DISCLAIMER: I believe the solutions to these problems are correct. However, since I am human, mistakes may have occurred. If you feel that there are errors or mistakes, please let me know ASAP.

For each of the following problems, you must apply the best available statistical test to the complete null-hypothesis significance testing methodology we have explored in class. For each analysis, you should:

- Specify a research hypothesis
- Specify a null hypothesis
- Identify a rejection region (alpha) and the direction (tail) of the test
- Name a critical value of $z$ or $t$ associated with rejecting the null hypothesis
- Select the best statistical test for the question and calculate your obtained value of $z$ or $t$
- Specify if you reject or fail to reject the null hypothesis based on your findings

1) You are a teacher at a local elementary school. You are interested in finding out if your teaching methods have had a positive impact on the academic functioning of the 37 third-grade students in your class. You decide to use a standardized measure of academic achievement to assess your students’ performance in comparison to the performance of the average third grader in the nation. You know that the achievement test you are using has an average score of 35.45, with a standard deviation of 4.22, for American third-graders. If your students scored, on average, a 37.01, can you conclude that they have performed significantly better or worse than the average third grade student in the United States?

2) You have been given a set of depression scores from undergraduates who have participated in a research opportunity to meet the requirements of their introductory psychology course. The school IRB (Interdisciplinary Review Board) wants to know if this measure is ethical—namely, they are concerned that individuals who complete this measure may score highly on the measure, and consequently, run a high risk of suicidal behavior or self-harmful behavior. The IRB’s policy is that measures of depression that might indicate suicidal behavior require a greater level of care and supervision when they are used. You know that the published population mean for college students on this measure is 3.67 (higher scores = more depression and an enhanced risk of suicidal behavior). Given the scores below, do you have any reason to believe that college students at your university score higher on depression than the average college student, necessitating more caution when this measure is used?

4  4  7  2  4  9  1  3  10  4  5  1

3) You are piloting a new psychotherapy to help individuals stop smoking. You decide to test the efficacy of your program by assessing individuals’ urge to smoke both before and after your therapy. Your measure of “urge to smoke” ranges between 1 and 5, with 1 being a low urge to smoke and 5 being a high urge to smoke. Given the scores below, do you have any reason to believe that your program can help people stop smoking

Before:  4  5  5  3  5
After:  3  2  1  2  4
4) You decide to expand on your anti-smoking psychotherapy by testing the efficacy of your psychotherapy in comparison to a no-treatment control group. This time, you collect a sample of participants and randomly assign them either to psychotherapy or a no-treatment control condition. Both groups have equivalent scores on your measure of urge to smoke at the beginning of the study. At the end of the study, you find the data listed below. Can you conclude that your psychotherapy is more effective in helping people quit smoking than a no-treatment control?

Treatment: 1 3 2 2 1 2 4 2 1 1
Control: 4 4 3 2 5 4 3 2 5

You decide that you want a rather conservative test of your hypothesis, so set your rejection region accordingly.
Answers

1) \(H_1:\) These students score higher or lower than the national average on the test (\(\mu_1 > \bar{x}\))

\(H_0:\) These students score the same as the national average (\(\mu_1 = \bar{x}\))

\(\alpha = .05,\) 2-tailed

\(z_{crit} = +/-1.96\)

**Test: z-test**

\[ z = \frac{(\bar{x} - \mu)}{\sigma} \]

\[ z = \frac{(37.01 - 35.45)}{4.22} \]

\[ z = \frac{(1.56)}{4.22} \]

\[ z = \frac{(1.56)}{.69} \]

\[ z = 2.26 \]

Reject the null. Your obtained value of \(z\) falls within the rejection region (greater than 1.96). Your students are scoring better than the national average... by a very small amount, at least.

