac{3}{5}\left(-\frac{1}{2}\right)(3)=\frac{3}{5}\left(-\frac{1}{2}\right)\left(\frac{3}{1}\right)$ $=-\frac{9}{10}$	$-\frac{3}{5}\left(\frac{1}{2}\right)(-3)=\left(-\frac{3}{5}\right)\left(\frac{1}{2}\right)\left(\frac{-3}{1}\right)$ $=\frac{9}{10}$	$-\frac{3}{5}\left(-\frac{1}{2}\right)(-3)=\left(-\frac{3}{5}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{1}\right)=-\frac{9}{10}$

$\frac{-6}{2}=\frac{6}{-2}=-\frac{6}{2}=-3$ **SO** $\mathbf{\frac{-a}{b}=\frac{a}{-b}=-\frac{a}{b}}$

$\frac{3}{5}\left(\frac{1}{-2}\right)(3)=\frac{3}{5}\left(-\frac{1}{2}\right)\left(\frac{3}{1}\right)=-\frac{9}{10}$	$\frac{3}{-5}\left(\frac{1}{2}\right)(-3)=\left(-\frac{3}{5}\right)\left(\frac{1}{2}\right)\left(-\frac{3}{1}\right)$ $=\frac{9}{10}$	$\frac{3}{-5}\left(\frac{-1}{2}\right)(-3)=\left(-\frac{3}{5}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{1}\right)$ $=-\frac{9}{10}$
$\frac{-9}{5}\left(\frac{-1}{-3}\right)=-\frac{\overset{3}{\cancel{9}}}{5}\left(\frac{1}{\underset{1}{\cancel{3}}}\right)=-\frac{3}{5}$	$\frac{4}{-15}\left(\frac{-5}{2}\right)=\left(-\frac{\overset{2}{\cancel{4}}}{\underset{3}{\cancel{15}}}\right)\left(-\frac{\overset{1}{\cancel{5}}}{\cancel{2}}\right)$ $=\frac{2}{3}$	$\frac{-9}{5}\left(\frac{4}{-15}\right)\left(\frac{1}{-3}\right)\left(\frac{5}{-2}\right)$ $\left(-\frac{\overset{3}{\cancel{9}}}{\underset{1}{\cancel{5}}}\right)\left(-\frac{\overset{2}{\cancel{4}}}{\underset{5}{\cancel{15}}}\right)\left(-\frac{1}{\underset{1}{\cancel{3}}}\right)\left(-\frac{\overset{1}{\cancel{5}}}{\underset{1}{\cancel{2}}}\right)=\frac{2}{5}$

Addition versus Multiplication.

$\frac{3}{-5}+\frac{-1}{2}=\frac{-3}{5}+\frac{-1}{2}=\frac{-6}{10}+\frac{-5}{10}=-\frac{11}{10}$	$\frac{3}{-5}\left(\frac{-1}{2}\right)=\left(-\frac{3}{5}\right)\left(-\frac{1}{2}\right)=\frac{3}{10}$
$\frac{-3}{-20}+\left(\frac{1}{-30}\right)=\frac{3}{20}+\left(-\frac{1}{30}\right)=\frac{9}{60}+\frac{-2}{60}=\frac{7}{60}$	$\frac{-3}{-20}\div\left(\frac{1}{-30}\right)=\frac{3}{20}\div\left(-\frac{1}{30}\right)=\frac{3}{20}\left(-\frac{30}{1}\right)=-\frac{9}{2}$

Mixed Numbers and improper fractions.

$3\frac{1}{2}=3+\frac{1}{2}=\frac{6+1}{2}=\frac{7}{2}$	$-3\frac{1}{2}=-\left(3+\frac{1}{2}\right)=-\frac{6+1}{2}=-\frac{7}{2}$

Exponents are a shorthand for the multiplication: $3^2=(3)(3)=9$ $\left(\frac{2}{3}\right)^3=\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)=\frac{8}{27}$

Exponent works only on the number directly under it: $(-3)^2=(-3)(-3)=9$

so the problem below is different from the one above $-3^2=-(3)(3)=-9$

Any number except 0 when raised to 0 is one: $a^0=1$ $\quad when\ a\neq 0$ (Example: $13^0=1$) and 0^0 **is undefined**

■ To proceed well, at this point you must know **everything so far AND:** number line and opposites
addition of positive and negative numbers
multiplication of positive and negative numbers
mixed numbers and exponents

Real numbers and their properties.

