INTRODUCTION

If you are a Chemist, please be good and skip the Introduction, as this section is all basic stuff found in any introductory Chemistry textbook.

I will first introduce a few concepts here, since this idea deals with a bit of Chemistry. Suppose one adds some sugar to a cup of water. This way, he has formed a sugar *solution*, formed, in this case, by water and sugar. The water itself is called the *solvent* and the sugar is called the *solute*. The properties of a solution differ considerably from those of the pure solvent. Those solution properties, which depend primarily upon the *concentration* of solute particles rather than upon their nature, are called *colligative properties*. Such properties include *vapor pressure lowering, osmotic pressure, boiling point elevation,* and *freezing point depression*. In this idea we are going to deal with osmotic pressure.

Osmosis is defined as a process in which a solvent moves through a *semipermeable membrane* into a solution. Taking our sugar & water example, a semipermeable membrane is a material that allows water molecules to pass through it, while retaining the sugar molecules. This can be understood if we think that the membrane’s *pore size* is large enough to allow water molecules to pass through it, but too small for the sugar molecules. At the microscopic level, one may regard a membrane as a multi-layered grid; the pore size can be taken as the distance in between grids.

The driving force behind osmosis is the thermal motion of water molecules. Thermal motion here refers to the constant “agitation” of a gas’s or a liquid’s constituent molecules. This randomized agitation leads to frequent collisions between the molecules themselves. From here, one can assign an average *kinetic energy* to each molecule.

In air, a molecule undergoes about 10 billion collisions per second. The average speed of an oxygen molecule at 25 Celsius is 482m/s, while that of hydrogen is 1920m/s. The velocity of a molecule being in a gaseous state equates its velocity while being in a liquid state (see Enrico Fermi, “Thermodynamics”, p. 122). This means that the average velocity of a water molecule in a liquid is of one the order of a few hundreds of meters per second (it may be just the speed of sound in water, at 25 Celsius, but if not, the exact value of it will not be required here). The distance between the molecules inside a liquid is much smaller than the distance between a gas’s molecules, so the rate of collisions between the molecules of a liquid is greater than the one inside of a gas. The rate of collisions is also directly proportional with temperature (T), as T is given by the average kinetic energy the molecules themselves. We may also need to assume in here that our system is closed, so no kinetic energy (heat) is lost in time.

We will deal in this idea with *nonelectrolytes*. An *electrolyte* is a substance that exists as ions in water, such as NaCl or HCl. For any nonelectrolyte, osmotic pressure is directly proportional to *molarity*. Molarity is a concentration unit, defined to be the number of moles of solute per liter of solution. The equation relating the osmotic pressure to molarity is the same as the ideal gas law:

\[ P = \frac{(nRT)}{V} = [M]RT \]
Where \( P \) is the \textit{osmotic pressure}, \( n \) is the \textit{number of moles}, \( R \) is the \textit{ideal gas law constant}, \( T \) is the \textit{Kelvin temperature}, and \([M]\) is the \textit{molarity}, defined as \( n/V \).

While there is no such thing as an ideal gas, I will ignore the corrections to the ideal gas law in this paper, as I’m trying to work at low concentrations, and, even if I were to not ignore them, they would be irrelevant for the idea we’re be talking about.

A \textit{saturated} solution is a solution in equilibrium with undissolved solute. A \textit{super-saturated} solution is a solution containing more solute than allowed at equilibrium; it’s unstable to addition of solute (if solute is added, it precipitates). An \textit{unsaturated} solution is a solution that contains less solute than the equilibrium value (i.e. in a water & sugar solution, there are free water molecules in the solution, unattached to the sugar molecules). We’ll deal in here with an unsaturated solution.

**UNDERSTANDING OSMOSIS**

We will need to understand what osmosis is and how it occurs, before we could go any further. Above, I just stated the law that governs osmosis; I did not \textit{explain} it, though. To explain it, we will need to understand it at a \textit{molecular level}. I will try in here to present the general accepted explanation for osmosis, and why this is wrong.

**First, Understanding Pressure**

First, we’ll need to understand what the term “\textit{Pressure}” means, before we could talk about osmotic pressure. If you put a barometer inside a gas-filled container, it will measure the pressure \( P = F/A = (m\cdot a)/A = m(dv/dt)/A = (dp/dt)/A \) (where \( A \) is the surface area of the barometer), so it’d measure how many molecules transfer their momenta to the barometer per unit of time. I will try to give a visual proof of this fact below. Consider the following two systems (containers, here):

![Figure 1](image)

System 1 has volume \( V \), it holds 1 mole of gas molecules of mass \( m \), at temperature \( T \). System 2 has volume \( V \), it holds 1 mole of gas molecules of mass \( M \), at temperature \( T \). So I chose the two containers to have the same volume, and to hold the same number of molecules, at the same temperature.

Let \( M = 9\cdot(m) \), so each molecule in (2) is 9 times as heavy as a molecule in (1).
About both systems, one can immediately write the ideal gas law, \( PV = nRT \). This law is true, regardless of molar masses (MM). Since \( V1 = V2 \), \( n1 = n2 \), \( T1 = T2 \), \( R = \text{const.} \), one realizes the pressures (P) in both containers are equal. Let’s try to see why.

The energy of a molecule at a certain T is given by: \( E = \frac{3}{2} kT \), where \( k \) is the Boltzmann’s constant, and T is the temperature in degrees Kelvin. That means, the energy of a molecule depends purely on its temperature, and not on its mass. Let’s ignore interactions in (1) and (2). That means, the energy is purely kinetic energy (KE). This says that \( KE1 = KE2 \) for each molecule of (1) and (2). So \( \frac{1}{2} m(v1)^2 = \frac{1}{2} M(v2)^2 \Leftrightarrow m(v1)^2 = 9m(v2)^2 \Leftrightarrow v1 = \sqrt{9} * v2 \). So if \( M = 9*m \), \( v2 = \frac{v1}{3} \), so 3 times smaller. That means, the big guys in (2) are moving 3 times slower than their fellows in (1). Let’s look at the momenta (\( p \)) of the molecules:

in (1), \( p1 = m*v1 \); in (2), \( p2 = M*v2 = 9*m*(v1)/3 = 3*m*v1 = 3*p1 = p2 \)

That means, while the molecules’ energies are equal, their momenta is not. So what does the barometer measure?

If the barometer were to measure the KE’s of the molecules, then we’d be in big troubles. In (1) we have n molecules moving 3 times as fast as the n molecules in (2), all having the same kinetic energies. A simple logic should tell that in (1) the KE transferred to the barometer per unit of time is greater than the one in (2), as each collision transfers the same KE to the barometer, but the collisions rate in (1) is 3 times greater than the one in (2), due to the fact that \( v1 \) is 3 times as great as \( v2 \). So KE can’t be responsible for pressure.

If the barometer were to measure the momenta of the molecules, then we are in business. For each collision with the barometer, each molecule in (2) transfers 3 times more momentum than one in (1) does, but the collisions rate in (2) is 3 times as small than the one in (1), so, per unit time, we get the same transfer of momentum to the barometer.

I hope this helps one understand that, for a gas, the molecules’ momenta is the one yielding the pressure, while their kinetic energy yields the temperature.

**Now, Onto The Osmotic Pressure Itself**

So we’ve seen that pressure in a gas is determined by the average momentum of all the gas’s molecules. Now, let’s try to see why osmosis occurs at all. I will first try to present the general-accepted theory, and we’ll look at what’s wrong with it. Let us consider the following setting:

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(see Appendix, Note #1)

In the above system, container (1) has a solution (sugar & water, let’s say) and container (2) is filled with the solvent (let’s say, water). Let’s suppose one can also put a lid on top
of (1) so that it can be closed, if necessary. In between (1) and (2) there is a membrane. So you pour the water in (2), then you pour the solution in (1), and you close the lid of container (1). Let’s start with both containers having the same water level, and they are at the same temperature, T. Initially, the pressures in both containers are equal, and that would be just the pressure of the column of water/solution in each of the containers. But soon enough, in (1), the osmotic effect will start to be felt. If we consider, for example, that in (1) we’d have a 1M solution (one-molar, i.e. one mole of solute per Liter solution) at 25 degrees Celsius, then the corresponding osmotic pressure is 24.7 atm. That’s quite a large pressure, the same as you’d feel if you were at 247 meters below the sea level, as 1 atm corresponds to a column of water of 10 meters.

One looks at this, and at the fact that the law governing osmosis is the same as the ideal gas law, and can’t stop but notice that this (osmotic) pressure is exactly the pressure that would be yielded by the solute’s molecules alone, if they were to be in the gaseous state. I mean, it’s dead easy, 1 mole of any gas, at 25 Celsius, and inside a 1-liter container, yields 24.7 atm in pressure, according to the above law. And this is what 1M means, that we have 1 mole of solute per liter of solution. People have looked at that, and got mislead by a few things, when trying to explain osmosis.

First of all, let me state that if (1) and (2) held gases, not liquids, then P inside (2) would be the partial pressure of the small molecules (o) from (2), while in (1) P would be the sum of the partial pressures of the small molecules (o), and of the big molecules (O). I’m sort of nuts in Thermodynamics, but I’ll state below what other people explained to me, and which I think makes sort of sense. For gases, there will be a net flow of (o) from (2) to (1), until the partial pressures of (o) on both sides of the membrane will be equal. That means, partial pressure of (o) in (1) = partial pressure of (o) in (2). However, one is left with (O) guys in (1) to yield an extra partial pressure, so if two barometers were put in our system, one in (1) and one in (2), the one in (1) would measure a difference in pressure from the one in (2), equal to the extra partial pressure of the (O) molecules, which happens to obey the ideal gas law, so, voila, the same as the osmotic pressure felt in (1). That works fine for gases, assuming ideality* (see Appendix, Note #2). So I’d guess osmosis occurs also in gases, not only in liquids. So people looked at this, and it made them conclude that this also goes true for osmosis in liquids. That means, somehow the (o) molecules on both sides of the membrane come to some sort of equilibrium, and then, all you’re left with, is the extra partial pressure of the (O) molecules in (1), which should be the osmotic pressure felt in (1).

