The Cournot Model

Assumptions

- Two firms A and B produce widgets
- The industry demand function is $D$
- Firm A produces $q_A$; firm B produces $q_B$
- Firm A takes its demand function as $D - q_B$

An important assumption, the heart of the Cournot model.
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Solving A’s problem

Symmetry

• Just as Firm A is choosing \( q_A \) to maximize profits, so too is Firm B choosing \( q_B \) to maximize profits.

• If B changes its output, A will react by changing its output.

A Reaction Function

• We do the mathematical approach first and then the graphical approach.
A Reaction Function

• The industry demand function
  \[ Q = 100 - 2p. \]

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  \[ P = 50 - (1/2)(q_A + q_B) \]

A Reaction Function

A’s demand function is then
\[ P = 50 - (1/2)(q_A + q_B) \]
• The firm’s profits are
  \[ \pi = Pq_A - 5q_A \]

A Reaction Function

\[ \pi = [50 - (1/2)(q_A + q_B)]q_A - 5q_A \]
A Reaction Function

\[ \pi = [50 - (1/2)(q_A + q_B)]q_A - 5q_A \]
\[ \pi = 50q_A - (1/2)q_A^2 - (1/2)q_Bq_A - 5q_A \]

A Reaction Function

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\[ \pi = 50q_A - (1/2)q_A^2 - (1/2)q_Bq_A - 5q_A \]

A Reaction Function

\[ \pi = 45q_a - \frac{1}{2}q_a^2 - \frac{1}{2}q_aq_b \]

A Reaction Function

\[ \pi = 45q_a - \frac{1}{2}q_a^2 - \frac{1}{2}q_aq_b \]
\[ \frac{d\pi}{dq_a} = 45 - q_a - \frac{1}{2}q_b \]

A Reaction Function

\[ \frac{d\pi}{dq_a} = 45 - q_a - \frac{1}{2}q_b = 0 \]
\[ q_a = 45 - \frac{1}{2}q_b \]

Symmetry

\[ q_A = 45 - (1/2)q_B \]

• There is a similar reaction function for B
\[ q_B = 45 - (1/2)q_A \]
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Solving for A’s Output

\[ q_A = 45 - \frac{1}{2}q_B \]
\[ q_B = 45 - \frac{1}{2}q_A \]
\[ q_A = 45 - \frac{1}{2}[45 - \frac{1}{2}q_A] \]

Solving for A’s Output

\[ q_A = 45 - \frac{1}{2}[45 - \frac{1}{2}q_A] \]
\[ q_A = 22.5 + \frac{1}{4}q_A \]
\[ \frac{3}{4}q_A = 22.5 \]
\[ q_A = \frac{4}{3}22.5 = 30 \]
\[ q_B = 30 \]

A Graphical Approach

\[ q_A = 45 - \frac{1}{2}q_B \]

• We want to use the reaction function to come to a graphical solution,
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A Graphical Approach

\[ q_A = 45 - (1/2)q_B \]

- When B produces nothing A should react by producing the monopoly output (45).
- When B produces the output of the competitive industry (90), A should react by producing nothing.

Similar rules apply for B’s reactions.

Graphing the Reaction Function

If B produces nothing, A acts like a monopoly. If B produces the competitive output, A produces nothing.
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Graphing the Reaction Function

If A produces the competitive output, B produces nothing.

If A produces nothing, B acts like a monopoly.

If A and B are off their reaction functions, they react and change output. Here B expands, A contracts.

If A is here, B wants to be here.

If B is here, A wants to be here.
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Equilibrium

The Basic Steps

• Plot the reaction functions
  – If B produces nothing, A behaves like a monopoly
  – If B produces competitive output, A produces nothing
• Solve for their intersection

End

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