Lecture 27—An aside on hyperconcentrated flows

Today I'd like to talk for a bit about hyperconcentrated flows. You remember these—they include debris flows, mudslides, and the like. Although we don't have to worry overly about hyperconcentrated flows in Ohio rivers, in some rivers, particularly those based on loess or volcanic terrains, hyperconcentrated flows are the major way that sediment is moved through the river. Way back in Lecture 1 I claimed that we wouldn't be talking about hyperconcentrated flows. This was for a reason—when we talked about velocity profiles in rivers, I wanted to be able to assume that water is a Newtonian fluid. This led us to an expression for viscosity in water, which we later amended to include turbulent viscosity. Here's the problem with hyperconcentrated flows—they don't act like Newtonian fluids. Instead of deforming as soon as shear is applied, they don't deform at all until a critical value of shear is reached.

$$\tau = k + \mu \frac{du}{dy}$$

See? Instead of just $\tau = \mu \frac{du}{dy}$, there's another factor, *k*, the yield strength of the fluid. Look back to Lecture 3—it looks just like the line I drew for a Bingham plastic. Soooo, what are the consequences of this change?

The biggest is that the velocity profile changes dramatically. Remember that:

$$\tau = \rho g h S$$

and that shear stress varies linearly in the flow, from a maximum of τ_0 at the base (that's $\rho g h S$) to 0 at the surface. Sooo,

$$\tau = \rho g (h - y) S$$

Now, if we combine this equation with the definition of a Bingham plastic, we get:

$$\frac{du}{dy} = \frac{1}{\mu}\left[\rho g S(h-y) - k\right]$$

which we can integrate:

$$u = \frac{\rho g S}{\mu}\left(hy - \frac{y^2}{2}\right) - \frac{ky}{\mu}$$

The curious thing here is that when $h - y = \frac{k}{\rho g S}$, there's no velocity gradient—above this point the velocity is constant because the yield strength is greater than the shear stress. What develops, then, is a rigid plug that scoots along on top of the flow. This plug is capable of moving very large objects, and is one reason debris flows often have huge boulders in them, or forests of trees.

For completeness sake, the maximum velocity occurs at the base of the rigid plug, and is defined as:

$$u_{\max} = \frac{\rho g S\left(h - \frac{k}{\rho g S}\right)^2}{2\mu}$$

and the average velocity is:

$$U = \frac{\tau_0 h}{3\mu} - \frac{hk}{2\mu}$$

Here's a question, though—can debris flows become turbulent? The answer is yes, they can. The *problem* is that we can't use just Reynolds number anymore. The problem is that the Reynolds number required for turbulence will *increase* with increasing yield strength, because that rigid plug will keep a lid on turbulent effects. We need yet *another* number that talks about the relative importance of yield strength vs. viscous stress. This number is called Bingham number and looks like this:

$$B = \frac{kh}{\mu U}$$

The Reynolds number that marks the transition from laminar to turbulent flow is defined by taking the ratio of Reynolds to Bingham number:

$$\frac{R}{B} = \frac{\left({\rho h U}/{\mu} \right)}{\left({kh}/{\mu U} \right)} = \frac{\rho U^2}{k}$$

The transition from laminar to turbulent flow occurs when *this* number (called Hampton number) is about equal to 1000.

Here's an example of working with debris flows. Consider a debris flow 5 meters thick, moving down a slope of 0.01. The bulk density is 2200 kg/m^3, the yield strength is 300 Pa, and the dynamic viscosity is 100 Pa·s. Is the flow turbulent or laminar?

What is the thickness of the rigid plug?

What's the maximum velocity?

When will it stop moving?