# Lecture 19—Bedforms and Bedform Stability Diagrams

Ok. We ended last time with Shields' ominous warning that the bed began to deform as soon as sediment started to move. What's up with that? Well, hey, you've all seen it, right? First grains begin to roll and hop, and form ripples. Ripple wavelength is mostly dependent on grain size, and only weakly on velocity. As hop length increases, dunes form, and as grains spend less and less time in contact with the bed, these dunes wash out and are replaced by a flat bed. Finally, if Froude number exceeds 1, antidunes form.

Here's *another* way to think about bedforms. What if we consider the bed to be the interface between a relatively low viscosity fluid (water) and a relatively high viscosity fluid (sand)? This deformable boundary would have shear on it, and could either be stable or unstable as a result of shearing. A stable interface would result in a planar bed, and an unstable interface would result in an undular bed. This sort of analysis happens quite a bit in fluid dynamics, and the instabilities are called Kelvin-Helmholtz instabilities.

Here's the thing—if some sort of perturbation to the interface occurs, the perturbation can either be damped (stability) or continue to grow (instability). As the bed undulates, it causes the same sorts of velocity and pressure differences that happen on an airplane wing. From our old friend Bernoulli, we know that as the interface swings up, it results in slower flow on the lower fluid, and faster flow on the lower fluid (the situation is reversed on the downswing). *But*, for conservation of energy, this results in an *increase* in pressure for a decrease in velocity. As a result, unless something acts to balance this increased pressure, the interface will continue to deform, and continue to accrue increased pressure. In natural fluids, the opposing forces are viscosity and the force of gravity. The effect of viscosity is effectively a kind of Reynolds number, where the length term is the initial amplitude of the perturbation, and the velocity term is the velocity difference between the two fluids.

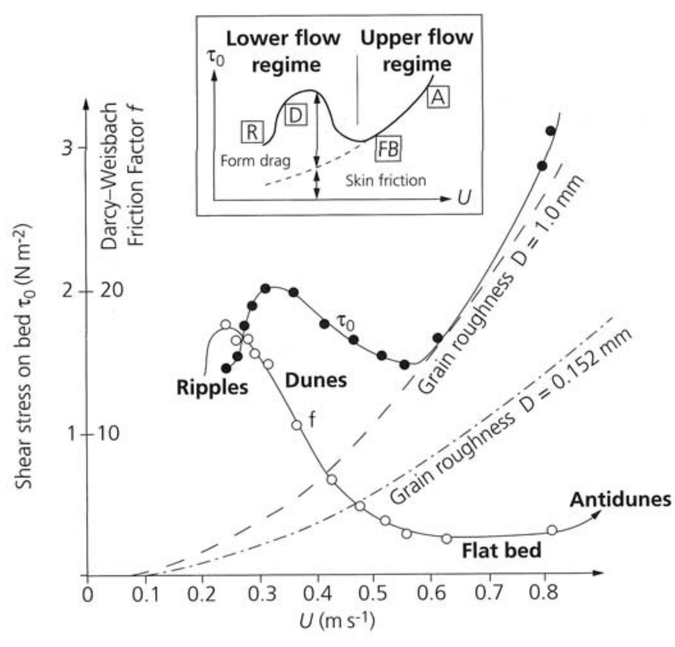
For faster moving fluids, the effect of gravity on the interface controls the stability of the flow. The velocity gradient across the interface tends to make the interface unstable (the greater the velocity change), while the density gradient tends to stabilize the interface. This results in a new dimensionless parameter:

$$Ri = \frac{\left(\frac{g}{\rho}\right)\left(\frac{d\rho}{dy}\right)}{\left(\frac{dU}{dy}\right)^2}$$

Called the Richardson number. For Ri<0.25, the bed is likely to be unstable.

**The Importance of Form Drag**

The reason we *care* about bedforms in this class is twofold: first, they impart drag on the flow, but drag *that does not result in sediment transport*. Only drag acting on the grains themselves causes sediment transport. So, when we attempt to do bedload calculations, we either have to pretend the bed is flat, *or* we have to rid ourselves of the part of drag caused by bedforms. The part caused by bedforms is often called *form drag*, and the part caused by the grains themselves is often called *skin friction* or *grain roughness* drag. Form drag often exceeds skin friction in magnitude. Take a look:



There’s a second thing, though. Because we’re geologists, one thing we’d like to be able to do is interpret past events. Bedforms are often preserved in sedimentary rocks (and occasionally in igneous and metamorphic rocks, too), and can give clues about past flow characteristics. We’ll talk about this more later. For now, suffice it to say we’d like to know some quantitative things about bedforms.

## Bedform Stability Diagrams

It would be nice if we could *predict* the bedform for any set of independent variables. Equally, if you can see the bedform (or the preserved remnant of that bedform), it would be nice to know something about the flow parameters. This sort of thing is the province of the bedform stability diagram.

