Section 12.4: Angles of Regular Polygons

- **Regular Polygon (Regular \(n\)-gon):** is a polygon with all \(n\) sides congruent and all \(n\) vertex angles congruent.

  - A regular \(n\)-gon has \(n\) rotations and \(n\) lines of symmetry.
  - A **vertex angle** also called an **interior angle** is formed by two consecutive sides.
  - A **central angle** is formed by the segments connecting consecutive vertices to the center of the regular \(n\)-gon.
  - An **exterior angle** is formed by one side and the extension of an adjacent side. A regular \(n\)-gon has \(2n\) exterior angles.
SECTION 12.4: ANGLES OF REGULAR POLYGONS

- **Formulas:** Let $n =$ the number of sides of the regular polygon.
  
  - Sum of the vertex angles for a regular $n$-gon $= (n - 2)180^\circ$
  
  - Measure of each central angle $= \frac{360^\circ}{n}$
  
  - Measure of each vertex angle $= \frac{(n - 2)180^\circ}{n} = 180^\circ - \frac{360^\circ}{n}$
  
  - Measure of each exterior angle $= \frac{360^\circ}{n}$

- **Note that the following hold for regular $n$-gons:**
  
  * measure of central angle = measure of exterior angle
  
  * central angle and vertex angle are supplementary
  
  * exterior angle and vertex angle are supplementary

**Example 1:** Suppose the measure of each central angle in a regular $n$-gon is $20^\circ$. How many sides does this $n$-gon have?

**Example 2:** Suppose the vertex angle of a regular $n$-gon is $162^\circ$. How many sides does this figure have?

**Example 3:** In a regular $24$-gon, what is the measure of the vertex angle? central angle? exterior angle?
• **Polygonal region:** is a polygon together with its interior.

• **Tessellation:** is an arrangement of polygonal regions having only sides in common that completely covers the plane.

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To form a rotation tessellation using a figure:
1. Cut out any figure.
2. Trace the figure on the paper.
3. Rotate the figure 180° around the midpoint of any side and trace the image again.
4. Continue rotating the figure 180° around the midpoint of each of its sides and tracing the new image until the entire paper is covered.
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- Any triangle and any quadrilateral will tessellate the plane.
- For a regular \( n \)-gon to form a tessellation, its vertex angle measure must be a divisor of 360, since a whole number of copies of the polygon must meet at a vertex to form a 360° angle. The only regular \( n \)-gons that tessellate the plane are 3-gons, 4-gons, and 6-gons since their vertex angles measure 60°, 90°, and 120° respectively. Below are the regular tessellations.
- The **vertex arrangement** is the configuration of regular polygons meeting at a vertex. The \((3,3,3,3,3,3)\) below the first figure indicates that six equilateral triangles meet at each vertex.
• **Semiregular Tessellations**: are tessellations using two or more regular polygons.

**Example 4**: Identify each of the following semiregular tessellations by giving its vertex arrangement.