2) \(H_1:\) These students score greater than average depression score

\(H_0:\) These students will scores of depression are equal to the national average

\(\alpha = .05,\) 1-tailed

\(t_{crit} = 1.796\)

**Test: One Sample t-Test**

\[ s = \sqrt{\frac{\sum x^2 - (\sum x)^2}{n - 1}} \]

\[ s = \sqrt{\frac{334 - 2916}{12 - 1}} \]

\[ s = \sqrt{\frac{334 - 243}{11}} \]

\[ s = \sqrt{\frac{91}{11}} \]

\[ s = \sqrt{8.27} \]

\[ s = 2.88 \]

\[ t = \frac{\bar{x} - \mu}{s/\sqrt{n}} \]

\[ t = \frac{(4.5 - 3.67)}{2.88} \]

\[ t = \frac{.83}{2.88} \]

\[ t = .83 \]

\[ t = 1.00 \]

Fail to reject the null hypothesis. There is no significant difference between the obtained depression scores and the national average.
3) \( H_1: \) Your therapy will help people stop smoking (reduce their urge to smoke)  
\( H_0: \) Your therapy will not reduce participants’ urge to smoke  
\( \alpha = .05, \) 1-tailed  
\( t_{crit} = 2.132 \)  
Test: Matched Samples \( t \)-Test  
\[
\begin{align*}
4-3 &= 1 \\
5-2 &= 3 \\
5-1 &= 4 \\
3-2 &= 1 \\
5-4 &= 1 \\
\text{Total} &= 10/5 = 2
\end{align*}
\]
\[
\begin{align*}
s_D &= \sqrt{\frac{\sum x^2 - (\sum x)^2}{n}} \\
s_D &= \sqrt{\frac{28 - 100}{5}} \\
s_D &= \sqrt{\frac{28 - 20}{4}} \\
s_D &= \sqrt{\frac{8}{4}} \\
s_D &= \sqrt{2}
\end{align*}
\]
\[
\begin{align*}
s_D &= 1.41 \\
t &= \frac{(D - 0)}{s_D} \\
t &= \frac{(2 - 0)}{1.41} \\
t &= 1.41 \\
t &= 2.24 \\
\text{Reject the null hypothesis. It appears that your program may have value after all.}
\end{align*}
\]
4) \( H_1: \) Individuals under the new psychotherapy will have a lower average urge to smoke than controls.  
\( H_0: \) There will be no difference in urge to smoke between treatment and control groups  
\( \alpha = .01, \) 1-tailed  
\( t_{crit} = 2.567 \)  
Test: Independent Samples \( t \)-Test (w/ pooled variance estimate)  
\[
\begin{align*}
\bar{x}_1 &= 1.90 \\
\bar{x}_2 &= 3.56
\end{align*}
\]
\[
\begin{align*}
s_1^2 &= \frac{\sum x^2 - (\sum x)^2}{n} \\
s_1^2 &= \frac{45 - 361}{10} \\
s_1^2 &= 45 - 36.1 \\
s_1^2 &= 8.9 \\
s_1^2 &= .99
\end{align*}
\]
\[
\begin{align*}
s_2^2 &= \frac{\sum x^2 - (\sum x)^2}{n} \\
s_2^2 &= \frac{124 - 1024}{9} \\
s_2^2 &= 124 - 113.78 \\
s_2^2 &= 10.22 \\
s_2^2 &= 1.28
\end{align*}
\]
\[
\begin{align*}
s_p^2 &= \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} \\
s_p^2 &= \frac{(10 - 1).99 + (9 - 1)1.28}{10 + 9 - 2} \\
s_p^2 &= \frac{9.99 + (8)1.28}{17}
\end{align*}
\]
\[
\begin{align*}
s_p^2 &= \frac{8.91 + 10.24}{17} \\
s_p^2 &= \frac{19.15}{17} \\
s_p^2 &= 1.13
\end{align*}
\]
\[
t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_p^2}{n_1} + \frac{s_p^2}{n_2}}}
\]
\[
t = \frac{1.90 - 3.56}{\sqrt{\frac{1.13}{10} + \frac{1.13}{9}}}
\]
\[
t = \frac{-1.56}{\sqrt{.11 + .13}}
\]
\[
t = \frac{-1.56}{\sqrt{.24}}
\]
\[
t = \frac{-1.56}{.49}
\]
\[
t = -3.18
\]

Reject the null hypothesis. The new psychotherapy appears to work.