That is basically wrong! Indeed, this explanation may work for gases. But it ain’t gonna’ work for liquids! How can one have equal partial pressures of the (o) guys, inside (1) and (2)? For gases, that could be done easily, you’d have a bunch of (o) molecules flow from (2) to (1) (assume (2) is very, very large, so this won’t change the density of (o) inside (2)), so the gas density in (1) will increase until the density of (o) in (1) = density of (o) in (2). Only this way can you have equal partial pressures in gases, as both (1) and (2) have same T. With gases, yes, you can compress a gas whenever and however you want. But try doing that to a liquid! Liquids like to be incompressible. Which means, there may be a minimal flow of (o)’s from (2) to (1), but, really, you’ll NEVER have the densities of the (o) guys in (1) equate the density of the (o) guys in (2), because inside (1) we have a liquid, and it’s very rude (and mean, too), it will not allow the (o) molecules from (2) to flow into it. See, if (1) is a 1-liter container (lid is closed),
having 200g solute and 800g water inside, then you’d need 200 more grams of water to get from (2) into (1), in order to have the same water-densities on both sides of the membrane. Well, you can forget that… You’ll never be able to get the required 200g of water, you can’t compress the liquid inside (1). So, in other words, the density of (o) guys in (1) is always going to be smaller than the density of the (o) guys in (2).

However, the partial pressures of (o) in (1) and the one of (o) in (2) need to be the same when the equilibrium is established in between (1) and (2), so that the rate of (o) (water) molecules flowing from (2) to (1) = rate of flow of (o) from (1) to (2). And since the water-densities are not equal in the two regions, (1) and (2), something else, beside density, must contribute to this process. We will try to understand what this is, by:

**Understanding Liquids**

There is a major difference in between gases and liquids. The molecular speeds are the same in gaseous state as they are in the liquid state, at the same T, and also both states exhibit chaotical motion. But in liquids we have the potential kick in. See, for gases, one can ignore the potential (interactions) in between the molecules, as the KE of the molecules is much greater than the attraction in between them. That means the molecules collide with each other and don’t stick together, as their KE overcomes easily any attractive forces. However, as I was saying above, there isn’t any real-life ideal gas. The potential in between the molecules, and the molecules’ size are actually responsible for the corrections to the ideal gas law (which I ignored), as PV=nRT only works 100% for non-interacting, rigid, point-sized molecules. At high densities this law breaks down and the corrections become really large, as the closeness of the molecules determines them to feel stronger the attractive forces in between them, while their volume takes up a more significant chunk out of the volume “V”, used in the ideal gas law.

Now, back to the liquids: in here the potential is due to the attractive forces in between the molecules, as well as to the repulsive forces in between them. A molecule in a liquid is much like a particle in a square-well potential (well, more precisely, something like a delta-function potential to the left, and a step potential to the right of the well). To escape the liquid, it needs to overcome the attractive forces in between the liquid’s molecules; to come closer to other molecules, it needs to overcome the repulsive forces in between the molecules. The attractive forces are the ones keeping the liquids together. And they are pretty big. For water, they are responsible for the surface tension, when the surface molecules are pulled in, toward the liquid, with great force; the attractive forces are big enough to make liquids (almost) incompressible, as liquids are already compressed by the attractive forces in between the molecules. The repulsive forces won’t let the liquids collapse. Just like with solids, you almost can’t compress a liquid, and very big forces are required to compress a liquid or a solid just by a tiny little bit. So that’s how it comes that you can pressurize water as much as you wish, only to find out that it’s density doesn’t change; the repulsive potential really kicks in here, and it won’t let the water molecules get any closer to each other. So you may apply huge pressures to water, and it’ll be compressed by a tiny, little bit; but that little bit of a decreased distance in between the water molecules stands for huge repulsive potentials. That’s why you can go 6-7 km below the ocean’s surface, and still find out that the water’s density is still about the same, 1 gram per cubic centimeter… This is what people knew for quite a
while, and that’s what your hydraulic breaks in your car do: since you can’t compress liquids, then you can use them to transmit pressure…

Oh, one more thing about the attractive forces: I said there was something like a step-potential for them. Well, at 50 Celsius, one water molecule out of 12,000 (12K) actually ends up having large enough KE to “jump” outside the well, so outside of the liquid, and to enter the gaseous phase* (see Appendix, Note #3). So molecules with enough KE can break the surface tension and leave the liquid; they call that evaporation, don’t they? Well, I’m just trying here to help you understand what evaporation actually is…

Before I end it, I’d need to mention again something about transmitting pressure into a liquid. In a liquid, the molecules are as close to each other as they’re going to get, so you can imagine them as being spheres, touching each other. To compress the liquid, you’ll need to compress the spheres; by contrast, when compressing a gas, where spheres are apart from each other, you decrease the average distance in between the spheres.

When you have a liquid comprised of two or more distinct substances mixed together (so, that would also include solutions) and you pressurize it, the pressure inside the liquid will distribute itself evenly. That means, if you’re inside a pressurized liquid, you’ll feel the same pressure all over around you. When you pressurize a solution, again, it’s a liquid, and pressure distributes itself evenly, so you will have both the solute’s and the solvent’s molecules feeling the same pressure. That means, if I were to put a 24.7 atm pressure on a solution, ALL the molecules of the solution will feel this pressure. You can’t tell me now that, for example, the water molecules feel this pressure, while the sugar molecules don’t, or vice versa; it’s a liquid, whatever you do to one molecule, is going to be transmitted to all the molecules surrounding it. For that reason, if inside the solution the osmotic pressure due to a 1M solution would build up to its corresponding pressure of 24.7 atm, that pressure would be felt all over inside the solution.

So, in Fig. 2, if a barometer would measure 24.7 atm inside (1), then you can’t say the barometer only measures the pressure of the (O) guys; because in a liquid, unlike in a gas, (O) and (o) molecules will have the same pressure! So that should put to shame all those who traditionally explain that the extra osmotic pressure in (1) is due to the (O) guys alone; they need to understand this, you can’t take a liquid, and tell it, ‘Listen, I want the (O) molecules inside of you to feel & exert a pressure which the (o) guys shouldn’t feel!’ If you apply a pressure of 24.7 atm to the solution in (1), then people will say, yes, all molecules, solute ones or solvent ones, are being pressurized by the same amount. But now, when due to osmosis, a barometer measures 24.7 atm inside the solution, then isn’t that actually the same thing as pressurizing (1) by the same amount? How can you tell this time that pressure decides itself to be due only to the (O) guys? How about the (o) guys? Can you have pressure inside a liquid, and not have it felt by each molecule of the liquid?…

So now we’ve seen that, in explaining osmosis, people have been going nuts, as they tried to apply the facts about gases to liquids, and, as I said, it ain’t workin’… So, you being ready now for a better explanation? If so, let’s try to:

**Really Explain Osmosis**

Let’s look closer at the membrane separating (1) and (2):
Looking at the molecular level, one realizes the membrane has a thickness, which is probably thousands of water molecules across, as water molecules are very small, so there would be something like long, warped tunnels in between one side and the other of the membrane. But we’ll ignore the thickness: we assume that the membrane is uniform, i.e. it has the same number/sizes of holes on side (1) as it has on side (2). That means, regardless of its thickness, the areas open to water flow (the “holes” in the membrane) are equal on both sides of the membrane, so they balance out, and whatever happens inside the wall (membrane) is irrelevant, it only matters what happens at the two surfaces of the membrane. Since liquids are incompressible, whatever hits the tunnel on one side is transmitted to the other side, so the length of the tunnel doesn’t matter, it can be totally ignored (in fact, due to this, the inside of the tunnels is actually still part of (2), if you’ll think well, so the only part of the membrane that matters is its surface facing the solution, where we go from (2) to (1)).

When we pour the water & the solution in (1) & (2), let’s see what happens. We need only to look at the membrane, the other walls of the two containers don’t matter. So on the right, in (2), we’ve got water molecules colliding with the membrane, and on the left, in (1), we’ve got water & sugar molecules colliding with it. Take the densities of (1) and (2) be approximately the same. The liquid in (2) only feels the pressure $P = qgh$, where $q =$ density, $g =$ gravitational const., $h =$ height of water column. The liquid in (1) feels this pressure, and the osmotic pressure. But where does this pressure come from?

Go back to the Figure 3 above. For the membrane, we need to ignore the area in between the holes, as there, molecules from the right or left hit the wall and bounce back, so nothing happens that can have any meaning to osmosis. So from now on we’ll look only at the sites of the wall that have holes into them.

Water molecules pass freely through the holes of the membrane, while sugar molecules can’t (they’re too big). That means, whatever (O) guys hit the wall, bounce back to the left. So they transfer their momentum/KE to the wall. Assume collisions with the wall are elastic* (see Appendix, Note #4). The wall is much, much larger than each individual molecule, so it’s like bouncing a ball on the ground, when the ball bounces back at the same speed/KE, but its momentum has been changed from $p$ to $-p$, so by an amount of $2p$, as in any normal elastic collision with an infinite wall. The wall doesn’t mind and doesn’t care if the (O) guys kick it. At most, they may cause the wall to ‘bend’ a bit to the right, due to this, if the membrane is flexible enough.

The very interesting thing you must realize here is that, each time an (O) hits the wall from the left, the (o)’s hit the same spot from the right, and they don’t transfer their KE/momentum to the wall; they transfer it to the (O) guys, and, thus, to the solution!

Now, the densities of the two liquids, (1) and (2), are the same, so the (O)’s are colliding with neighboring molecules at all times, whether they are inside (1), or whether they are approaching the wall, or whether they depart from the wall. But it’s which (O)
guys are colliding with the (O)’s that matter. Right at the wall, right when an (O) hits the wall, the molecules on its top & bottom colliding with it balance out; the ones on the left, collide with it & transfer their momentum to the wall, just as the (O) does, and the ones on the right, coming from (2), collide with the (O) and transfer their momentum to the solution. It’s like in a boxing match; the (O) guy, coming from the left, can’t punch the guys at its right, as the wall stops it, so it can only punch the wall; on the other hand, the (o) guys at its right punch it all the time, so the (O) boxer is big but can never throw a punch, while his (o) small adversaries keep constantly punching him.