Real Numbers

/ \

Rational Numbers

- fractions (¾, -5/1…)

|

Integers (…-2, -1, 0, 1, 2…)

|

Whole numbers {0,1,2,3…}

|

Natural numbers {1,2,3..}

Irrational numbers (π, $\sqrt{2}$, $\sqrt{5}$)

(irrational numbers have infinite number of decimals that do not repeat)

Example: $\pi=3.14159\ldots$

$\sqrt{2}=1.41421356\ldots$

$\sqrt{3}=1.73205\ldots$

Observe that any fraction can be converted to decimal by long division.

Some fractions, such as $\frac{1}{3}$ **result in infinite number of decimals BUT they repeat.**

$$\frac{1}{3}=0.333\ldots=0.\overline{3}$$

As long as decimals repeat the number is rational number because it can be represented as a fraction.

Order of Operations: PEMDAS

P	**parenthesis (), [], {}, | |**
E	**exponents**
MD	*** and / (order is left to right)**
AS	**+ and – (order is left to right)**

Note: Observe that these two problems are the same. Make sure to know how to work with both decimals and fractions.

Compute:

$-13^0 + 36/9(3) = -1 + 4(3) = -1 + 12 = 11$

$-0.5 - 0.3(-0.2) = -0.5 + 0.06 = -0.44$

$\begin{array}{r} 0.3 \\ \times 0.2 \\ \hline 0.06 \end{array}$ $\begin{array}{r} -0.50 \\ +0.06 \\ \hline -0.44 \end{array}$

$-\frac{1}{2} - \underbrace{\frac{3}{10}\left(-\frac{1}{5}\right)}_{+\frac{3}{50}} = -\frac{1}{2} + \frac{3}{50} = \frac{-25}{50} + \frac{3}{50} = -\frac{22}{50}$

$= -\frac{11}{25}$

Absolute Value is a type of parenthesis (the distance from 0 so it is ALWAYS greater than or equal to 0).

$\|2\| = 2$ *also* $\|-2\| = 2$	$\|3.6\| = 3.6$ *aslo* $\|-3.6\| = 3.6$

Commutative property of addition and multiplication:

$\mathbf{a + b = b + a}$ $\mathbf{a * b = b * a}$

Examples: $5 + 7 = 7 + 5 = 12$ $3 * 5 = 5 * 3 = 15$

$(x+1) + 4 = 4 + (x+1) = x + 5$ $(2 * 5) * 3 = 3(2 * 5) = 30$

Associate property of addition and multiplication:

$\mathbf{(a + b) + c = a + (b + c)}$ $\mathbf{(a * b) * c = a * (b * c)}$

Examples: $(2 + 1) + 9 = 2 + (1 + 9) = 12$ $(3 * 2) * 5 = 3(2 * 5) = 30$

$(2 + 1) + (9 + 5) = 2 + (1 + 9) + 5 = 17$

Distribution property: $\mathbf{a(b + c) = ab + ac}$ and $\mathbf{a(b - c) = ab - ac}$

Example showing that this works for numbers: $2(3 + 7) = 2(3) + 2(7) = 6 + 14 = 20$

BUT when there are no variables do parenthesis first: $2(3 + 7) = 2(10) = 20$

The distribution is used when variables are involved: $2(x + 7) = 2(x) + 2(7) = 2x + 14$

Example when we can simplify: $2(x + 7) - 5 = 2(x) + 2(7) - 5 = 2x + \underbrace{14 - 5} = 2x + 9$

$0.2(x + 0.7) - 1.5 =$

$0.2(x) + 0.2(0.7) - 1.5 =$

$0.2x + 0.14 - 1.5 = 0.2x - 1.36$

$\begin{array}{r} -1.50 \\ +0.14 \\ \hline -1.36 \end{array}$

$\frac{2}{3}\left(x + \frac{3}{8}\right) - \frac{1}{2} =$

$\frac{2}{3}x + \frac{\cancel{2}^{1}}{\cancel{3}}\left(\frac{\cancel{3}}{\cancel{8}_{4}}\right) - \frac{1}{2} = \frac{2}{3}x + \frac{1}{4} - \frac{1}{2}$

$= \frac{2}{3}x + \frac{1}{4} - \frac{2}{4}$

$= \frac{2}{3}x - \frac{1}{4}$

Identity and inverse elements for addition and multiplication.