First, nearly all bedform stability diagrams are empirical. Someone with a flume, or a natural channel, puts sediment in and sees what bedform occurs for given flow parameters. So, here’s my question: if all bedform stability diagrams are empirical, why do people still make new ones? Doesn’t it make sense that this would have been *done* by now?

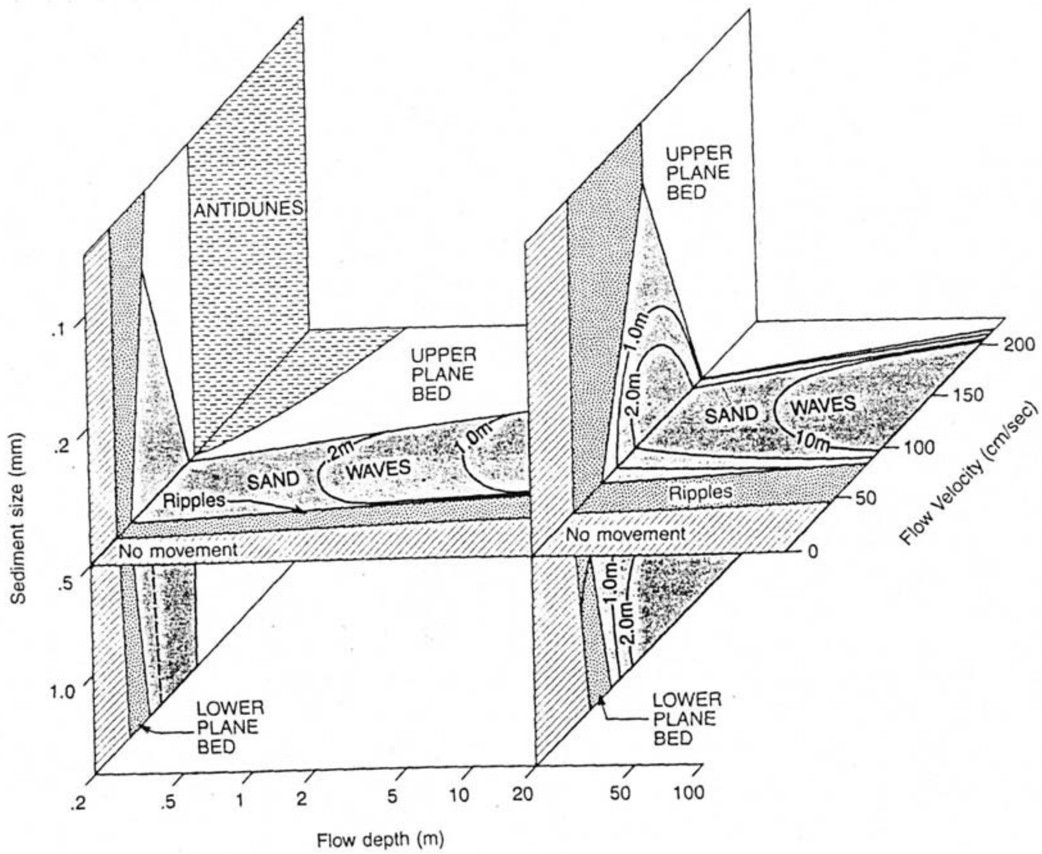
Here’s the answer. The big issue becomes what to use as axes. Let’s come up with a list of variables that may influence what bedform occurs:

| | | |
|---|---|---|
| $D$ | bed grain size (median and otherwise) | [L] |
| $U$ | flow velocity | $[LT^{-1}]$ |
| $h$ | flow depth | [L] |
| $g$ | acceleration of gravity | $[LT^{-2}]$ |
| $\rho_s$ | sediment density | $[ML^{-3}]$ |
| $\rho_f$ | fluid density | $[ML^{-3}]$ |
| $\mu$ | dynamic viscosity | $[ML^{-1}T^{-1}]$ |