We just said above, momentum is the one creating/responsible for pressure, so this means, for each collision of the (O)’s with the wall, the molecules hitting an (O) from the right and transferring their momentum to it, will induce a pressure inside the solution, as pressure is due to momentum transfer of molecules. Thus, you get the solution inside (1) to be pressurized. Now, the (O) guys collide with the wall freely, and since they move at the same speed as they would move with in the gaseous phase (even though their mean paths are much, much shorter), they will collide with the wall at the same rate at which they would collide with it, if they were to be in the gaseous form* (see Appendix, Note #5). For each collision with the wall, there will be a net transfer of momentum (therefore pressure) from (2) to (1), so the amount of transferred momentum from (2) to (1) is only dependent upon how often (O)’s collide with the wall. If they collide with the wall according to the ideal gas law, then the net transferred pressure from (2) to (1) will obey the same law, as they’re interdependent. That’s why I think one gets the ideal gas law applied to osmosis; not because only the (O)’s inside of (1) will transfer their p to a barometer, but because only they determine how much pressure is transferred to the solution, and they collide with the membrane following the ideal gas law...

Next issue: I said, right when an (O) collides with the wall, the (o)’s hitting it from its left transfer their momentum to the wall as well, and then bounce back from the (O). The (o)’s at its right transfer also their momentum to the (O). The (O) sees no difference, for it gets punched from all sides at equal rates. But you know what happens? Because the (o)’s at the right transfer their momentum to the (O), they will tend to move into the solution, from (2) into (1). They will do that, because their buddies, the (o)’s at the left, don’t punch back; instead, they punch the wall, just as the (O) does. Drawing it,

![Figure 4](image-url)

I delimited the area of the wall covered by the (O) molecule by the line of “*” characters, so it would be big, but I didn’t need to draw the molecule itself. All I need to look here at is the fact that when (O) collides with the wall it covers the hole into the wall + some extra area, as it’s bigger than the hole (otherwise it’d pass through), and in doing so, whatever hits (O) from the left transfers its momentum to the wall. So if you look at the hole’s area of the wall, the (o)’s at the right will tend to move in, as their transfer of
momentum to (1) is not balanced by a similar transfer of momentum of the (o)’s from the left to (2). This is again the same style of boxing game: the (o)’s at the left punch the wall with no avail, while the (o)’s at the right punch without getting hit back, so they’ll push together, as the (o)’s on the left can not balance them, all they can do is punch the wall itself (only for an instance, as long as (O) hits the wall). So now we can see that for each collision of the (O)’s with the wall, there tends to be a net flow of (o)’s from the right to the left, or from (2) into (1). Actually, this should also be the obvious result of the net transfer of momentum which takes place right there, as it’d be natural that if a molecule transfers its momentum from the right to the left, then it will also tend to push in, toward the left. If you can’t believe that, have someone hit you with a heavy ball, moving at a high speed, and tell me if you had or not the tendency to fall backwards, as the ball transferred its momentum to you…

So now we see that at the collision sites of the (O)’s with the membrane, there tends to be a net flow of (o)’s from (2) into (1). So how is this going to be balanced out?

Well, the net transfer of the (o)’s momentum from (2) to (1) induces an extra pressure inside (1), so (1) now get to be pressurized. The pressure inside (1) can be a lot larger than the pressure in (2); just as I said above, you can get 24.7 atm in (1) due to a 1M solution. Since (1) is a liquid under pressure, in spots (the holes into the wall) where only water molecules collide with each other, the water molecules on the left will tend to push away the molecules to their right, and move from (1) into (2), as they are pushed by this extra pressure, which in turn creates an additional repulsive potential in between the (o)’s inside (1). So now you can see that, even though the water molecules inside (1) have a lower density than the water molecules in (2), they are being pressurized, and that way they can balance out the in-coming water molecules from (2). That way, you can, indeed, get the partial pressures of the water inside (2) and the one of the water inside (1) be the same.

Besides using the repulsive potential, one can also understand pressure as if you’d have a bunch of marbles in a box, the ones at the bottom feeling the weight/pressure of the ones on top of them. So a molecule’s momentum would change from \( p = m \cdot v \) (unpressurized) to \( p = (n \cdot m + m) \cdot v = (n+1) \cdot m \cdot v \) (pressurized), where \( n \) stands for the number of molecules laying exactly on top of our molecule. This is much like the ocean, as if you’d be 7 km below the surface, you’d feel the weight/pressure of the water on top of you; so if you wish, you can visualize this as if the (o)’s in (1) were ‘heavier’ than their buddies in (2), so it’s like billiard balls, the heavier ones push away the lighter ones, even though they may have the same speeds. So in spots/holes (of the wall) where only (o)’s collide, there will be a net transfer of momentum from (1) to (2), as this time the holes are open to the water flow from (1) going to (2), and, if it weren’t for the (O)’s to keep colliding with the membrane & get ‘punched’ all the time, the osmotic pressure would be lost in time, due to this transfer of momentum from (1) to (2)…

Well, they call this a dynamic equilibrium, which sort of tells you that the rate at which (o)’s flow from (1) into (2) equates the rate at which they flow from (2) into (1). It’s same stuff like evaporation; if you pour some water into a closed container, after a while, the water vapor will have the same pressure as the vapor pressure of the liquid underneath, so then there’ll be a dynamic equilibrium, meaning that as many molecules enter the liquid from the vapor as they do otherwise.
Suppose we have a container (A), immersed in water (B), and the walls of the container are made of a semipermeable membrane at the bottom (until level “O”), and glass at the top (C). Assume that inside the container (A) there is a 1-Liter, 1M solution to start with. This rends an osmotic pressure of about 24 atm. That is, if container (A) were to be closed with a lid, let’s say at level “O”, the pressure inside container (A) will build up to 24 atm. Of course, this assumes that the solute inside container (A) will not diffuse through the walls of the membrane and cancel any osmotic effect. At this very large pressure, the membrane walls of the container will tend to break, so they have to be supported on some form of porous substrate (e.g. ceramic). These types of supports are already in use, but, anyway, we will not need to use such membrane walls.

Assuming there is no lid over container (A), water will move from the pure water area (B) into the container. This is due to the fact that, when the solute’s molecules collide with the membrane, the (o)’s from (B) keep punching the solution in (A), thus building up pressure, so the molecules in (A) have two choices: one is to go into (B) through the holes where only (o)’s collide with each other, and the other one is for the molecules on top, in (A), to go upward, in (C). Since they collide with nothing on top, they will tend to go easier upward, in (C), rather than going into (B) by pushing the (o)’s who try to enter (A) from (B). Thus, the liquid level inside the glass cylinder will rise, as the solution inside the container becomes more and more diluted. For the sake of argument, assume that the level will rise until it reaches height “P”; that, by this time,
there have been 7 more liters of water flowing through the membrane wall into the container, so the molarity of the solution inside (A) and (C) became 0.125 M (as we have now 1+7=8 Liters solution, but still one mole solute). The corresponding osmotic pressure has dropped by now to 24/8=3 atm, same pressure as the one due to a 30 meters column of water. And assume that at level “P” the system “dies” there, reaching its equilibrium position. That is, the pressure of the column of water above the water level “O”, inside the cylinder, will equate the osmotic pressure corresponding to the 0.125 M solution.

Oh, something I forgot to say about the dynamic equilibrium: now, let’s be clear here, there is a flow of water from (A) into (B) at all times. But this flow will be smaller than the one from (B) into (A), until the equilibrium will be reached. Once that happens, nothing can disturb this equilibrium state unless, let’s say, we have a piston on top of the cylinder and we apply a pressure from above; this would force pure water molecules back into the (B) area, so the rate out will be greater than the rate in. This process is called reverse osmosis and it is used on large scales by Arabic countries to obtain drinking water from seawater. Also, if we lift up (A) out of (B), then we have cut the rate in, but the rate out will still be left there (well, hmm, that’s a bit complicated, but just take it to be a simple logical argument for now).

Now, Just A Few Assumptions

Assume that we use a solute whose molecules are very large-sized, in comparison with the water molecules. That is, the solute is a substance whose molar mass is rather great (at least 500 grams/mole), but which is still being soluble in water. This type of a solute would probably be an organic compound, whose molecules would have (OH)-groups attached to them, and that would make it soluble in water, as most organic compounds are oil-soluble rather than water-soluble. It may as well be a protein, as proteins are very large-sized, too. Of course, it would take a greater mass of this organic compound to yield the same osmotic pressure as salt, for example, would yield. To form a 1M solution, one would need only 58.5g NaCl/Liter solution (but because salt is an electrolyte this would yield in fact an osmotic pressure of 48 atm). To form a 1M solution using the organic compound, it would take at least 500g/L solution, so we would have more solute and less water in the solution. But as the water level will rise in the cylinder (C) and the solution will become more diluted, more of the compound will be able to dissolve inside the container. Assuming that we use this large-sized compound, then we can also choose a membrane with a very large pore size, which will still retain the solute’s molecules inside (A), while allowing water molecules to pass rather freely. Going back to our sugar (C12H22O11) solution: sugar molecules can be considered as being sort of large. Sugar (or sucrose) has a molar mass (MM) of 342g/mole. We can dissolve as much as 204 g sugar per 100 g water at 20 degrees Celsius. This means that we can form a sugar solution of 1.96M. Any further amount of sugar added to this solution will not dissolve.

So we have now the huge solute molecules inside (A) & (C). Each solute molecule will attach itself to a certain number of water molecules, thus making it “look” even bigger. We also use the membrane with the largest pore size possible, which will still retain the solute inside the container. Imagine something like a grid, water molecules
as small balls passing freely through the grids, and solute molecules, attached to the water molecules, as “giants” who can’t make it through the grid. Sure, this means that osmosis can occur faster. Also, besides the pore size, certain materials have properties which fit them better to be used as membranes, and this has to do with the capacity of the materials to absorb water, which means, they form bonds with the water molecules. Such membranes are called hydrophilic membranes. For example, paper or cardboard behave as semipermeable membranes, while allowing water molecules to flow quite easily through - if their structures wouldn’t deteriorate rather fast when placed in water. By contrast, a cucumber’s skin, which is a semipermeable membrane also, will not allow water to pass through as easy, though it is more resistant to water. We want to use in our experiment the type of a material that would be as permeable to water as possible. Now, imagine we have a container made out of this very permeable membrane-call it X. And we fill that container with water. Sure, if X were to be paper, or cardboard, it wouldn’t “hold” the water. It would allow water to pass through, just due to gravity, and soon enough there would be no water left in our fictive container. But we may not be able to make X out of paper. Our goal is, though, to make X out of a material which still behaves like paper: a material that would allow water to pass through it, just due to gravity. It’s somehow like a home water filter, where you pour the water in the top container, and, in time, it will be filtered, and will leak into the lower container. But a home water filter uses active carbon, rather than a semipermeable membrane, so I’m only using this as an illustration, nothing more.

