Lecture 26—Dimensional Analysis and Bed Load

The last category of bed load equations we'll look at are those derived by identifying "functional groups" and assessing their effect on bed load. Basically, the researcher comes up with a list of variables, eliminates those which overlap, then simplifies until a reduced set can be non-dimensionalized and used to make the equation. Sound familiar?

The approach we'll look at today comes from Brownlie (1981). For variables, Brownlie chose:

dependent: $c = \dfrac{g_b}{\rho Q + g_b}$

independent:

V	velocity
R	hydraulic radius
g	gravity
ρ	fluid density
ρ_s	sediment density
ν	fluid viscosity
σ_g	sediment standard deviation
D_{50}	median grain size
S	slope

From these, he identified the following groups:

$$c = f\left(F_g, \frac{R}{D_{50}}, S, \sigma_g, R_g, \frac{\rho_s - \rho}{\rho}\right)$$

where

$$F_g = \frac{V}{\sqrt{\left(\dfrac{\rho_s - \rho}{\rho}\right) g D_{50}}}$$

and through fitting to data, to the formula:

$$c = 7100 c_f \left(S^{\frac{1}{3}} F_g - S^{\frac{1}{3}} F_{g0} \right)^2 \left(\frac{R}{D_{50}} \right)^{\frac{-1}{3}}$$

where c_f=100 for the lab and 127 for the field.

F_{g0} is the grain Froude number in lower regime flow (flow Froude number < 1) at slope S which is at critical shear (*i.e.* the V,R combination that produces $\tau_0=\tau_c$).

$$F_{g0} = 4.6 \tau_c^{0.53} S^{-0.14} \sigma_g^{-0.16}$$

Note that Brownlie's equation ends up being very like Yang's stream power:

$$F_g S^{\frac{1}{3}} \propto V S^{\frac{1}{3}}$$

So, we've seen a cook's tour (or rogue's gallery) of approaches to bed load equations. How well do they do? The answer seems to be that you can have an equation that works equally poorly all of the time, or one that works really well some of the time, and really badly the rest of the time. Your decision to use a particular bed load equation, then, will come down to your experience and the knowledge you have about the way they work. As a last note, you might take a look at two papers:

Nakato, T., 1990, Tests of selected sediment-transport formulas, *Journal of Hydraulic Engineering*, v. 116, n. 3, p. 362-379.

and

Gomez, B., and Church, M., 1989, An assessment of bed load sediment transport formulae for gravel bed rivers, *Water Resources Research*, v. 25, n. 6, p. 1161-1186.