Additive identity is **0** : $a + 0 = 0 + a = a$ $\qquad -3 + 0 = -3$

Additive inverse of a is $-a$: $a + (-a) = (-a) + a = 0$ $\qquad 3 + (-3) = 0$

(additive inverse is the opposite number)

Multiplicative Identity is 1 : $a * 1 = 1 * a = a$ $\qquad -5(1) = -5$

Multiplicative Inverse of **a** is $\frac{1}{a}$: $a * \frac{1}{a} = \frac{1}{a} * a = 1$ where $a \neq 0$ $\qquad -5\left(-\frac{1}{5}\right) = 1 \qquad \frac{4}{7}\left(\frac{7}{4}\right) = 1$

(multiplicative inverse is a reciprocal)

Zero property: Any number multiplied by 0 is 0: $0 * 9 = 0 \qquad y(0) = 0 \qquad (y + 3)0 = 0$

- To proceed well, at this point you must know **everything so far AND:**
  - **Order of operations - PEMDAS**
  - **Commutative and associative properties**
  - **Distributive property**
  - **Neutral and inverse elements for + and ***
  - **Zero Property**

ALGEBRA: We indicate unknown numbers (variables) with letters: **a, b, c,…. x, y, z**

Terms are products of variables and numbers:

$-x$ is a product of -1 and x $\qquad 3x$ is a product of 3 and x

y is a product of 1 and y $\qquad -0.42y$ is a product of -0.42 and y

Like terms have same variables raised to same powers.

$3x$ and $-x$ are like terms

y and $3x$ are not like terms

Combine like terms (use reverse distribution to combine like terms.)

$3x - x = (3-1)x = 2x$	$3x - x - 1 = (3-1)x - 1 = 2x - 1$
$-5x + x = (-5+1)x = -4x$	$-3b - 7 - 7b = -3b - 7b - 7 = (-3-7)b - 7 = -10b - 7$
$-0.42y + y + 5 = (-0.42 + 1)y + 5 = -0.58y + 5$	$\frac{2}{7}m - \frac{2}{3}m = \left(\frac{2}{7} - \frac{2}{3}\right)m = \left(\frac{6}{21} - \frac{14}{21}\right)m = \frac{-8}{21}m = -\frac{8m}{21}$
$-0.33a - 1.75a + 3.12 = (-0.33 - 1.75)a + 3.12$ $= -2.08a + 3.12$ -0.33 -1.75 -2.08	$\frac{-1}{6}p - \frac{3}{8}p = \left(-\frac{1}{6} - \frac{3}{8}\right)p = \left(-\frac{4}{24} - \frac{9}{24}\right)p$ $= -\frac{13}{24}p$

Equation properties.

General equation property: If $a = b$ then $b = a$

$3x - 1 = 5$ then $5 = 3x - 1$

Additive property of equality: If $a = b$ then $a + c = b + c$

$3x - 1 = 5$ then $3x - 1 + 1 = 5 + 1$ $\quad 3x = 6$

Multiplicative property of equality: If $a = b$ then $a * c = b * c$

$3x = 6$ then $3x * \frac{1}{3} = 6 * \frac{1}{3}$ $\quad x = 2$

Solve and check equation in one variable.

Simplify: $3 - 2(x - 1) = 3 - 2x + 2 = 5 - 2x$

Solve for x:

$$3 - 2(x - 1) = 11$$
$$3 - 2x + 2 = 11$$
$$5 - 2x = 11$$
$$-5 \qquad -5$$
$$-2x = 6$$
$$\frac{-2x}{-2} = \frac{6}{-2} \qquad x = -3$$

Check:

$$3 - 2(-3 - 1) = 11$$
$$3 - 2(-4) = 11$$
$$3 + 8 = 11$$

Solve for x: $5(2 - x) - 14 = 16 - x$

$$\underline{10} - 5x \underline{- 14} = 16 - x$$
$$\cancel{-4} - 5x = 16 \cancel{-x}$$
$$\cancel{+4} + x \quad +4 \cancel{+x}$$
$$-4x = 20$$
$$\frac{-4x}{-4} = \frac{20}{-4}$$
$$x = -5$$

Check

$$5(2 - (-5)) - 14 = 16 - (-5)$$
$$5(2 + 5) - 14 = 16 + 5$$
$$5(7) - 14 = 21$$
$$35 - 14 = 21$$
$$21 = 21$$

True statement

so the solution $x = -5$ is correct.