Now go back to Figure 5. Imagine that now, besides the container (A)’s walls, the cylinder’s walls (C), are made out of that kind of a membrane, X. So X’s pore size is as large as possible, while it retains the huge, solute molecules inside. So we start from immersing (A) in the water (B). Water level will rise, due to osmotic pressure, and will stop at height “P”. We have now 7 liters of 0.125M solution inside the top cylinder whose walls are made of X. The height of the column of water (from “O” to “P”) exerts, at level “O”, a pressure of 3 atm. Also, this solution MUST be unsaturated, condition which shouldn’t be hard to attain at this concentration (that would also depend upon how many water molecules will attach themselves to each solute molecule). We may also use higher temperatures for the whole system to enhance solubility (take the sugar example: at 80 degrees Celsius, 362 g sugar dissolve in 100 g water, rendering a 2.29M solution). So in a 0.125M sugar solution there would be plenty of free water molecules. However, for even larger molecules, so closer to real-life experiments, probably even 0.01M would still do, as this very-low molarity will still render a 2.47 meters column of water (so 0.247 atm).

SO WHAT!?

Well, I’ll tell ya what, I’ll tell ya what will definitely happen in the experimental setting from above, and then I’ll also tell you what I want to happen, and if and how that would be possible.

So Let’s See What’s So Interesting About The System Described Above
Now it’s time for me to confuse you a little bit, and we’re going to observe something very interesting, which happens to be irrelevant for my idea (😊), but still interesting to pursue as an:

**Observation**

The system in Figure 5 is a closed system. That means I have a larger container, not drawn (I’m too lazy to do it), which encloses everything and insulates it from the exterior world. To make things more realistic, the height of “P” is about 2.4 meters, as really the substance I’m thinking of using in here won’t do much better than giving me a 0.01M solution. So let’s look at a few things in here. At level “O”, over the solvent’s surface, there’s an equilibrium established in between the (o)’s exiting the water from (B), going into the vapor phase, into (D), and the (o)’s entering the water* (See Appendix, Note #3). Which is normal, that’s what you’ll always have in a closed system. Below, at the membrane’s (X) two interfaces, there is an equilibrium established in between the (o)’s entering (A) from (B), and the (o)’s entering (B) from (A), once the water level reaches height “P”. Nice… But do you think there’s also an equilibrium on top, at “P”, in between the (o)’s entering the solution, and those leaving the solution & going into the vapor, (D) (so we don’t have a lid covering the top of the cylinder at “P”)? I know you’d be tempted to say ‘yes’, but hold on, you’d be all wrong! I’ll tell you why: the vapor pressure ($P_v$) of a solution is lower than the vapor pressure of the solvent! Which means, have a closed system, put in two containers, one has pure water, the other one has some solution. In (a long) time, the water will move from the pure water container into the one with the solution, assuming it’s large enough. How did we do that? Since the $P_v$ of pure water is greater than the $P_v$ of the solution, there wasn’t an equilibrium at the solution’s surface; instead, the vapor above the solution had a larger pressure than the $P_v$ of the solution itself, so more (o) molecules entered the solution than they left it… The law governing this lowering of $P_v$ is given by: $\Delta P = (X_2)^*P_0$.

In here, $\Delta P = P_0 - P_2$ = the vapor pressure lowering, so the difference in between $P_0 =$ the vapor pressure of the pure solvent (water), and $P_2 =$ the $P_v$ of solution; $X_2 =$ the mole fraction of solute [ $X_2 = n_{\text{solute}} / (n_{\text{H}_2\text{O}} + n_{\text{solute}}) \text{, or } X_2 = (\#\text{moles solute}) / (\text{total} \#\text{moles in solution})$ ] . So in Figure 5, the $P_v$ of the solution below “P” is lower than the $P_v$ of the water at level “O”, 2.4m below “P”. So there are more water molecules entering the solution than leaving it, at “P”. Which means, what? If you pour pure water on top, at “P”, you’re breaking the equilibrium by increasing the height of the column of water inside (C), so now there will be a net flow of (o)’s (through X) from (A) into (B) until the equilibrium height “P” is reached again. So if more (o)’s enter at “P” than they leave, you’ll have an overall net flow of (o)’s downward through (C), starting at “P”. Hey, but wait a minute, that’s some organized flow, and it’s spontaneous, so it’s order out of chaos! No, no, no way, something must be wrong here! 😊 Well, I’ll tell ya, that can happen, you can have order out of chaos sometimes take place (the Government kind of people won’t believe to you on that, though 😊)

Let me though stop confusing you 😊… I purposely ignored something. “P” is 2.4m above “O”. The (o)’s exiting (B) at “O” & going into (D) have some KE, at least enough to allow them to break the surface tension at “O”. By the time they get to height “P” they will loose some KE, as they gain potential energy (U). It’s like this, you shoot a ball upward, and you’ll watch it slow down, stop & then come back at you with the same speed, but as the ball goes up, it looses KE and gains U (just conservation of energy
So by the time (o)’s escaping (B) reach “P”, they are a bit slower. Which means, they are cooler, as the speed of a molecule determines its temperature. And this does make sense: as you go higher into the Earth’s atmosphere, it gets cooler… So when the (o)’s reach “P”, they have become slightly cooler and that lowers their $P_v$, as $P_v$ is T-dependent! You can compute how much cooler (but I won’t do it, it’s not important) from $U = mgh = \text{loss in KE}$, and using $KE = \frac{3}{2}kT$, to find $T$ from the final KE. So the (o)’s at “P” do not have the same $P_v$ as the (o)’s at “O”, and this is a very small, but real, difference. On the other hand, the (o)’s inside (C), are not any cooler. They don’t loose any KE while going up all the way to “P”, because they don’t do this on their own; they are pushed from below due to osmosis, so they have the same $T$ as the molecules in (B), below “O”. So in the equation \( \Delta P = P_o - P_2 = (X_2)P_o \), one would need to plug in the real $P_o$, the one at “P”, rather than the one at “O”.

However, note that $P_o$ at height “P” (in (D)) is a constant, as it’s only height-dependent, while the mole fraction of solute, $X_2$, is not. To reach height “P” (2.4 meters), one needs a 0.01M solution, and here molarity is constant. But depending on the MM of the solute used, $X_2$ can have many values. Remember, $X_2 = \frac{n_{\text{Solute}}}{(n_{H_2O} + n_{\text{Solute}})}$. While for a 0.01M solution $n_{\text{Solute}}$ is the same, just 0.01 moles, $n_{H_2O}$ varies: it gets smaller as you increase the MM of the solute, for our 0.01M solution; so, overall, $X_2$ will be greater for solutes with greater MM, and that means that $\Delta P$, the vapor pressure lowering, gets greater as you increase the MM of the solute. So while $P_o$ is lowered by a fixed amount, $\Delta P$ is MM-dependent, which means, you can use a solute with a great MM and thus get a greater $\Delta P$, which means, $P_2$, the $P_v$ of the solution, may still be smaller than $P_o$, the pressure of the water vapor at height “P”. That, inevitably, will lead to the net flow of (o)’s downward through (C) I was just talking about…

Before I leave this specific topic, one more thing: I ignored in here the collisions in between the molecules. As a water molecule exits the solvent & enters the vapor phase it will collide with many, many other molecules, before reaching the height “P”. So I am only taking averages, and I’m saying that, on the average, no matter how many collisions water molecules may go through, they will loose $U = mgh$ out of their initial KE. The same goes for molecules going down through (C), once they entered the solution at “P”: there would be zillions of collisions going on inside the liquid. If one were to ignore these collisions and take just averages, then any molecule entering the solution at “P” will increase its KE by the same amount it lost while it was going up when it was in the vapor. Otherwise, we’d violate C.O.E., as if the molecules go up with one speed (in (D)), and come back to the same height (in (A)) at any a lower speed, so having a lower KE, then they must have spent their energy somewhere, and I don’t see where or how that would be spent… Remember, what goes up, must come down 😐. One can also realize that a molecule entering (C) at “P” has $U$, and by the time that molecule reaches “O”, the $U$ it had is gone, so unless it goes into KE, then where else would it have disappeared?

This C.O.E. is also a key issue, in case one would want to use such a downward flow of (o)’s through (C). In case you come in and put a lid inside (C) at height “Q” (separating the top & the bottom parts of the solution in (C)), then, due to the difference in vapor pressures I just talked about, (o)’s enter (C) from vapor, and the level of the solution increases in (C), beyond height “P”, until it stops, as adding (o)’s will decrease the solution’s molarity in the top side of (C), so thus increasing $P_2$ (the $P_v$ of the solution), while the extra height of the column of water will decrease $P_o$ of the vapor.
When the new equilibrium height is established, one can open the lid at level “Q” and use the downward flow of (o)’s in (C) to do some work. But that comes at a price; if you let a brick drop from your roof, it hits the ground at KE = U initial = mgh. But if you use its U to do work & turn a wheel, you’d slow the brick down, and thus you’ve used up its energy. So if you’d use the U’s of the individual (o)’s entering (C) from above, they’d loose KE (so temperature) by the time they get to “O”. Trying to escape this, one may realize that you could try to use not the individual U’s but the collective ones: a bucket of water can be put to do work while going down some height, and that wouldn’t change the water’s temperature inside the bucket, so you’d treat the water inside the bucket as a mass added to the bucket. Well, this option still doesn’t come for free either: as I said, the (o)’s entering (C) on its top are cooler than the (o)’s at “O”, so this time the water in the bucket will be cooler & thus the (o)’s getting down from (C) into (A) will cool (A) down, which in turn will cool (B) down, so the whole system will cool down, so you’d need to use up a heater. However, one can notice that we can supply heat to the system and create a pretty effective “heat engine” 😊, as no heat is lost in the process, all the heat (KE of (o)’s) lost goes into turning our fictive wheel inside (C)... So that’s somethin’ one can pound on for some time, and think, while it’d be dead slow, think it’d be an extremely efficient heat engine... You could open & close the lid at “Q” using buoyant forces, the wheel would be dead easy to do (propeller-shaped, inside (C)), or else one can use the rising & lowering of the solution’s level in (C) (above “Q”), to have buoyant forces move up & down some object (some mini-submarine 😎) to have it turn some wheel etc. etc. but, again, this is just an observation, not my real idea, so I’m not going to waste time explaining this stuff. Just wanted to help you see what can happen at “P” in between the solution and the water vapor, and wanted to help you see that equilibrium may be a hard thing to attain at this particular interface.

