Section 11.3: Permutations/Combinations

- **Factorial:** The factorial of a positive integer $n$, denoted by $n!$, is the product of all positive integers less than or equal to $n$. By definition, $0! = 1$.

- **Permutations:** In counting problems, arrangements are called permutations. The number of permutations of $n$ distinct items taken $r$ at a time is denoted $n^P_r$ or $P(n, r)$.

  **Permutations Formula:** The number of permutations, or arrangements, of $n$ distinct things taken $r$ at a time, where $0 \leq r \leq n$, is given by

  $$n^P_r = \frac{n!}{(n-r)!} = n(n-1)(n-2) \cdots (n-r+1)$$

- **When to use permutations:** Permutations are only applied when
  * repetitions are not allowed, and
  * order is important

**Example 1:** Evaluate the following permutations.

(a) $5^P_3 =$

(b) $8^P_5 =$

(c) $7^P_7 =$
Example 2: How many non-repeating three-digit numbers can be formed from the set \{2, 5, 6, 7, 9\}?

Example 3: Ann, Bill, Carla, Dave, and Eugene are all members of a club. How many ways can they arrange themselves in a row for a picture?

Example 4: How many different ways could first, second, and third place finishers occur in a race with seven runners competing?

Example 5: Suppose certain account numbers are to consist of two letters followed by four digits and then three more letters, where repetitions of letters or digits are not allowed within any of the three groups, but the last group of letters may contain one or both of the letters used in the first group. How many such accounts are possible?
• **Combinations:** The number of combinations of \( n \) distinct items taken \( r \) at a time is denoted \( _nC_r \) or \( \binom{n}{r} \) or \( C(n, r) \).

**Combinations Formula:** The number of combinations of \( n \) distinct things taken \( r \) at a time, where \( r \leq n \), is given by

\[
_nC_r = \frac{n!}{r!(n-r)!} = \frac{n(n-1)(n-2) \cdots (n-r+1)}{r(r-1)(r-2) \cdots 2 \cdot 1}
\]

• **When to use combinations:** Combinations are only applied when
  * repetitions are not allowed, and
  * order is not important.

**Example 6:** Evaluate the following combinations.

(a) \( _9C_4 = \)

(b) \( _5C_2 = \)

(c) \( _7C_6 = \)

**Example 7:** Ann, Bill, Carla, Dave, Eugene, and Frank are all members of a club. How many ways can they select a 3 member committee?
Example 8: A common form of poker involves hands of five cards each, dealt from a standard deck of 52 playing cards. How many different 5 card hands are possible?

Example 9: Five cards are dealt at random from a standard deck of 52 card. Find the probability that the hand contains the following:

(a) three clubs and two non-clubs.

(b) one black card, three diamonds, and a heart.

Guidelines for selecting a counting method:

1. If selected items can be repeated, use the Fundamental Counting Principle.
2. If selected items cannot be repeated, and order is important, use Permutations.
3. If selected items cannot be repeated, and order is not important, use Combinations.