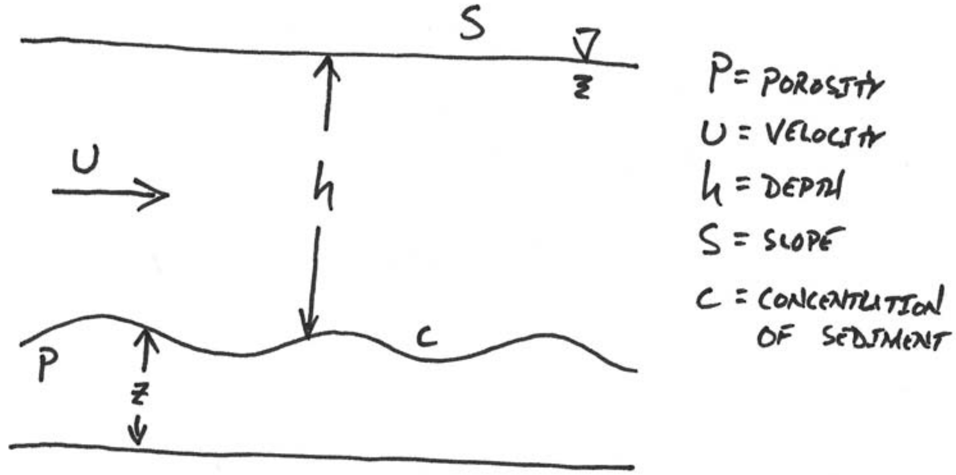
Lecture 20a—The Exner Equation

This is one of those “a” lectures. I would be remiss if I didn’t introduce you to the Exner equation, which is an *extremely* common way of approaching sediment transport theoretically. On the *other* hand, the math is a little dicey, and in the end, it only talks about things as a sort of theory, and that doesn’t tend to fly well here. As a result, I present to you, *for your notes only*, the Exner equation.

Equations for unsteady sediment transport in a wide rectangular channel



For the fluid, we have:

Continuity:

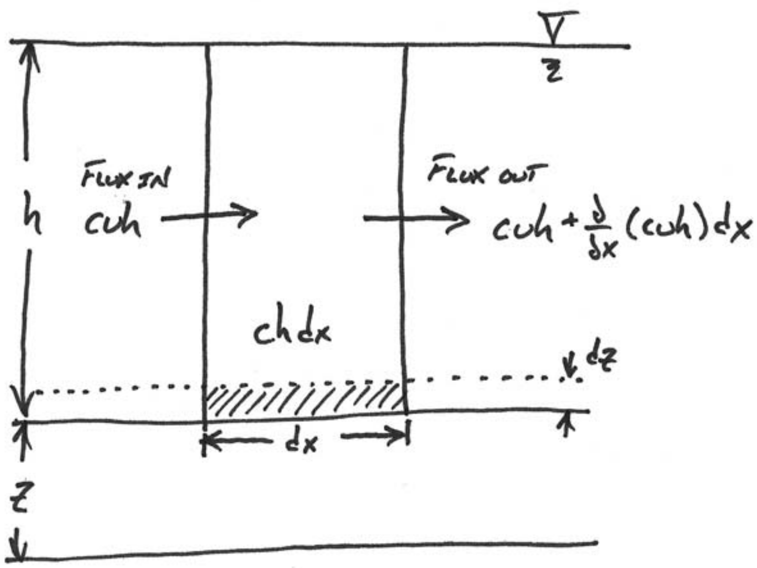
$$\frac{\partial h}{\partial t}dx = -\frac{\partial}{\partial x}(uh)dx$$

Momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \text{acceleration}$$

But we need more relationships to deal with the sediment.

Material gets stored in two places in the water column
- --on the bottom (as deposition)
- --in suspension



SO

$$(1-P)\frac{\partial z}{\partial t}dx + \frac{\partial}{\partial t}(ch)dx = -\frac{\partial}{\partial x}(cuh)dx$$

since $c=q_s/q$, $q=uh$, $q_s=cuh$

and as a special case,

$$(1-P)\frac{\partial z}{\partial t} = -\frac{\partial q_s}{\partial x}$$

when suspended sediment is minimal.

We could make this a bedform by taking,

$$\eta(x,t) = f(x-ct)$$

integrating w/ respect to time yields:

$$N\frac{\partial \eta}{\partial t} = n(-c)f'(x-ct) = -\frac{\partial q_s}{\partial x}$$

and with respect to x yields:

$$q_0 + ncf(x-ct) = q_s$$

so, if c>0, bedforms move downstream:

$q_s = q_{max}$ at $\eta = \eta_{max}$

$q_s = q_{min}$ at $\eta = \eta_{min}$

If c<0, bedforms moving upstream

$q_s = q_{max}$ at $\eta = \eta_{min}$

$q_s = q_{min}$ at $\eta = \eta_{max}$

$$q_{\max} - q_{\min} = nc(\eta_{crest} - \eta_{trough})$$