**Observation #2**

There is one more interface we didn’t look at. Remember, in **Figure 5**, we were assuming that we’ll make the walls of the top cylinder, (C), out of X (membrane). That means (o)’s will be free to pass through X on both sides. But now, on one side of the membrane, we have a solution (so a liquid), and on the other side, we have water vapor (a gas). So what do you think it will happen? Well, let’s take a look at it:

![Figure 6](image_url)

This time you will have to bear with me and to please excuse the poor quality of the drawing above, and you’ll need to use a bit your imagination, as I can’t draw on a keyboard... 😎 So left region (1) is the top cylinder, (C), and it has the solution inside it; the water vapor is on the right side, in region (2), or (D), from **Figure 5**. I tried to draw the membrane’s holes by showing the (o) molecules in bold “o” letters (o), being present inside the membrane, while the walls of the membrane are drawn as empty areas. I ignored most of the thickness of the membrane, but I couldn’t ignore all of it, and I’ll tell
you why I did that below. The solute’s giant molecules are drawn as “O”, in (1), and you’ll just have to use your imagination and imagine that they were larger than the holes themselves (couldn’t think of any way to draw the solute molecules any larger….). At the interface in between region (2) and the holes, I drew “)” to stand for the (liquid) water surface. This interface is in between a liquid, and a gas. See, inside the membrane, water stays as liquid. The holes in the membrane are thousands of (o) molecules across, and maybe hundreds of (o) molecules in diameter, so the surface tension will hold our (o) molecules together to form an interface in the shape of “)”, so curved away from the membrane, or in the shape of “(”, so curved toward the membrane, and let me explain why this would be so. Inside a liquid there’s basically two types of forces. Taking water as our example, the forces in between the (o)’s themselves are called cohesive forces, while the forces in between the (o) molecules and the walls of the container, or any other solid, are called adhesive forces. The cohesive forces are the very ones responsible for the attractive potential in between the water molecules I was talking about when we were trying to understand liquids. For water, the cohesive forces are due to the dispersion forces (in between induced dipoles), to the dipole forces (in between adjacent polar molecules) and to the hydrogen bond forces (unique to molecules in which nitrogen, oxygen, or fluorine are bounded to hydrogen). For water, the adhesive forces in between the (o)’s and most solids are in general greater than the cohesive forces in between the (o)’s alone. That’s how you can have water inside a cup, and if you look carefully at the edges of the liquid, you’ll see that the liquid level rises at the cup’s walls, due to the fact that adhesive forces exceed cohesive forces, and they pull the water up a tiny bit. Well, that’s true about a horizontal water surface; in our case it’s vertical, and due to the liquid inside (C)’s pressure, the interface may as well take the “)” shape. So because the adhesive forces in between the (o)’s and the membrane’s walls are going to generally be greater than the cohesive forces in between the (o)’s themselves, if one drops flat a membrane on the surface of a still water container, the water getting through the holes of the membrane will probably form an interface with the gas/vapor (from above the liquid) in the form of a curved interface pointing toward the membrane, but, here, the water is being a bit pressurized, so the interface may also be curved pointing away from the membrane. In both cases, the effect will be the same, and we’ll look at what that may be.

Assuming you do understand the drawing from above, let’s see what we’ve got in here. First, let me make the assumption that our membrane holds water, so it’s like a cucumber’s skin, you can make a container out of this stuff and it will hold water pretty well. Let’s see if there is an equilibrium at the top membrane. For the heck of it, in Figure 5, assume our membrane starts at height “Q”, so (C) is made up of glass in between “Q” and “O”, and above “Q” it’s made up of X. Again, in Figure 6, we need to ignore the areas of the membrane which are impenetrable to water molecules (so, the walls), and we’re only concerned with the membrane’s holes, as only there there’s a contact in between (C) and (D).

The first thing one needs to realize in here is that we cannot ignore anymore X’s thickness, as we did in Figure 2. As I was saying above, in Figure 2, no matter how thick the membrane (X) from in between the containers (1) and (2) was, the water inside the X’s holes behaved as if it were part of (2), as it was just pure solvent, just like there was inside (2). All collisions of the (o)’s from (2) with the X’s holes were transmitted through the holes all the way to (1), so whatever collided with a hole (into X) from the
right side, was transmitted all the way to the left side of the hole, due to the fact that liquids are incompressible and they transmit pressure. So I ignored the thickness of the membrane and I did consider the inside of X’s holes to still be part of (2), and only at the left side of the membrane we passed from (2), the pure solvent, to (1), the solution, so only the left side of the holes into X were drawn, as if X had no thickness at all.

But, really, that’s not the case, X is surely thousands of (o)’s across. Looking at Figure 6, one needs to realize that, indeed, we do have a solution into (1), and water vapor into (2), but inside the membrane we don’t have any of these; inside the membrane’s holes we have pure water, so inside the X there’s really a new region, and you can call it the region (3). So now (1) and (2) are not in direct contact, but rather they communicate with each other through the help of (3)... So, what could happen now?

Well, when only (o)’s collide with a hole from the left (so from (1)), they transmit their momentum/pressure through the (o)’s that fill the hole all the way to the right, where the hole ends into an interface, let’s say of the shape “>”. The surface tension won’t allow the (o)’s at the very interface to escape into (2), so these (o)’s would be hit from the left, bounce to the right, but the surface tension would behave like a spring and thus it’d pull them back in, so they’d bounce back, toward the left, and return the very same momentum (p) they received to the solution in (1), if we assume the restoring forces holding our (o)’s into the liquid behave like an elastic spring. These restoring forces are not magic, they don’t come from nowhere. We’re not violating conservation of momentum in here. If you hit the interface’s molecules from the left, and they bounce back at you, then the interface somewhat behaved like a wall. This wall feels a net transfer of p from the left to the right, and unless the wall is rigid, it will also move of its own to the right. What happens here is that the adhesive forces in between (o)’s and the hole’s walls* (see Appendix, Note #6) are pulling back the water molecules, from near to near, so even though the (o)’s at the interface may be far from the hole’s walls, they will be held in place through cohesive forces in between them and the (o)’s at their very next left, and those, on their turn, will be held by the same types of forces to other (o)’s, and, eventually, all (o)’s will be attracted by the hole’s walls through adhesive forces, keeping all the liquid inside the hole together. So that’s why our “wall” (the interface) is rigid and it won’t move altogether toward the right: it is attracted to X’s walls (so to the membrane itself), and X is rigid, it doesn’t move…

Eventually, if the momentum transmitted from the left is high enough, the (o)’s at the very interface receiving this momentum would escape the liquid from (3) and would go into the vapor, into (2). As we said, every one in 12,000 water molecules will exit the liquid and go into the vapor (at 50 Celsius), so that’s perfectly normal. But for every molecule that goes into the vapor from (3), another (o) from (2) goes into (3), as we’re having here the same vapor-liquid equilibrium I was talking about before. So, to sum it up, in areas of the X where only (o)’s hit the holes from the left, we have a net transfer of p to the X itself (as X’s holes’ walls are part of X and they finally get all the p transferred to the interfaces) for most of the times, and for every one in 12,000 collisions of the (o)’s from (1) that hit the X’s holes, there’s a loss of p, as the (o)’s escaping (3) and going into (2) take that with them. On the other hand, from the right, for every (o) molecule that escapes (3) and goes into (2), an (o) molecule from the vapor in (2) goes into the liquid in (3), transferring its p to the solution in (1) (just our usual liquid-vapor equilibrium), so the (o)’s exiting and the (o)’s entering (3), at the interface, balance out, and the net
transfer of p to the solution in (1) due to them is zero. So all we’re left with is the net transfer of the (o)’s p (hitting the holes and transferring their p to the interfaces) to X, just like it happens in areas where X doesn’t have holes, and where the net p transfer of the (o)’s inside the liquid, to the membrane X, is not balanced out by a similar p transfer of the (o) molecules from the vapor to X, as the (o)’s in the vapor have a much smaller density. But the X can hold all that and it doesn’t mind, (it’s like having water inside a cup, the cup is constantly hit by (o)’s from only one side, but it doesn’t care…) and if this is so, then nothing happens, really, when (o)’s hit the holes from the left.

Well, that actually is not 100% true. If you consider the holes into X to be long cylinders filled with water, then the right interface of one of such cylinders can either have a shape like “(“ or like “)”. Now, this is going to be especially true about the interfaces with shapes like “(“; looking through the cylinder, from the right side, one needs to see that the (o)’s at the interface that are closer to the cylinder’s walls feel the adhesive forces, besides the cohesive ones. If the adhesive forces are greater than the cohesive forces, then the (o)’s being close enough to the hole’s walls are less likely to ever exit (3). Why? Because the surface tension may be greater towards the hole’s walls. See, the surface tension exists because the (o)’s at the very interface are attracted towards the liquid’s molecules, by the (o)’s from inside the liquid, and in the gas there are no (o)’s to counter-balance that attraction. But such attraction comes normally from cohesive forces. If, close to the hole’s walls, it’s the adhesive forces keeping our (o)’s into the solution at the very interface, then the surface tension at such points will be higher than normal, given that adhesive forces are greater than cohesive forces. This will mean that as you go toward the walls of our cylinder, the surface tension will increase at the interface in between (3) and (2). So if you look from the right, our interface may look like a circle; you’d need to draw an inner ring around the edges of the circle, and for the area of the interface enclosed by the ring, the surface tension would be increased. That means that, for this ring-shaped area, we’ll have an unbalanced flow of (o) molecules, as the (o)’s from (3) who are inside that ring-shaped area of the interface will be stuck inside the liquid, having harder times escaping it, while the (o)’s getting into (3) from (2) are still coming at the same rate as they do in all the rest of the interface’s area (they don’t know what happening inside (3)), so, overall, more (o)’s may flow into (3) from (2)…