Exponents

$a^1 = a$	$a^m a^n = a^{m+n}$ $2^3\, 2^2 = 8 * 4 = 32$ $2^{3+2} = 2^5 = 32$	$(a^m)^n = a^{mn}$ $(2^3)^2 = 8^2 = 64$ $2^{3*2} = 2^6 = 64$	$(ab)^n = a^n b^n$ $(2 * 3)^2 = 6^2 = 36$ $2^2 * 3^2 = 4 * 9 = 36$
$a^0 = 1$ $\quad$ *for* $a \neq 0$ 0^0 is undefined	$\frac{a^m}{a^n} = a^{m-n}$ $\frac{2^3}{2^2} = \frac{8}{4} = 2$ $2^{3-2} = 2^1 = 2$	$a^{-n} = \frac{1}{a^n}$ $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ $\qquad$ $(-3)^{-2} = \frac{1}{(-3)^2} = \frac{1}{9}$ $(-3)^{-3} = \frac{1}{(-3)^3} = \frac{1}{-27} = -\frac{1}{27}$	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\left(\frac{5}{3}\right)^2 = \left(\frac{5}{3}\right)\left(\frac{5}{3}\right) = \frac{25}{9}$ $\left(\frac{5}{3}\right)^2 = \frac{5^2}{3^2} = \frac{25}{9}$

Radicals (radicals are expressions that have a square root, cube root… in general: $\sqrt[n]{b}$)

$\sqrt[n]{b} = a$ $\quad$ Definition: A number $\boldsymbol{a}$ is ***n-th root of the number*** $\boldsymbol{b}$ if $\boldsymbol{a^n = b}$

$$\sqrt[index]{radicand}$$

When the index is missing ($\sqrt{b} = a$), it is assumed that $n = 2$ or "square root".

$\sqrt[n]{0} = 0$ (because $0^n = 0$)

Practice: $\sqrt{4} =$ $\qquad$ $\sqrt{16} =$ $\qquad$ $\sqrt{25} =$ $\qquad$ $\sqrt{36} =$ $\qquad$ $\sqrt[4]{16} =$

Fact 1: **Radicals with even indexes,** like square root ($\sqrt{b}$), fourth root ($\sqrt[4]{b}$) …
have always two roots, when squared, result in radicand: a positive (**principal**) root and **negative root.**

Example: $\sqrt{81} = 9$ $\quad$ this is because $\quad 9^2 = 81$ $\quad$ (9 is a **principal** square root of 81)
$-\sqrt{81} = -9$ $\quad$ this is because $\quad (-9)^2 = 81$ $\quad$ (-9 is **negative** square root of 81)

Fact 2: **Radicals with even indexes,** like square root ($\sqrt{b}$), fourth root ($\sqrt[4]{b}$) … , and negative value under the root are **NOT real numbers** (this is because any number raised to even exponent is positive).

Examples: $\sqrt{-4}$ is not a real number; $\qquad$ $\sqrt[4]{-16}$ is not a real number

Fact 3: **Radicals with odd indexes have always only one root and the sign of the radicand and radical are always the same.**

$\sqrt[3]{-27} = -3$ $\quad$ because $(-3)(-3)(-3) = -27$ $\quad$ as opposed to $\quad$ $\sqrt[3]{27} = 3$ $\quad$ because $3(3)(3) = 27$

Fact 4: Another way to write roots: $\sqrt{25} = 25^{\frac{1}{2}} = (5^2)^{\frac{1}{2}} = 5^{2*\frac{1}{2}} = 5^1 = 5$ $\quad$ **Rewrite:** $\sqrt[3]{7} = (7)^{\frac{1}{3}} = 7^{\frac{1}{3}}$

Multiplying Polynomials (use distributive property to multiply; in case of two binomials we FOIL)

$(a+b)^2 = (a+b)(a+b) = a^2 + 2ab + b^2$	$(a+b)(a-b) = a^2 - b^2$
$(a-b)^2 = (a-b)(a-b) = a^2 - 2ab + b^2$	

Factoring Polynomials (make a product out of polynomial; this is used when simplifying fractions).

$a^2 + 2ab + b^2 = (a+b)^2$	$a^2 - b^2 = (a+b)(a-b)$
$a^2 - 2ab + b^2 = (a-b)^2$	