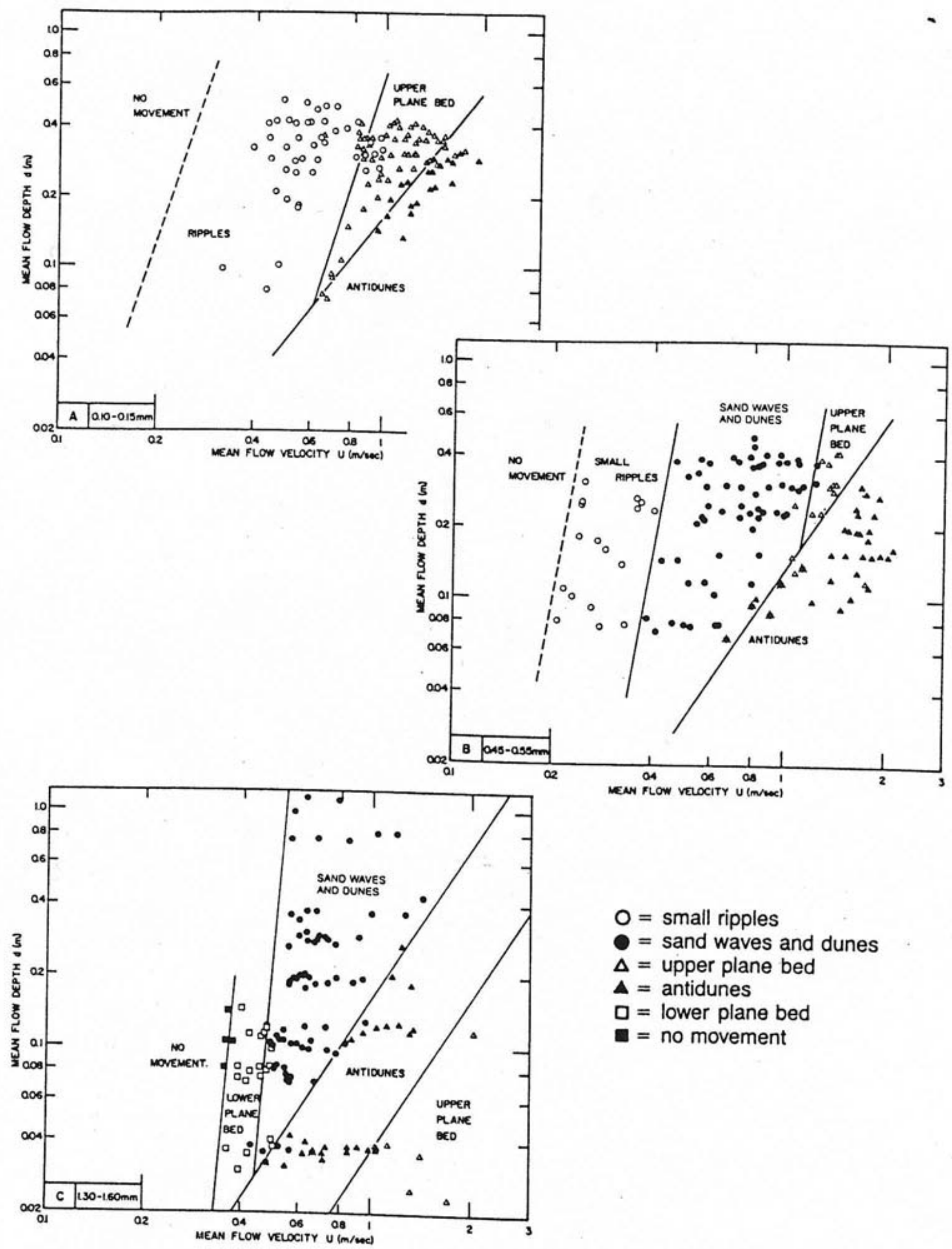
That’s seven variables, and three dimensions—we should end up with four dimensionless groups to characterize this system. One way of doing this is:

$$\frac{\rho_s}{\rho_f} \qquad U\sqrt[3]{\frac{\rho_f}{\mu g}} \qquad h\sqrt[3]{\frac{\rho_f^{\,2} g}{\mu^2}} \qquad D\sqrt[3]{\frac{\rho_f^{\,2} g}{\mu^2}}$$