But what happens at the sites in which the (O)’s hit the membrane? This is where the interesting story begins… When an (O) hits a hole, being larger than the hole, it won’t pass through, and it will transfer its p to the wall (X). At the very same time, whatever (o)’s from (2) are hitting the same hole from the right, at the interface, transmit their p (through (3)) to the (O)’s, and, thus, to the solution. Since the (O)’s can’t punch back, these (o)’s coming from (2) will also tend to move into the solution, though not themselves, but they will hit the (o)’s at the interface, from the right, and their punches will be transmitted through (3) all the way to the left, to (1), and the (o)’s laying in between (3) and (1) will actually be the ones to enter the solution in (1)… Hey, wait a minute, that sounds familiar, doesn’t it?! That’s exactly osmosis itself, that’s how the osmosis actually occurs! This is just the old story about the net transfer of momentum (and therefore pressure) from the vapor in (2) to the solution in (1). So in between the solution in (C), and the vapor in (D), there is actually the same phenomenon which is happening all the way down, inside the liquid, in between (A) and (B)!!! Doesn’t that sound scary and weird?© Anyway, this is for sure, we now are smart enough to
understand what causes osmosis, and because we understand that, we know that osmosis is bound to occur in between (D) and (C)… However, with a small difference, though. It is true that each time an (O) hits the wall, there will be a net transfer of p from the right to the left, and that the (O)’s are hitting the wall in (C) at the very same rate they are hitting the wall in between (A) and (B), as (C) and (A) have the same molarity, and the same temperature. So the fact that (O)’s have the same concentration in (C) just like they do in (A), should tell someone that $P = [M]RT$ also applies to this osmotic effect in between (C) and (D), as the rate at which (O)’s hit the walls determines this law (we saw that above). But now, in (D), we don’t have a liquid, but a gas, whose density is 12,000 times smaller than the pure water’s density. And that does matter. That basically says that, each time an (O) hits the wall from the left, in (D) there will be only one molecule hitting the same hole from the right, where there used to be 12,000 of them, if this were to happen down, in between (A) and (B). So for every collision of one (O) with X, there will be 12,000 times less p transferred to the solution. Of course, in time, it will all add up, and still make the law $P = [M]RT$. But the rate at which osmosis occurs now has dramatically changed. The rate now is 12,000 times slower than the rate at which osmosis occurs down below, in between (A) and (B). See, the rate at which osmosis will occur is also determined by how many (o)’s hit a hole from the right. Each time an (O) hits the wall, all the (o)’s at the right of the membrane will do the job we need them to do, so it really matters how many of them there are there, on the right side, ready to do the job. Because now only one out of 12,000 is left, and only that one creates the osmotic effect, the rate at which osmosis will occur will be 12,000 times slower. So we’ve seen now that the rate at which osmosis occurs depends on: a) the collisions rate (which is T and concentration-dependent) of the (O)’s with the walls, inside the solution, and, b), the water density of the pure solvent container, while the law governing osmosis, $P = [M]RT$, is independent of the concentration (water-density) of the pure solvent, as it only depends upon the collisions rate of the (O)’s with the wall.

One more thing: since osmosis is bound to occur in between (D) and (C), then there will be the same good old flow of (o)’s going downward, in (C), I was talking about in before, regardless of whether the $Pv$ difference on top, at “P”, would also create such a flow on its own or not, so, yes, again, it’s order out of chaos… Since the height of (C) is less than 2.4 meters, my best guess is that, for all practical purposes, one can ignore the fact that the (o)’s entering the solution from (D) into (C), above level “Q”, through the X in between “Q” and “P”, are cooler than the (o)’s from inside (C), and we explained above why they’d be cooler (C.O.E.). Since “Q” would be only 20 cm or so high, the (o)’s entering (C) from (D) would be warmer at level “Q”, and cooler as you go up, but, really, a few centimeters in height could be ignored and thus still say PV=nRT, so one could approximate T for the (o)’s coming from (D) to be the same with T inside (C)...

**Observation #3**

![Figure 7](image-url)

O ooo|____|oIo o
Oooooo|_____oI ←Hole o
Oooooo|___|oI o

**Figure 7**

C 1 Oooo|____|oI o 2 D
OooOooooI ←Hole o
Ooo|____|oI o
Membrane
In Figure 7 we have basically about the same thing as we have in Figure 6. With only one difference: this time, X is hydrophilic, so it gets wet, just like your clothes do when you jump all dressed into the swimming pool. So X here has the same pore size as the X in Figure 6 had, but now there’s no more cucumber skin kind of membrane; now we made it out of paper, or something like it. Which means, on the side of the X facing the vapor, we will have a layer of water molecules forming, as X is hydrophilic, and it not only attracts the (o)’s through adhesive forces; it also likes bonding with water, because that’s what hydrophilic substances do. So our membrane now got wet. And on its side facing (2), the water molecules attached to X form a new interface with (2), which I tried to draw as “I”. I mean, all the outer surface of the X is wet, so water in its liquid form is present not only inside X’s holes, but also on the outer surface of X (that would also increase the region (3) to enclose this new area, having pure water into it). So what has this changed? Well, before, when we used the cucumber skin membrane, only the holes into the membrane meant something. The membrane’s walls were impenetrable to the (o)’s hitting X from the right: they’d hit the wall, and then bounce back, into the vapor. But now, those wall areas are wet, and there’s a water interface in there. Which means, (o)’s from the right, from the vapor, will hit X, and, regardless of whether they collided at a hole’s site or not, they’ll encounter a water interface. And here we have the attractive potential kicking in, once they hit the water interface, so chances are, they will enter the liquid. So for the holes’ sites, everything is just about the same physical situation as it is in Figure 6, we have osmosis occur, and for this part of the membrane’s area we already said what there was to be said. But looking at the wall’s area of the membrane, one can see a few differences. The (o)’s from (1) still see just walls like before, so for them, a hydrophilic X doesn’t make a difference, a bunch of them will form bonds with X, but they can’t pass through the wall. But the (o)’s from (2) now hit these walls areas and they enter the liquid. Their p is transferred to the (o)’s at the very interface, and through them, to the wall (so not to the solution!), so it’s almost like before, when they’d hit the wall and bounce back. Well, this time they do hit the wall, but can’t bounce back, they are now into the liquid and are attracted to the wall. They may slide along the (o)’s that are bonded to X and when they’ll encounter a hole, they may get into it, or where else could they go? Going back into (2) wouldn’t be much of an option along the areas of the wall, as there, the surface tension is again higher than at the holes’ sites, due to the extra adhesive forces in between themselves and the walls. So for this case, we may get a faster-occurring osmosis, as we increased the number of (o)’s getting from (2) into (3), but I’m not so sure. See, (o)’s entering (3) in the areas of the interface along the walls, transfer their p to the wall itself instead of transferring it to the solution in (1), so am not sure what could really happen here. As long as their p isn’t transferred to (1), then they can’t push in. It’s the same physical case as the following: if in Figure 6 I come in from the right, so from (D), at the hole’s site, and, with a minuscule syringe, I pour really slowly some water into (3), right through the interface, what would happen? Well, only two things can happen: either my water from the syringe will move into (1), or it will break the surface tension and will drop/flow along the right side of the wall. If I push lots of water, very quick, through the syringe, then surely the later option will take place. But (o)’s hitting X in (D) and entering (3) have such a low density in (D), that this flow will for sure be very slow. And in that case, remember, in Figure 6 the
water interface curved like “)” was in equilibrium with the solution in (C), and when (o)’s tried to break the surface tension, at the interface, they were pulled back into (3). So now some extra (o)’s added to the hole, in (3), will push their way both to the left and to the right. To the left it’s OK; they’ll enter the solution. To the right, they’ll need to break the surface tension if they want to go anywhere, and that may be hard to do, as the surface tension is kept in place by the hole’s wall, so I think the (o)’s trying to go to the right will be kicked/held back to the left and will end up entering (C). That is, if the wall attracts strong enough the interface, because otherwise the interface will altogether move to the right and there will be a drop forming and dripping down, along the right side of X.

FINALLY, MY CRAZY IDEA

I apologize for having to be so long until now, but I couldn’t have started with my idea directly, as we needed to first be real smart ☺️, and understand deeply what osmosis is, how it works, and what it does, before we could make any predictions about it…

I am going to use now something very similar with what I was using in Figure 7, I will use a hydrophilic membrane, but this time, I will make one change: I will increase its pore size. So let’s see what it could happen now:

Now you really have to use your imagination. I drew the (O) molecules as “O”. In my drawing, there are a lot smaller than the X’s holes, but you need to imagine they’d actually be bigger… So now X’s pore size is pretty large, about 20 nanometers (0.02 microns), and the (O)’s have a MM of about 50,000 g/mole, so that’s why I said I can’t expect a better molarity than 0.01M (and you though that 500g/mole was pretty big, huh?☺️). At least, these are the materials I have available to use for now, though I have no lab settings to try it out…

So now it’s time for us to become real mean, and very rude. And that would require us to make X out of a material that doesn’t hold water. Which is exactly the type of membrane I have already, you can make a small container out of it, pour some water in, and pretty soon all the water will be gone, as gravity pulls it out, through X. May I also mention that my X is hydrophilic, and the solute with the 50K molar mass (MM) is EXTREMELY water-soluble, I took years to find it, and nothing else I’ve seen before is so soluble in water, and neither its formed solutions are hazy, so it’s really ideal for us!
So, let’s see, what we’ve got now? Well, first of all, I’ll need to explain what it means for a membrane to NOT hold water. You all know that a cup made out of paper will let all the water leak out, in time. How is that happening? Well, if the cup were to be made out of X, then X’s pore size is so big, that water molecules just flow easily through the X’s holes. That means, if we’d have, in Figure 8, just pure water inside (C) (which you can consider to be a container/cup made out of X if you wish), (o)’s from (C) will flow through the X’s holes, and let’s try to understand this flow. So now you are using again your imagination and you imagine that in Figure 8 there’s ONLY water inside (C). The X’s holes are open to (o)’s. The pressure of the column of water inside (C) pushes the (o)’s into these holes. When they get on the right side of X, well, X is hydrophilic, and a layer of (o)’s will form on its exterior surface, as X forms bonds with the water molecules (just like a paper container gets “wet”). The surface tension WILL keep these (o)’s inside the liquid, so inside (3). But if the holes are too large, (o)’s coming from (C) will slide along the (o)’s attached to the wall, and will flow along the wall, while still staying inside the liquid, in (3). The surface tension will still be there, but this time the molecules at the interface aren’t attached tightly enough to the wall, and they will altogether flow, so the whole interface will move downward. Now, the interface is still tied enough to the wall so that there may be no water drops forming and dripping, thus breaking the interface; but it’s not tied enough to stop gravity from pulling the whole interface downward. And I hope that all this makes sense to you: if you put water in a paper container, you’ll see exactly this, water from inside your cup will flow out, ALONG THE WALL, so you will see the whole interface moving downward, while the surface tension will keep all the (o)’s from behind the interface inside the liquid. In my drawing, in Figure 8, I assumed that only one layer of (o)’s was sliding along the (o)’s attached to the wall, but, really, that’s just the minimum; there could be many (o)’s flowing along the wall, and in between the (o)’s attached to the wall and those at the very interface, there may be bunches of water molecules/layers flowing. For this case, there will be a differential flow along X: the water flow at the very interface will be faster than the flow closer to the wall, due to the attraction in between the (o)’s closer to the wall, and the wall itself, which will slow down their flow. I hope that now you understand better what you already probably knew, and that is, how a paper cup lets all its water out.

