## Lecture 12a—A Brief Example

PROBLEM: For uniform flow in a wide channel, find the relative change in discharge per unit width, d$q$/$q$, for a small relative change in depth, d$d$/$d$ by using the Manning equation:

$$v = \frac{1}{n} R^{\frac{2}{3}} S^{\frac{1}{2}}$$

where $v$ is depth-averaged velocity, $R$ is hydraulic radius, and $S$ is channel slope. You may assume that the channel slope and the bottom roughness remain constant.

SOLUTION: Since $n$ and $S$ are constants, let:

$$k = \frac{\sqrt{S}}{n}$$

so

$$v = kR^{\frac{2}{3}}$$

For a wide rectangular channel, however, $R \approx d$, so:

$$v = kd^{\frac{2}{3}}$$

Last, to convert to discharge, we need $Q=vA$, but for discharge per unit width, $q=vd$, so $v=q/d$.

$$\frac{q}{d} = kd^{\frac{2}{3}} \Rightarrow q = kd^{\frac{5}{3}}$$

Taking logarithm of both sides,

$$\log q = \log k + \frac{5}{3} \log d$$

and differentiating:

$$\frac{dq}{q} = \frac{5}{3}\frac{dd}{d}$$

for constant *k*.

So, assuming *n* and *S* to be constant, and $R \approx d$, a 10 % change in *d* will result in a 16.6% change in *q*.