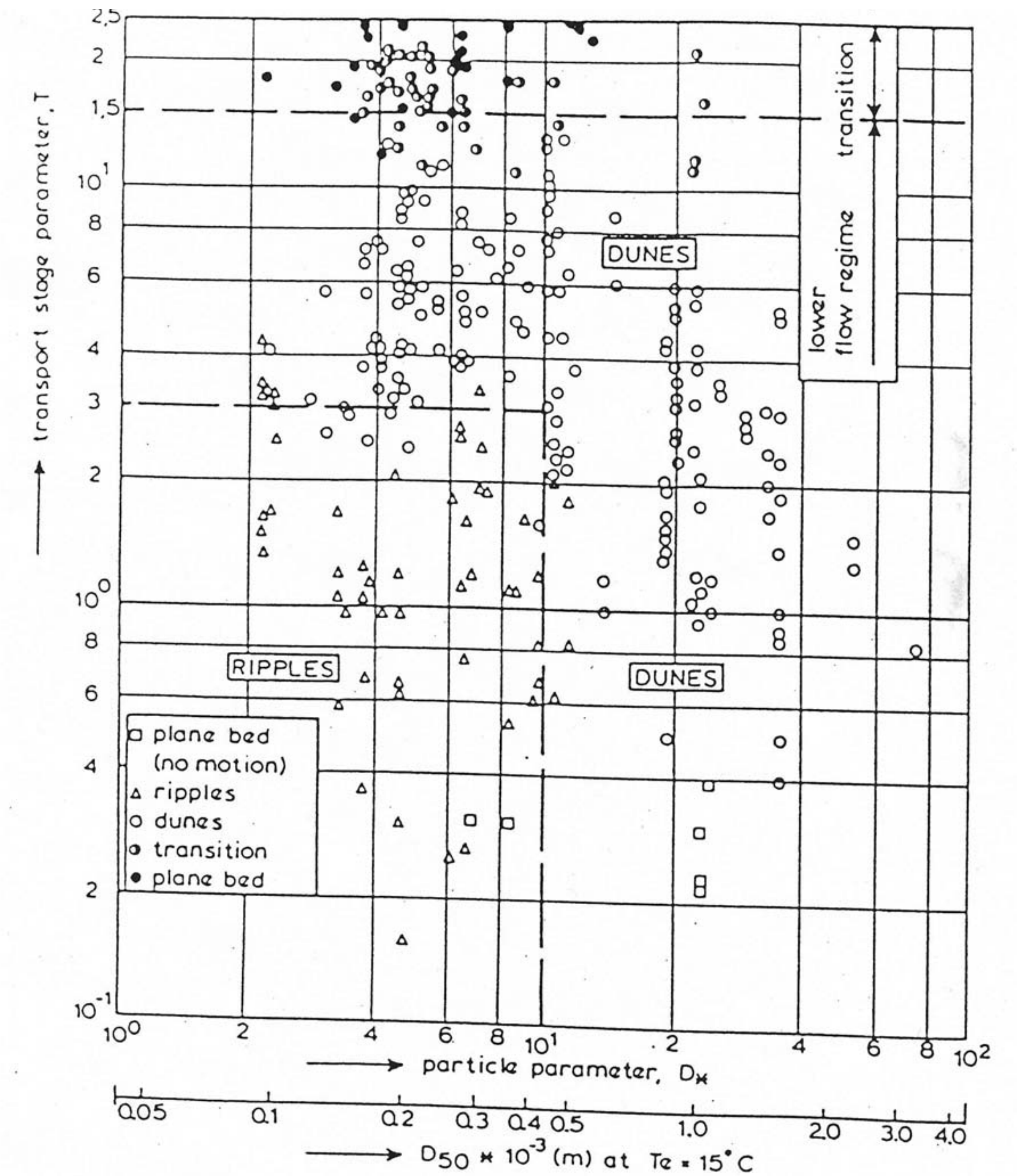
often called the density ratio, the dimensionless velocity, dimensionless depth, and dimensionless grain size, respectively. So, on the one hand, we could make a series of charts that attempt to demonstrate any two (or at best three) of these variables, *or* we could start simplifying the system. The easiest simplification would be to say that the sediment is constrained to be quartz, and therefore to have a constant density ratio. This removes one of our four groups. If we hold fluid density and viscosity constant (essentially we say that all the experiments are done in water of a given temperature) and we claim that we're only doing this on Earth, then we're down to just three groups, and we could even have those be dimensional—they're just *U*, *h*, and *D*. This is why most bedform stability charts are dimensional, and why most of them plot any two of these axes. After that, it's a crap shoot. Here are three end members. The first is by Rubin and McCulloch and is one of the nicest schematic diagrams out there.




You can see qualitatively that ripples become less common for larger grain sizes, and that dune wavelength increases as a function of depth. The problem here is only that it's hard to interpolate between the parts they've given. If you don't have 0.5 mm sand, it's going to be hard to use this graph (unless you hit a flow depth of 0.2 m or 20 m). For people who really need quantitative data, some variant on this graph, by Harms, Southard, and Walker, is used.




Instead of a smooth function as happens in nature, these guys use a sort of piecewise lumping of similar grain sizes. Effectively, these are a series of horizontal slices through Rubin and McCullough's diagram. Great! Now we've at least got a better diagram to work from. If you want one that includes density effects, probably the best one is a Delft Hydraulcs product, Van Rijn's bedform predictor.




Here $D_*$ is familiar to you, but $T$ is not. This is the *transport stage parameter* and is defined most simply as:

$$T = \frac{(\theta - \theta_c)}{\theta_c}$$

Where $\theta$ is the Shields' parameter for the flow, and $\theta_c$ is the critical Shields' parameter. Effectively, this is saying how much *extra* shear stress the flow has over what it needs just to move the grains. Although van Rijn is somewhat cumbersome, it gets used in part because of its easy integration into bedload transport models that *also* rely on excess shear. More on this later…

Perhaps the nicest set of diagrams comes from Southard and Boguchwal. They're so nice, in fact, that you're going to have to read a paper about them.