Now we go back to Figure 8. The only difference now in between Figure 8 and what I just talked about is that, in Figure 8, you’ve got the solute molecules in. That is some good news, and some bad news. The good news is, we need those (O)’s in there, as they are the ones responsible for osmosis, so for keeping the water level up, all the way to height “P”. The bad news is, this decreases the water concentration in (C), so, basically, one has to count out all the holes/sites where (O)’s are hitting X from the left, and thus you’re left with fewer holes into the membrane through which (o)’s from the left, from (C), are free to pass into (3), and then flow on the outer side of X. Moreover, those sites that we just counted out, the ones at which (O)’s hit X’s holes, are responsible for creating a water flow backwards, from the right to the left, as we’ve seen in the Figures 6 or 7, as osmosis in between the vapor in (D) and the solution in (C) occurs at these very sites. But, again, this time the rate at which osmosis occurs is really small, it’s 12,000 times as slow as the osmosis’ rate in between two liquids. Also, there is a difference in here from how things worked in Figure 7: in areas of the interface (so on the right side of X) that face a wall part of X, the (o)’s from (D) entering the liquid in (3) don’t get to
enter anymore into (C), but rather, once they entered the liquid, in (3), they transfer their p to the wall, and then they will also be carried along with the flow that’s going on, on the right side of X, so there is one less flow of molecules, from the right to the left, to count in. The only (o)’s who will really try to get into (C) from (D) are those colliding right at the holes’ sites, as they transfer their p to the (o)’s inside the holes, and through them, to the solution in (C).

Again, the interface will maintain the equilibrium in between (o)’s entering (3) from (D), and those going into (D) from (3), as this is the good, old, liquid-vapor equilibrium, so really the net transfer of p to the solution is only due to the sites where (O)’s (in (C)) collide with the wall, as I said in Observation #2. So, to summarize it, osmosis will occur in between (D) and (C), but at a rate that’s 12,000 times smaller than the regular osmosis’s rate in between two liquids. Now, the flow rate of (o)’s through X is material-dependent, and it is a constant of the material; a more permeable membrane has a higher flow rate, while a less permeable membrane has a lower flow rate. Whether there were any vapors or not into (D), this rate would still be the same, as the holes into X still have the same size. However, larger pore size also means more area of the X is occupied by holes, while the opposite is true about the area occupied by X’s walls. This increase in holes’ areas will allow more (o)’s from (D) to transfer their p to (C), rather than just transferring it to X, so a more permeable membrane also means a faster osmosis rate, in between (D) and (C). But that is not to worry about: the water-density in (D) is much, much smaller than the one in (C), so the incoming (o)’s from (D) won’t stop the water flow through X, in its going from the left to the right. It’s like this, if you fill with water your paper cup, and then put it into a water-vapor container, it will still leak all its water out, as the water in the vapor phase isn’t dense enough to counterbalance the flow of (o)’s flowing from inside your cup outside, through X.

So WHAT am I trying to say? I’m trying to say that, if the flow rate of (o)’s through X, flowing out of (C) (due to the pressure of the column of solution inside (C), so due to gravity), is greater than the rate at which osmosis occurs in between (D) and (C), then we will be left with a net flow rate of (o)’s from (C) to (3), so outside the solution, and this net flow will occur along the outer side of the membrane enclosing (C). We can then collect this water flow at level “Q” and then let the water drop back into (B), but now we can use its U to turn a wheel, placed right under “Q”, so, yes, thus doing WORK! Of course, these (o)’s that are flowing back into (B), from (3), at level “Q”, will be free to enter back into (A), due to osmosis, and repeat the whole process all over again. So this was my crazy idea!!!

FURTHER COMMENTS ON MY CRAZY IDEA

As I said, the flow rate through X is material-dependent. If we had pure water in (C), then all the water would flow out, due to gravity. I’m not sure this would help, but in trying to understand this, let’s look at this example: I give you a cubic cup full of water. In an instance, so in an extremely small time unit, I remove one of its sidewalls, so just assume it suddenly disappears. We all know the water will flow pretty quick out.
But I want you to understand why. In the instance the wall disappeared, there will be a water interface formed, right there, in between the liquid and the gas surrounding our cup. The (o)’s right at the wall’s site have nothing to collide with anymore, so they will try to escape the interface, into the gas. The cohesive forces, though, will keep them inside the liquid, but the whole interface will move, and we’ll have the water in our cup flow out. What I’m trying to say is, there could be a heck lot of a water flow, underneath an interface. So, in my idea, I don’t expect the water from (C) to go into the vapor phase, into (D), by escaping the liquid, nor am I saying that reverse osmosis, or anything else, would occur. I do know that osmosis does occur in between (D) and (C), and that this would imply there is a water flow, from (D) into (C). All I am saying is, if my membrane doesn’t hold water, then there also is a new type of a flow involved, which does not exist with a membrane that would actually hold water. So this extra flow has its own rules, being material-dependent only, and there’s no law, nor limit, to stop this flow from being faster than the flow inward, due to osmosis. And this is basically because the water density inside (C) will be greater than the water density in (D), so if you look at a single hole, more (o)’s hit the hole from the left than they do from the right, in time, which would immediately tell you there will be a net flow of (o)’s from the left to the right, assuming the hole’s walls don’t bond so tight to the water liquid that they would prevent such flow from occurring. So even with a cucumber skin container, if you just put pure water in it, it holds water not because the water densities on its sides are equal; in fact, they are not, especially if you put this container in vacuum... The only problem is, its holes’ walls attach so well to the water that water can never escape into flowing out. So I guess the whole trick, and all the difference in here, would be made by whether X holds or not pure water, and on how fast does water flow through such a membrane.

Now, if the water from (C) does flow out: in the solution, the water-density will be lowered, while we’re increasing the solution’s molarity. As water flows out of (C), through X, the solution level in (C) will drop below “P”. Some solute molecules will then start moving downward, from (C) to (A) (they call this diffusion), as (C) now has a higher concentration of solute molecules than (A) does. Since our equilibrium height is “P”, any a lower water column in (C) will trigger more molecules to get into (A), from (B), in trying to re-establish this equilibrium height. On the other hand, the (O)’s diffusing down, from (C) into (A), will increase the molarity of (A), thus triggering even more (o)’s to get into (A) from (B) (due to osmosis). So once (o)’s flow out of (C), the solution will sort of go “OK, never mind, no problem!” and, with the help of osmosis, it will get back other (o)’s into it, from (B), so it looks like this thing would keep on going forever...

However, there may be something that may stop the whole thing: since there is a difference in pressures in between (C) and (D), it may be that the solute’s molecules will end up being “stuck” on the inside side of the X wall. See, when an (O) hits the wall from the left, it will bounce back, but then it will be punched continuously by the (o)’s at its very left, from the inside of the solution. If the (o)’s at its right flow through X, instead of staying there and punching (O) from the right, the (O) will feel a net transfer of p from the left to the right, so it may end up being stuck at the hole’s site, thus blocking the hole completely. Well, if we do this at higher T’s, the thermal motion of our (O) may overcome this, and it may return back into the solution, so if we’d have our system be at 90 degrees Celsius, the solute’s molecules will have a higher KE, (so a greater velocity)
and their agitation may overcome the difference in pressures. Otherwise, they will stick to X, thus inhibiting the water molecules to pass through X anymore, and the liquid level inside (C) will drop until a certain height and will stay there, while no water will flow out of (C) anymore. Because there would be no more holes left into X, they’ve all been blocked by the (O) molecules... While the membrane I have will stand easily T’s of 90 Celsius, the solute may break apart, so actually I may not even be able to run the system under high temperatures; anyway, I have no clue yet what T’s my solute could take... But if one thing could kill this idea for good, then this is what that thing could be, as, indeed, for the case of a water flow, the (O)’s WILL feel a net transfer of p from the left to the right, which is REALLY bad news... Will their thermal motion overcome this? I have no clue, let’s only hope so.... Even if they do, since close to the walls there is a net flow outward, the (O)’s bouncing off the wall’s holes may be taken away by this flow, and get back to the wall, only to bounce off of it again, thus moving in small circles, close to the wall, and never being free to travel back into (C) (though this case is a lot better than when the (O)’s would completely block the wall’s holes...). Of course, the third option is that the (O)’s may hit the wall, bounce off of it and get back into the solution altogether, escaping the above-stated problems. Now, this is where the funny part begins at: a faster flow of (o)’s through X will be more likely to cause the (O)’s to stick close (or to the) wall, while a slower flow of (o)’s will be less likely to take away the (O)’s and keep them close to the wall, or to stick them into the wall’s holes. So now we see that the flow rate through X is a double-edged sword: on one hand, it allows the (o)’s to flow out, while on the other hand it may cause the (O)’s to stick to X, so I guess if the latter one would be a problem, one would need to be more careful, and use an X that is less permeable to water, thus slowing down the water flow L, from (C) to (D)...

Again, experiment will show if indeed the (O)’s could cause such a problem or not...

