Lecture 18a—Slope Effects on Initiation of Motion

When we began talking about initiation of motion, we stated (more or less explicitly) from the beginning that the bed didn't have a slope. What happens if the bed *does* have a slope? We'll consider two different problems, the side wall:

and the sloping channel:

Before we start, remember that we had the following:

Now, let's consider the side wall of a channel:

Where

$$A = F_g \sin\phi = c_2 g(\rho_s - \rho)D^3 \sin\phi$$

$$B = F_D = \tau_w c_1 D^2$$

So,

$$F_2 = F_g \cos\phi = c_2 g\left(\rho_s - \rho\right)D^3 \cos\phi$$

$$F_1 = \sqrt{B^2 \cos^2 \beta + \left(A + B\sin\beta\right)^2} = \sqrt{B^2 + A^2 + 2AB\sin\beta}$$

$$= \sqrt{{c_1}^2 {\tau_w}^2 D^4 + F_g \sin^2\phi + 2c_1\tau_w D^2 F_g \sin\phi \sin\beta}$$

And,

$$\frac{F_1}{F_2} = \tan\alpha$$

SO

$$\frac{\sqrt{{c_1}^2 {\tau_w}^2 D^4 + F_g \sin^2\phi + 2c_1\tau_w D^2 F_g \sin\phi \sin\beta}}{F_g \cos\phi} = \tan\alpha$$

For ϕ, β=0 (flat bed):

$$\tau_w = \frac{c_2}{c_1}\left(\rho_s - \rho\right)gD\tan\theta$$ whew!

For $\phi \neq 0$, $\beta \neq 0$:

$$(\quad)\tau_{wc}{}^2 + (\quad)\tau_{wc} + (\quad) = 0$$

which ends up as:

$$\frac{\tau_{wc}}{\tau_c} = \frac{-\sin\phi\sin\beta}{\tan\alpha} + \sqrt{\left(\frac{\sin\phi\sin\beta}{\tan\alpha}\right)^2 + \cos^2\phi\left(1 - \frac{\tan^2\phi}{\tan^2\alpha}\right)}$$

where α is commonly taken as the angle of repose.

When β=0,

$$\frac{\tau_{wc}}{\tau_c} = \cos\phi\sqrt{1 - \left(\frac{\tan\phi}{\tan\alpha}\right)^2}$$

How about the second case?

We can't use the same derivation (try it); let's take a simpler one:

So,

$$F_1 = \tau_\beta c_1 D^2 + F_g \sin\beta$$

$$F_2 = F_g \cos\beta$$

and,

$$\tau_\beta c_1 D^2 + F_g \sin\beta = F_g \cos\beta \tan\alpha$$

finally,

$$\frac{\tau_\beta}{\tau_c} = \cos\beta - \frac{\sin\beta}{\tan\alpha}$$