Looking through things, I am using osmosis just as a “pump”, to get the water level up. Then, the next worry is how to get the water out. If X won’t let pure water to flow through, just due to gravity-so if X holds water-there’s no way in the world how this idea will work. Unless I’d apply a pressure from above, thus spending energy. Well, what I mean through flow due to gravity is, P=qgh inside a liquid, so pressure increases linearly with the height of the column of water above you, and at depth h, you feel the weight of all the water molecules lying on top of you, which, of course, is determined by gravity; so since gravity is responsible for the pressure inside the liquid, it also is the one responsible to the flow through the membrane. Though this idea would also work in the outer space, you can put a lid at “P” inside (C), and you’ll still have exactly the same pressure, but now, without gravity, the flow out won’t go anymore back into (B), but it’ll wonder wherever else it wants to. So gravity at least does the nice job of pulling the (o)’s coming out of (C), back into (B)... ☺

Now, remember, ever since we placed (A) into (B), with the 1M solution inside (A), there was a flow out, which, of course, was surpassed by the flow in, until the equilibrium was reached. Well, the top (C) area should suggest you the question: and what happens if you suddenly cut the rate in? Does the flow out stop right away? I don’t think so... While the flow out now may be smaller than it would be at the equilibrium, it will be there, no matter what, as this is what a membrane is supposed to do, it allows (o)’s to flow through it. The only issue is, when placed in between two liquids, our X will always allow water to flow in and out, as no surface tension is involved; but if X is
placed in between a liquid and a gas, then the surface tension kicks in, and, while X will still behave nicely, and allow (o)’s to pass through, they’ll all stop at the interface. if X holds water. Which, of course, doesn’t stop the liquid and the gas from exchanging water molecules, but it would stop the liquid from flowing altogether outside, along the wall, in this particular case. So, really, the key issue in here is what the flow rate through X is...

The whole idea is based on the fact that water molecules have KE. They always collide with each other elastically, but they move utterly chaotically. If we can somehow force them to follow a certain path in time, in which the starting point and the ending point are the same, then we are in business. Now, I won’t go through this one, but there’s a chance that, when (o)’s pass from (B) to (A) due to osmosis, they will cool off a little bit, basically because when they’ll enter (A), the (O)’s just bounced off the wall and they’re moving away from it, so the (o)’s coming from (B) and hitting them will slow down after colliding with the (O)’s. But one can also argue that, when the (O)’s were approaching the wall, the (o)’s coming from (B) actually increased their own p after colliding with the (O)’s, so, really, I’m not sure there will be a cooling or not. If there will be a cooling, well, no problem, we’d just put a heater into the system, and still have a darn efficient heat engine...

Feel free to argue that all these can’t be. It’s good for the sake of the idea itself that one would try to understand it and to prove it wrong. As on my side, all I’ll say is, since water molecules have no governmental traffic laws to enforce them to stay inside the solution, when the membrane’s walls are open to their flow, then you’ll waste your time trying to disprove that particular aspect of it. Maybe somewhere else I made a mistake… Good luck with it! 😊

If you’ll ever feel like trying this crazy thing at home, you may find some useful information at the very end of my paper* (See Appendix, Note #7).

APPENDIX

Note #1:
KE or p transfer: throughout my paper, I always say that molecules collide with a membrane/interface from the left or from the right, which is not true. If one were to consider the x-axis in each figure as being horizontal, then molecules hitting a wall have momenta (p) along x, y, and z-directions. I just ignored the y & the z-directions, as it’d make things too complicated, and, besides, only the px, so the x-component of each molecule’s momentum, is going to count in all our experiments.

Note #2:
No ideal gas: An ideal gas is a gas whose molecules have no size, so they are point-like, and which do not interact with each other.

Such a gas does not exist. Molecules have sizes, and there also is a potential in between them at all times, and this holds true even for rarefied gases.

In a container holding two ideal gases, Y and Z, you can add the partial pressures of each individual gas, for a few reasons. Since the two gases are ideal, their constituent molecules have no size, and therefore occupy no space of their own inside the container. That means that the gas Z sees the container as being empty, or void, of any other gas’s
molecules, and thus, Z will exert a P (pressure) on the container’s walls equal to its own partial pressure, and same goes true for the Y gas.

In reality, this is far from true. Let the volume of the container be V. The molecules of Y and of Z have sizes of their own. Let Vz be the volume occupied by all the Z’s molecules, and Vy be the volume occupied by all the Y’s molecules. Thus, the gas Z does not have V available, but, in fact, V–Vz–Vy, and the same goes true for the gas Y (V–Vz–Vy is the empty space available in the container). The partial pressures of each gas comes from the ideal gas law, PV = nRT. This law doesn’t hold true in reality, as one has to substitute in the above equation, for each gas, V–Vz–Vy, instead of using V. That would increase P in the ideal gas law, so the partial pressures for each gas would be higher than the ideal gas law would normally yield. In a dense gas, this correction to the ideal gas law becomes even greater.

The interactions in between the container’s molecules also play a role in the real-life gases. The ideal gas law assumes molecules do not interact, so they just bounce off each other, after colliding 100% elastically. Again, that’s not true: rather than viewing a gas’s molecules as being elastic balls, a more realistic picture would be, actually, sticky balls. Each time they collide, they tend to stick to each other, and the interaction term becomes really large at higher densities, breaking the law down by lowering the P term in the equation PV=nRT.

Note #3:

Liquid-vapor equilibrium: At 50 Celsius, one can calculate the densities of the water liquid and of the water gas, when they are in equilibrium. Assume you have a closed container, half-filled with water, at 50 Celsius. The vapor pressure (Pv) of the liquid is: PV = 92.5mm Hg, so (92.5/760) atm. From the ideal gas law, one can calculate the density of (o)’s into the vapor: PV = nRT => n/V = P/RT. A few constants: Na = Avogadro’s number = 6.0221*(10^23) molecules/mole R = Rydberg’s constant = 0.082056 L*atm/(mol*K).

At T = 50 + 273.15 = 323K (degrees Kelvin), we get, for the water vapor:

n/V = [(92.5/760)/(0.082056*323.15)]*[6.0221*(10^23)] = 2.764352092*(10^18) molecules/cm^3 = water density in the vapor phase. (1)

For the water liquid, one can also calculate the same quantity. At 50 Celsius, the water density is close to q = 0.996 g/cm^3 => n/V = (0.996g/cm^3)*(1 mole/18g)*(Na/l mole) = 3.33222866*(10^22) molecules/cm^3 = water density in the liquid phase. (2)

Taking the ratios of the equations (1) and (2), one gets:

(# water molecules in the liquid phase)/(# water molecules in the vapor) = 12,054.3, or, roughly, 12,000. This ratio is the same with the densities in between the two phases ratio; I won’t do the entire math, but qvapor = 0.083g/L and qliquid = 996g/L, you take the ratio of the two densities and you’ll get 12,000.

Note #4:

Elastic Collisions: we’re going to talk in here about the collisions in between the molecules themselves, as well as about the collisions in between the molecules and the walls. There really isn’t such a thing as an elastic collision. In any material the collisions in between its constituent molecules leads to radiation (occurring at a rate that’s
proportional to its $T$ raised to the fourth power, according to the Stephan-Boltzmann law), which basically is photons being emitted after each collision. This would inevitably lead to cooling, and if any material were to be placed in vacuum, at 0 K, it will radiate all its heat (energy) until it will reach 0 K itself. So in order to have elastic collisions we only need to assume the walls inclosing our system are 100% isothermal, so any photon emitted will bounce off the walls, back into our system, only to be absorbed again. Such walls will permit the molecules of our system to collide as much as they want to, without decreasing their temperature.

**Note #5:**

**Collisions of (O)’s with the walls:** So we know that the average velocity of (O)’s in the liquid phase is the same with the one in the gaseous phase, while the mean free path of (O)’s is much smaller inside the liquid. We need to see why the collisions rate of (O)’s with the walls, inside the solution, is the same with the collisions rate of (O)’s with the walls in the gaseous state. To do that, let’s do what Physicists always do: let’s simplify things a bit. So consider only one (O) molecule, inside a one-dimensional (1-D) microscopical container; (O) can only move in the $x$-direction, so it exhibits 1-D motion. Once we understand how things work for such a case, we can always extend the results to $n$ molecules, moving in 3-D. Drawing it,

![Figure 9](image)

The box’s walls are drawn as “I”, the middle and the quarters of the box are marked as “,” symbols, the (O) molecule is marked as “O”. Now it’s time for some Statistical Mechanics (boy, how I hated that course!) to come into play; I am not very good at it, so just bear with me as we go along. Let the size (length) of our 1-D box be $d$, and let the (O) molecule travel a distance equal to $d$ in a time $t$. That will cause the probability ($P$) of the (O) molecule to hit a wall in a time interval equal to $t$, to be: $P(t) = Po = 1$, which means, in a time $t$, no matter where we start at, the (O) will hit one of the box’s walls. Now I come in and cut the (O)’s path in two, by sticking some object at the middle of the box, and I trap the (O) in between the right wall and my object. The (O) can’t penetrate this object, so to the (O) it looks like the box shrank to half its size. Since the (O)’s speed is the same, it will collide with the object and with the right wall now, hitting the right wall at twice the rate it did that in the first place. Then I can come in again and cut its path by another half, and now the collisions rate of the (O) with the right wall has increased by another factor of two. So if I keep cutting the (O)’s path’s length all the time, thus dividing the original path of the (O) molecule into $n$ equal intervals, the (O) has two options: if it lies into the $n$th interval, right close to the wall, it will collide with the wall at a rate that’s $n$-times greater than the original rate (so $n$ collisions instead of one), or, if it lies into an interval further from the wall, it will not collide with the wall at all. Since the $P$ for the (O) to be into *any* of the $n$-intervals is the same, then the $P$ for the (O) to be into *any one interval* is $P_{total}/n$. So now the $P$ for the (O) to hit a wall in the same time $t$, is $P(t) = (P_{total}/n)^n (n$ collisions in each interval) = $(Po/n)^n n = 1 = Po =$ original probability, as $P_{total}$ is just $Po$, for the $t$ time interval. So we’ve seen now that if we cut the (O)’s path into $n$ intervals, the $P$ for the (O) to hit a wall in time $t$ is still 1, the
same as if there were no cuts at all in the (O)’s free path, so that proves that the collisions rate of the (O) with the walls is the same, both in the gaseous phase, and inside a solution.

**Note #6:**
The holes’ walls: I know this expression sounds kind of puzzling. If you *imagine* a hole into X as a long cylinder full of water, then the cylinder’s walls are the hole’s walls, so they are inside the membrane, and I don’t know how much better to describe them…

**Note #7:**
They have a membrane filter specification table:
And here’s a page with a few specific membranes and lists of the solutes they can filter: [http://www.millipore.com/analytical/amicon/pa-425-1.html](http://www.millipore.com/analytical/amicon/pa-425-1.html)